STEP 3

Review the Knowledge You Need to Score High

CHAPTER

Motion in a Straight Line[1]

IN THIS CHAPTER

Summary: The entire goal of motion analysis is to describe, calculate, and predict where an object is, how fast it's moving, and how much its speed is changing. In this chapter you'll review two separate approaches to make these predictions and descriptions: graphs and algebra.

KEY IDEA

Definitions

- The cart's **position** (x) tells where the cart is on the track.
- The cart's **speed** (v) tells how fast the cart is moving.[2]
- **Acceleration** (a) tells how much the object's speed changes in one second. When an object speeds up, its acceleration is in the direction of its motion; when an object slows down, its acceleration is opposite the direction of its motion.
- **Displacement** (Δx) tells how far the object ends up away from its starting point, regardless of any motion in between starting and ending positions.
- The **graphical analysis of motion** includes **position-time graphs** and **velocity-time graphs**. On a position-time graph, the slope is the object's speed, and the object's position is read from the vertical axis. For velocity-time graphs, the speed is read from the vertical axis, and the slope is the object's acceleration.

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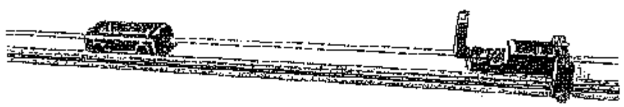
[1]We're discussing motion with constant acceleration; that covers pretty much all motion in a straight line in AP Physics 1 except things attached to springs.

[2]Strictly speaking, speed is the magnitude—the amount—of the velocity vector. Velocity tells how fast something moves, as well as in which direction it moves. I tend to use "speed" and "velocity" interchangeably, especially in this unit; the distinction between the two is not important here.

- The **five principal motion variables** are:
  - v_0 initial velocity
  - v_f final velocity
  - Δx displacement
  - a acceleration
  - t time
- In any case of accelerated motion, when three of the five principal motion variables are known, the remaining variables can be solved for using the kinematic equations.
- **Free fall** means no forces other than the object's weight are acting on the object.
- A **projectile** is an object in free fall, but it isn't falling in a straight vertical line. To approach a projectile problem, make two motion charts: one for vertical motion and one for horizontal motion.

Introduction to Motion in a Straight Line



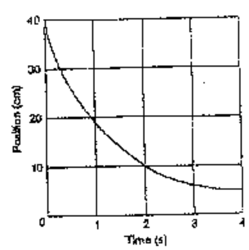
Pretty much all motion problems can be demonstrated with a cart on a track, like in the diagram above. The motion detector can read the location of the cart up to 50 times each second. This detector can make graphs of position or velocity versus time.

The entire goal of motion analysis is to describe, calculate, and predict where the cart is; how fast it's moving; and how much its speed is changing. You'll use two separate approaches to make these predictions and descriptions: graphs and algebra.

Graphical Analysis of Motion

Before you start any analysis, tell yourself which kind of graph you're looking at. The most common mistake in studying motion graphs is to interpret a velocity-time graph as a position-time graph, or vice versa.

Position-Time Graphs



Example 1: The preceding position-time graph represents the cart on the track. The motion detector is located at position $x = 0$; the positive direction is to the left.

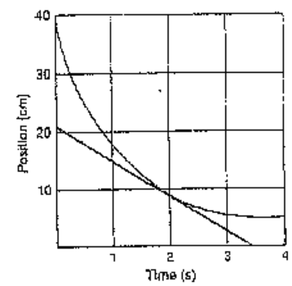
An AP Exam question could ask all sorts of questions about this cart. How should they be approached? Use these facts, and reason from them.

FACT: In a position-time graph, the object's position is read from the vertical axis.

Look at the vertical axis in Example 1. At the beginning of the motion, the cart is located 40 cm to the left of the detector.[3] After 2 s, the cart is located 10 cm left of the detector. Therefore, in the first 2 s of its motion, the cart moved 30 cm to the right.

FACT: In a position-time graph, the object's speed is the slope of the graph. The steeper the slope, the faster the object moves. If the slope is a front slash (/), the movement is in the positive direction; if the slope is a backslash (\), the movement is in the negative direction.

Wait a second, how can I tell the slope of a curved graph? Just look at how the graph is sloped at one specific place on the graph. If I want to know how fast the object is moving after 2 seconds of motion, I take the slope drawn in the following figure.



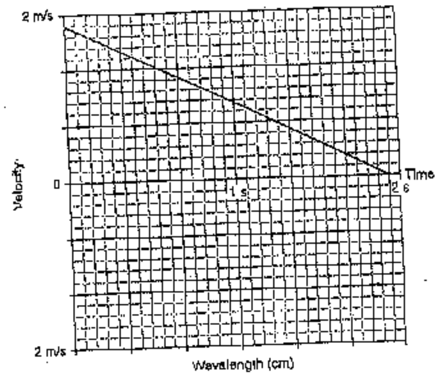
Mathematically, this means you divide (change in y-value)/(change in x-value) to get a speed that's a bit less than 6 cm/s. The exact calculation is not something to obsess over. It's more important to think in terms of comparisons.

You're not often going to be asked "Calculate the speed of the cart at $t = 2$ s." Instead, you'll be asked to describe the motion of the cart in words and to justify your answer. When you describe motion, use normal language that your grandparents would understand. Avoid technical terms like "acceleration" and "negative." Justify your answer with direct reference to the facts.

Referring back to Example 1, because the slope was steeper at earlier times and shallower at later times, the cart must be slowing down. The car is moving to the right the whole time, because the slope is always a backslash.

[3] Why not 40 cm to the right of the detector? Because the position value is +40 cm and the positive direction is left.

Velocity-Time Graphs



Example 2: The preceding velocity-time graph represents a different cart on the track. The positive direction is to the left.

FACT: In a velocity-time graph, the object's speed is read from the vertical axis. The direction of motion is indicated by the sign on the vertical axis.

In Example 2, at the beginning of the motion, the vertical axis reads 1.8 m/s. This means that initially, the cart was moving 1.8 m/s to the left. After one second, the cart was moving about 0.8 m/s. A bit less than two seconds into the motion, the vertical axis reads zero, so the cart stopped.

FACT: In a velocity-time graph, the object's acceleration is the slope of the graph.

You could do the rise/run calculation to find the amount of the acceleration, or you could use the definition of acceleration to see that the object lost 1 m/s of speed in one second, making the acceleration 1 m/s per second.[4] The cart in Example 2 was slowing down and moving to the left. When an object slows down, its acceleration is opposite the direction of its motion; this cart has an acceleration to the right.

The Mistake

Acceleration is *not* the same thing as speed or velocity. Speed says how fast something moves; acceleration says how quickly speed changes. Acceleration doesn't say anything about which way something is moving, unless you know whether the thing is speeding up or is slowing down.

[4]Textbooks and problems on the AP Exam will write this as 1 m/s². Well, that's silly—what the heck is a "second squared," anyway? When you see that notation, read it as "meters per second per second." I suggest you always write the units of acceleration as "m/s per second." Then you'll be far less likely to make "The Mistake."

Someone who says, "This car has an acceleration of 4 m/s per second, so it is moving at about a jogging pace," has made "The Mistake." The car is speeding up or slowing down by 4 m/s every second; the car could well be an Indy 500 racecar traveling 94 m/s right now, but only 90 m/s a second later.

Someone who says "The cart in Figure 2 has a negative acceleration, so it is moving to the right" has made "The Mistake." An acceleration to the right means either speeding up and moving right, or slowing down and moving left. While the cart's acceleration is negative—after all, the slope of the line is a backslash—the car was slowing down, making the velocity's direction opposite the acceleration's direction. The acceleration is right, and the velocity is left.

It takes a lot of practice to avoid "The Mistake." Just continually remind yourself of the meaning of acceleration (how much an object's speed changes in one second), and you'll get there.

FACT: The object's displacement is given by the area between the graph and the horizontal axis. The location of the object can't be determined from a velocity-time graph; only how far the object ended up from its starting point can be determined.

To find how far the cart in Example 2 moved, take the area of the triangle in the graph,[5] giving about 1.6 m. Since the cart's velocity as read from the vertical axis was positive that whole time, and the positive direction is left, the cart ended up 1.6 m left of where it started. But exactly where it started, no one knows.

Algebraic Analysis of Motion

Example 3: A model rocket is launched straight upward with an initial speed of 50 m/s. It speeds up with a constant upward acceleration of 2.0 m/s per second until its engines stop at an altitude of 150 m.

Sometimes you'll be asked to analyze motion from a description and not a graph. Start your analysis by defining a positive direction and clearly stating the start and the end of the motion you're considering. For example, take the upward direction as positive,[6] and consider from the launch to when the engines stop.

Next, *make a chart* giving the values of the five principal motion variables. Include a plus or minus sign on every one (except time—a negative time value means you're in a Star Trek–style movie). If a variable isn't given in the problem, leave that variable blank.

The Five Principal Motion Variables for Your Chart

v_0 initial velocity
v_f final velocity
Δx displacement
a acceleration
t time

[5]The area of a triangle is (1/2) base × height.
[6]Could I have called the downward direction positive? Sure. Then signs of displacement, velocity, and acceleration would all be switched.

For Example 3, the chart looks like this:

v_0	+50 m/s
v_f	
Δx	+150 m
a	+2.0 m/s per second
t	

The acceleration is positive because the rocket was speeding up; therefore, acceleration is in the same direction as the motion, which was upward. Upward was defined as the positive direction here.

KEY IDEA

FACT: In any case of accelerated motion when three of the five principal motion variables are known, the remaining variables can be solved for using the kinematic equations.

In Example 3, we know three of the five motion variables; therefore, we can find the others, and the physics is *done*.

Whoa there. Um, how is the physics "done"? Don't we have to plug the numbers in to the kinematic equations, which incidentally you haven't mentioned yet?

Well, remember the AP Physics 1 revolution: While you will occasionally be asked, say, to calculate how much time the engines run for, you'll just as often be asked something that doesn't involve calculation. For example, "Is it possible to determine the running time of the engines?" Or, "When the engines have run for half of their total run time, is the rocket at a height greater than, less than, or equal to 75 m?"[7]

More to the point, actually doing the math here is, well, a *math* skill not a physics skill. As long as your answers are reasonable—a model rocket will likely burn for a few seconds, not a few thousands of seconds—the exam is likely to award close to full, or sometimes even full, credit for a correct chart and for recognizing the correct equation to use.

KEY IDEA

FACT: To calculate the missing values in a motion chart, use the three kinematic equations listed as follows. Choose whichever equation works mathematically. Never solve a quadratic equation. If the math becomes overly complicated, try solving for a different missing variable first.

Kinematic Equations

1. $v_f = v_0 + at$
2. $\Delta x = v_0 t + \frac{1}{2}at^2$
3. $v_f^2 = v_0^2 + 2a\Delta x$

[7]The answer is *less than* 75 m. The rocket is speeding up throughout the time when the engines burn. In the second half of the burn time, the rocket is (on average) moving faster, and so it covers more distance.

Continuing with Example 3, we can use equation (3) to solve for the final velocity of the rocket[8]; this is about 56 m/s.[9] Then we can use equation (1) to get the time before the engines shut off, which is 3 seconds. If you had tried to use equation (2) to solve for time, you would have gotten a quadratic; that's why I said to use equation (3) and then (1).

Objects in Free Fall

KEY IDEA

FACT: When an object is in free fall, its acceleration is 10 m/s per second[10] toward the ground. "Free fall" means no forces other than the object's weight are acting on the object.

Let's do more with the rocket in Example 3. When the engines stop, the rocket is moving upward at 56 m/s. The rocket doesn't just stop on a dime. It keeps moving upward, but it slows down, losing 10 m/s of speed every second.

Try making a chart for the motion from when the engines stop to when the rocket reaches the peak of its flight. We'll keep the positive direction as upward.

v_0	+56 m/s
v_f	0 (The peak of flight is when the object stops to turn around.)
Δx	
a	−10 m/s per second (There is negative acceleration because free-fall acceleration is always *down*.)
t	

Three of the five variables are known so the physics is *done*.

Now, be careful that you keep grounded in what's physically happening, not in the algebra. For example, you might be asked for the maximum height that the rocket in Example 3 reaches.

Good job recognizing that you need equation (3) to solve for Δx. Dropping the units during the calculation, that gives $0^2 = 56^2 + 2(-10)(\Delta x)$. Solving with a calculator you get about 160 m for Δx.

Wait a second! Think what the 160 m answer means—that's the distance the rocket goes between when the engines stop and when the rocket reaches its highest height. That's not the height above the ground, just the additional height after the engines stop! The actual maximum height is this 160 m, plus the 150 m that the rocket gained with its engines on, for a total of 310 m.

If you were blindly plugging numbers into equations, you would have totally missed the meaning behind these different distances. The AP Physics 1 Exam will repeatedly ask targeted questions that check to see whether you understand physical meaning. Calculation? Pah. In comparison, it's not so important.

Projectile Motion

A *projectile* is defined as an object in free fall. But this object doesn't have to be moving in a straight line. What if the object were launched at an angle? Then you treat the horizontal and vertical components of its motion separately.

[8]This calculation requires a calculator, of course: $v_f^2 = (50\ m/s)^2 + 2(2\ m/s\ per\ second)(150\ m)$. See, here's another reason you're probably not going to have to actually carry out this math—you're not likely to need a calculator more than a few times on the entire exam.

[9]Not 55.67764363 m/s. Don't make me ask your chemistry teacher to talk to you about significant figures again. Just use two or three figures on all values, and we'll all be happy.

[10]No, stop it with the 9.8 m/s per second. The CollegeBoard is very clear that you can and should use a free-fall acceleration of 10 m/s per second. Really. Limited calculator use, remember?

Example 4: A ball is shot out of a cannon pointed at an angle of 30° above the horizontal. The ball's initial speed is 25 m/s. The ball lands on ground that is level with the cannon.

KEY IDEA

FACT: A projectile has no horizontal acceleration and so moves at constant speed horizontally. A projectile is in free fall, so its vertical acceleration is 10 m/s per second downward.

To approach a projectile problem, make *two* motion charts: one for vertical motion and one for horizontal motion.

KEY IDEA

FACT: To find the vertical component of a velocity at an angle, multiply the speed by the sine of the angle. To find the horizontal component of a velocity at an angle, multiply the speed by the cosine of the angle. This always works, as long as the angle is measured from the horizontal.

Here are the two charts for this ball's motion in Example 4. Consider up and right to be the positive directions. Let's consider the motion while the ball is in free fall—that means, starting right after the ball was shot, and ending right before the ball hits the ground. Note that the initial vertical velocity is (25 m/s)(sin 30°) = 13 m/s. The initial horizontal velocity is (25 m/s)(cos 30°) = 22 m/s. You needed to use your calculator to get these values.

Vertical	Horizontal
v_0 + 22 m/s	v_0 + 13 m/s
v_f	v_f
Δx 0	Δx
a −10 m/s per second	a 0
t	t

Two entries here are tricky. Remember that displacement only means the distance traveled start to end, regardless of what happens in between. Well, this ball landed on "level ground." That means that the ball ends up at the same vertical height from which it was shot; it didn't end up any higher or lower than it started. Thus, vertical displacement is zero.

Second, the final vertical velocity is unknown, not zero. Sure, once the ball hits the ground it stops; but then it's not in free fall anymore. The "final" velocity here is the velocity in the instant before the ball hits the ground.

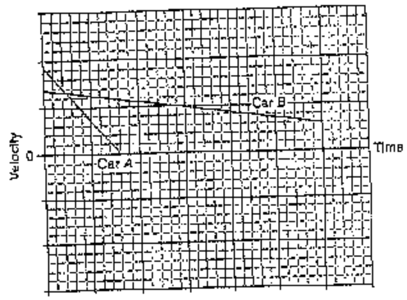
KEY IDEA

FACT: The horizontal and vertical motion charts for a projectile must use the same value for time.

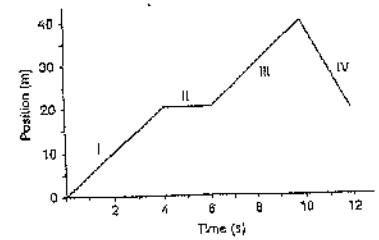
The vertical chart is completely solvable, because three of the five variables are identified. Once the time of flight is calculated from the vertical chart, that time can be plugged into the horizontal chart, and *voila*, we have three of five horizontal variables identified; the chart can be completed.

Practice Problems

Note: Extra drills on describing motion based on graphs can be found in Chapter 18.



1. The velocity-time graphs represent the motion of two cars, Car A and Car B. Justify all answers thoroughly.
   (a) Which car is moving faster at the start of the motion?
   (b) Which car ends up farther from its starting point?
   (c) Which car experiences a greater magnitude of acceleration?



2. The following questions refer to the preceding position-time graph, which is the readout of an eastward-pointing motion detector. Justify all answers thoroughly.
   (a) Rank the speed of the object in each of the four labeled regions of the graph, from fastest to slowest. If the object has the same speed in two or more regions, indicate so in your ranking.
   (b) What total distance did the object travel in the 12 s, including all parts of the motion?
   (c) How far from the object's starting point did the object end up after the 12 s?
   (d) Which of the following objects could reasonably perform this motion?
      (A) A baby crawling
      (B) A sprinter
      (C) A car on the freeway
      (D) A jet airplane during takeoff
      (E) An amoeba in a petri dish
3. A ball is dropped from rest near Earth. Neglect air resistance.[11] Justify all answers thoroughly.
   (a) About how far will the ball fall in 3 s?
   (b) The same ball is dropped from rest by an astronaut on the Moon, where the free-fall acceleration is one-sixth that on Earth. In 3 s, will the ball on the moon fall:
      (A) One-sixth as far as the ball on Earth
      (B) One-36th as far as the ball on Earth
      (C) The same distance as the ball on Earth
      (D) Six times as far as the ball on Earth
      (E) Thirty-six times as far as the ball on Earth

Data Table

Projectile	Initial Horizontal Speed (m/s)	Initial Vertical Speed (m/s)	Time of Flight (s)
A	40.0	29.4	6.00
B	60.0	19.6	4.00
C	50.0	24.5	5.00
D	60.0	19.6	4.00

4. Four projectiles, A, B, C, and D, were launched from and returned to level ground. The preceding data table shows the initial horizontal speed, initial vertical speed, and time of flight for each projectile. Justify all answers thoroughly.
   (a) Rank the projectiles by the horizontal distance traveled while in the air.
   (b) Rank the projectiles by the maximum vertical height reached.
   (c) Rank the projectiles by the magnitude of their acceleration while in the air.

[11] *Always* neglect air resistance, unless it is extremely, abundantly, and unambiguously clear from the problem's context that air resistance is important (i.e., talking about "terminal velocity").

Solutions to Practice Problems

1. (a) On a velocity-time graph, speed is read off of the vertical axis. At time = 0, Car A has a higher vertical axis reading than Car B, so Car A is moving faster.
   (b) Displacement is determined by the area under a velocity-time graph. Car A's graph is a small triangle; Car B's graph is a trapezoid of obviously larger area. Both cars' graphs are always above the horizontal axis, so both cars move in the same direction the whole time; Car B moves farther away from its starting point.
   (c) Acceleration is the slope of the velocity-time graph. Car A's graph is steeper, so its acceleration is larger. [Sure, the slope of Car A's graph is negative, but that just means acceleration is in the negative direction, whatever that is; the question asks for the "magnitude" of the acceleration, meaning the amount, regardless of direction.]

2. (a) IV > I = III > II. Speed is the steepness on a position-time graph, without reference to direction (i.e., whether the slope is positive or negative). Segment IV is steepest. Segments I and III seem to be the same steepness. Segment II has 0 slope, so it represents an object that doesn't move.
   (b) It's a position-time graph, so read the vertical axis to figure out where the object is at any time. The object travels from its original position at $x = 0$ m to $x = 40$ m, then backtracks another 20 m. The total distance traveled is 60 m.
   (c) It's a position-time graph, so read the vertical axis to figure out where the object is at the beginning and after 12 s. At the beginning the object was at $x = 0$ m; after 12 s, the object was at position $x = 20$ m. The object traveled 20 m. (If your justification didn't explicitly mention that the object started at $x = 0$ m, or that you must find the *difference* between the final and initial positions, then it's incomplete.)
   (d) At its top speed in segment IV, the object travels 20 m in about 2 s. That's a speed of 10 m/s. If you're familiar with track and field, you'll know that the best sprinters run the 100-m dash in somewhere in the neighborhood of 10 s, so the sprinter is an obvious choice. It might be easier[12] to approximate a conversion to miles per hour. The result of 1 m/s is a bit more than 2 miles per hour. This object goes between 20 and 25 miles per hour. This is the speed of a car on a neighborhood street. There is no way a baby or an amoeba can keep up; takeoff speeds for most airplanes are at least in the high tens of miles per hour; and you'd be a danger to yourself and others if you drove on the freeway at 25 miles per hour.

3. (a) Use the equation $\Delta x = v_0 t + \frac{1}{2}at^2$ with $v_0 = 0$ and $a = 10$ m/s per second. You should get about 45 m.
   (b) In the equation we use in (a), the time of 3 s is still the same, as is v_0. The only difference is the acceleration a, which is in the numerator and is neither squared nor square rooted. Therefore, reducing a by one-sixth also reduces the distance fallen by one-sixth. That's choice A. (By the way, if your answer is A but your justification included "setting up a proportion" or anything without specific reference to this equation, your answer is incorrect.)

4. (a) Horizontal speed remains constant throughout a projectile's flight. Use $\Delta x = v_0 t + \frac{1}{2}at^2$ horizontally with the acceleration term equal to zero. That means you're multiplying the horizontal speed by the time of flight. This gives D > C > A = B.
   (b) Regardless of the time of flight, the vertical speed is directly related to the maximum height reached. Why? Use $v_f^2 = v_0^2 + 2a\Delta x$ vertically with $v_f = 0$ and $a = 10$ m/s per second. The bigger the v_0, the bigger the Δx. So A > C > B = D.
   (c) Easy—all objects in free fall have a downward acceleration of 10 m/s per second. A = B = C = D.

[12] For an American, who doesn't usually know from meters per second (m/s), anyway. If you'd prefer km/hr, multiply speeds in m/s by about 4 to get km/hr.

Rapid Review

- In a position-time graph, the object's position is read from the vertical axis.
- In a position-time graph, the object's speed is the slope of the graph. The steeper the slope, the faster the object moves. If the slope is a front slash (/), the movement is in the positive direction; if the slope is a backslash (\), the movement is in the negative direction.
- In a velocity-time graph, the object's speed is read from the vertical axis. The direction of motion is indicated by the sign on the vertical axis.
- In a velocity-time graph, the object's acceleration is the slope of the graph.
- In a velocity-time graph, the object's displacement is given by the area between the graph and the horizontal axis. The location of the object can't be determined from a velocity-time graph; only how far it ended up from its starting point can be determined.
- In any case of accelerated motion when three of the five principal motion variables are known, the remaining variables can be solved for using the kinematic equations.
- To calculate the missing values in a motion chart, use the three kinematic equations listed below. Choose whichever equation works mathematically.

  (1) $v_f = v_0 + at$

  (2) $\Delta x = v_0 t + \dfrac{1}{2}at^2$

  (3) $v_f^2 = v_0^2 + 2a\Delta x$

When an object is in free fall, its acceleration is 10 m/s per second[16] toward the ground. "Free fall" means no forces other than the object's weight are acting on the object.

A projectile has no horizontal acceleration, and so it moves at constant speed horizontally. A projectile is in free fall, so its vertical acceleration is 10 m/s per second downward.

To find the vertical component of a velocity at an angle, multiply the speed by the sine of the angle. To find the horizontal component of a velocity at an angle, multiply the speed by the cosine of the angle. This always works, as long as the angle is measured from the horizontal.

The horizontal and vertical motion charts for a projectile must use the same value for time.

CHAPTER 10

Forces and Newton's Laws

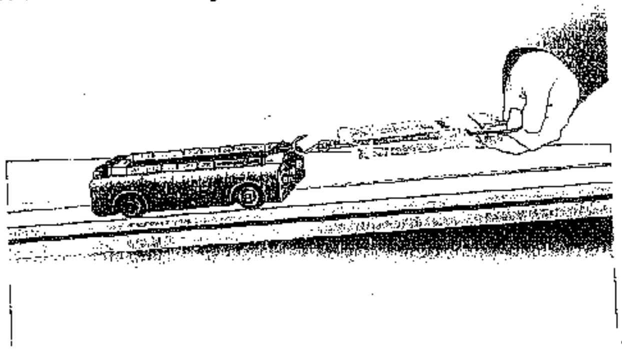
IN THIS CHAPTER

Summary: A force is a push or a pull applied by one object on another object. This chapter describes the construction and use of free-body diagrams, which are key to approaching problems involving forces.

Definitions

- A **force** is a push or a pull applied by one object and experienced by another object.
- The **net force** on an object is the single force that could replace all the individual forces acting on an object and produce the same effect. Forces acting in the same direction add together to determine the net force; forces acting in opposite directions subtract to determine the net force.
- **Weight** is the force of a planet on an object near that planet.
- The **force of friction** is the force of a surface on an object. The friction force acts parallel to the surface. **Kinetic friction** is the friction force when something is moving along the surface and acts opposite the direction of motion. **Static friction** is the friction force between two surfaces that aren't moving relative to one another.
- The **normal force** is also the force of a surface on an object. The normal force acts perpendicular to the surface.
- The **coefficient of friction** is a number that tells how sticky two surfaces are.
- **Newton's third law** says that the force of Object A on Object B is equal in amount and opposite in direction to the force of Object B on Object A.
- **Newton's second law** states that an object's acceleration is the net force it experiences divided by its mass, and is in the direction of the net force.

Describing Forces: Free-Body Diagrams



A force is a push or a pull applied by one object and experienced by another object. A force in the laboratory is often measured by a spring scale, as in the preceding picture. In AP Physics 1 we have to understand two aspects of forces. First, we have to describe the application of the force: What are the objects involved, and how much force is applied and in which direction? Next, we have to connect the net force acting on an object to that object's change in velocity.

Start with correct language: an object can "experience" a force, but an object cannot "have" a force. Don't let yourself say, "Ball A has a bigger force than Ball B"—that means nothing. "The net force on Ball A is bigger than on Ball B" is fine, as is "The Earth pulls harder on Ball A than on Ball B."

The canonical method of describing forces acting on an object is to draw a free-body diagram. A free-body diagram should include two elements:

(1) A labeled arrow representing each force, with each arrow beginning on the object and pointing in the direction in which the force acts
(2) A list of all the forces acting on the object, indicating the object applying the force and the object experiencing the force

On the AP Exam you'll be asked something like, "Draw and label the forces (not components) that act on the car as it slows down." This means "Draw a free-body diagram."

KEY IDEA

FACT: Only gravitational and electrical forces can act on an object without contact.[1]

Example 1: A car moving to the right on the freeway applies the brakes and skids to a stop.

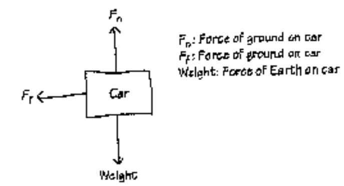
Pretty much always start with the force of the Earth on the object, which is commonly known as its weight. Don't call this force "gravity"—that's an ambiguous term. Weight acts downward and doesn't require any contact with the Earth in order to exist.[2] Draw a downward arrow on the dot, label it "weight," and in the list write "Weight: force of Earth on the car."[3]

[1]In AP Physics 1, anyway.
[2]Chapter 15, on gravitation, explains more about how to find the weight of an object in a gravitational field.
[3]Why not call it "force of gravity on the car?" Well, because all forces must be exerted by an *object* on another object. Since when is "gravity" an object? ☺

Any other forces must be a result of contact with the car. What's the car touching? It's touching just the ground. Since the car is touching the ground, the ground exerts a normal force perpendicular to the surface. Draw an upward arrow on the dot, label it something like "F_n," and in the list write "F_n: The force of the ground on the car."[4]

Since the car is sliding along the ground, the ground exerts a force of (kinetic) friction. By definition, kinetic friction must always act in the opposite direction of motion—the car skids right along the ground, the friction force acts to the left. Draw a leftward arrow on the dot, label it "F_f," and in the list write "F_f: The force of the ground on the car."[5]

The car is not in contact with anything else, so we're done.



Whoa there! The car is moving to the right, so what about the force of its motion?

There's no such thing as the "force of motion." All forces must be exerted by an identifiable object; and all nongravitational and nonelectrical forces must be a result of contact. The car is not in contact with anything that pushes the car forward.

Then how is the car moving to the right?

It just is. It is critically important to focus *only* on the problem as stated. Questions about what happened before the problem started are irrelevant. Perhaps at first the car was pushed by the engine, or pulled by a team of donkeys, to start it moving; perhaps it had been in motion since the beginning of time. It doesn't matter. All that matters is that when we tune in to the action, the car is moving right and slowing down.

If the car had been pulled by a team of donkeys to start it moving, wouldn't we put the force of the donkeys on the car on the free-body diagram?

No, because the free-body diagram includes only forces that act *now*, not forces that acted earlier, or forces that will act in the future. If donkeys pulled the car, the force of the donkeys would appear on the free-body while the donkeys were actually pulling. After they let go and the car is slowing down, the donkeys might as well have never existed.

While it's important to learn how to draw a free-body diagram, it's just as important to learn how to *stop* drawing a free-body diagram. Don't make up forces. Unless you can clearly identify the source of the force, don't include the force.

[4]It makes no difference what you label the arrow, as long as you define the label in a list. You want to call it N instead of F_n? Be my guest.
[5]Yes, both the friction force and the normal force are properly listed as the force of the ground on the car.

Exam Tip from an AP Physics Veteran

If you see a problem involving forces, try drawing a free-body diagram for each object in the problem, or for a system including multiple objects. A free-body diagram will always be useful, even if you're not explicitly asked to make one.

Determining the Net Force

To determine the net force on an object, treat each direction separately. Add forces that point in the same direction; subtract forces that point in opposite directions. Or, if you know the acceleration in a direction, use $F_{net} = ma$.

FACT: When an object moves along a surface, the acceleration in a direction perpendicular to that surface must be zero. Therefore, the net force perpendicular to the surface is also zero.

In Example 1, the net force horizontally is equal to the force of friction, because that's the only force acting in the horizontal direction—there's no other force to add or subtract. Vertically, the net force is equal to the normal force minus the weight. But since the car is moving along the surface, the vertical acceleration and the vertical net force on the car are zero.

We can conclude, then, that the normal force on the car is equal to the car's weight. This isn't a general fact, though—a normal force is *not* always equal to an object's weight. If more vertical forces are acting, or if the surface is changing speed vertically (as in an elevator), the normal force can be different from the weight.

FACT: The friction force is equal to the coefficient of friction times the normal force.

$$F_f = \mu F_n$$

A good AP question might describe a second car, identical in mass and initial speed to the car in Example 1, but on a wet freeway. The question might ask you to explain why this second car skids to a stop over a longer distance.

The coefficient of friction is a property of the surfaces in contact. Here, since a wet road is less "sticky" than a dry road, the coefficient of friction has decreased. But since the second car is identical to the first, its weight and thus the normal force of the surface on the car is the same as before. Therefore, by the equation $F_f = \mu F_n$, the new car experiences a smaller force of friction.

With a smaller net force on the second car, its acceleration is also smaller by $F_{net} = ma$. Then the distance traveled during the skid depends on the car's acceleration by the kinematics equation (3), $v_f^2 = v_0^2 + 2a\Delta x$. Take the initial speed v_0 to zero and solve for Δx to see that acceleration a is in the denominator of the equation. Thus, a smaller acceleration means a larger distance to stop.

Static and Kinetic Friction

You may have learned that the coefficient of friction takes two forms: static and kinetic friction. Use the coefficient of static friction if something is stationary, and the coefficient of kinetic friction if the object is moving. The equation for the force of friction is essentially the same in either case: $F_f = \mu F_N$.

The only strange part about static friction is that the coefficient of static friction is a *maximum* value. Think about this for a moment—if a book just sits on a table, it doesn't need any friction to stay in place. But that book won't slide if you apply a very small horizontal pushing force to it, so static friction can act on the book. To find the maximum coefficient of static friction, find out how much horizontal pushing force will just barely cause the book to move; then use $F_f = \mu F_N$.

Newton's Third Law

FACT: The force of Object A on Object B is equal in amount and opposite in direction to the force of Object B on Object A. These two forces, which act on different objects, are called Newton's third law companion forces.[6]

In Example 1, then, what's the Newton's third law companion force to the normal force? It's tempting to say, "Oh, the weight." After all, the weight is equal to the normal force and is opposite in direction to the normal force. But that's wrong.

To find the companion force, look at the description of the force in the free-body diagram, and reverse the objects applying and experiencing the force. The normal force is the force of the ground on the car, and that acts upward. Therefore, the third law companion force is the force of the car on the ground, acting downward.

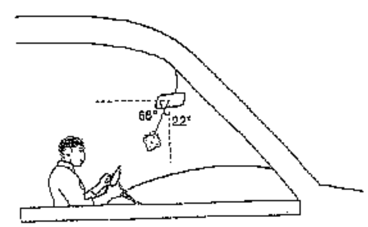
FACT: If the net force has both a vertical and a horizontal component, use the Pythagorean theorem to determine the magnitude of the net force, and use the tangent function to determine the direction of the net force.

In most AP problems, though, the net force will be zero in one or both directions. In Example 1, the magnitude of the net force is equal to only the magnitude of the friction force, because the vertical forces must subtract to zero.

If this were an AP problem, chances are it would ask about the connection between the net force and the change in the object's speed. We'll revisit this example later.

Forces at Angles

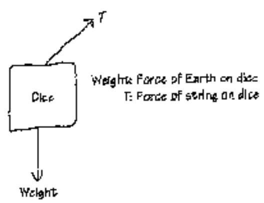
A force at an angle is drawn on a free-body diagram just like any other force. But when you're ready to do any analysis on the free-body diagram, start by breaking the angled force into components.



[6] Or, sometimes, this is called a Newton's third law force pair.

Example 2: A pair of fuzzy dice is hanging by a string from your rearview mirror, as shown in the preceding figure. You speed up from a stoplight. During the acceleration, the dice do not move vertically; the string makes an angle of $\theta = 22°$ with the vertical. The dice have mass 0.10 kg.

No matter what this problem ends up asking, you'll want to draw a free-body diagram. What forces act on the dice? Certainly the weight of the dice (the force of the Earth on the dice) acts downward. No electrical forces exist, so all other forces must be contact forces. The only object actually in contact with the dice is the string. The string pulls up on the dice at an angle, as shown in the picture. I labeled the force of the string on the dice "*T*," which stands for "tension"—which means the force of a string.



What about the force of the car on the dice?

What about it? The car is not in contact with the dice, the string is. It's the string, not the car, applying the force to the dice.

Determining the Net Force

It's likely that you'd be asked to determine the net force on these dice. But the tension acts both up *and* to the right. How do you deal with the vertical and horizontal forces, then?

FACT: When a force acts at an angle θ measured from the horizontal:

- The vertical component of that force is equal to the amount of the force itself times $\sin\theta$.
- The horizontal component of that force is equal to the amount of the force itself times $\cos\theta$.

Before you do any further work to find the net force, break all individual forces into horizontal and vertical components as best as you can. Here, the weight is already vertical. The tension becomes two separate components: $T\sin 68°$ goes in the vertical direction, and $T\cos 68°$ goes in the horizontal direction. Now you're ready to answer any possible problem.

Exam Tip from an AP Physics Veteran
Do *not* put force components on the same diagram as the force itself. You won't earn full credit. First, draw all forces at whatever angle is appropriate. Then, on a *separate diagram*, redraw the forces with the angled forces broken into components.

FACT: The acceleration of an object is F_{net}/m. This is the same thing as saying

$$F_{net} = ma$$

Whoa. I get that the net force is the horizontal Tcos68°, and that I can write that Tcos68° = (0.1 kg)(a). But the problem didn't give me an acceleration, it didn't give the tension—I'm stuck to solve for anything. The College Board screwed this problem up, right?

It's vanishingly unlikely that the problem is unsolvable as posed. You obviously can't ask questions of the College Board during the AP Exam. So if you're absolutely sure the exam is screwed up, you can just state where you think the problem is unclear, make up the information you need, and do your best. Chances are, though, that you need to find a creative alternate way to solve the problem.

Look at the problem statement: It said that the dice have a mass of 0.10 kg. This means that the weight of the dice is 1.0 N.[7] Since the dice are moving only in a horizontal direction, vertical acceleration (and the vertical net force on the dice) must be zero. Forces in opposite directions subtract to determine the net force. Here, that means that the up force must equal the down force of 1.0 N. The up force is $T\sin 68°$, which equals 1.0 N. Plug in from your calculator that the sine of 68 degrees is 0.93, and then solve to find the tension is 1.1 N.

Now deal with the horizontal direction. The horizontal net force is $T\cos 68°$, which is (1.1 N)(0.37) = 0.41 N. Since there's no vertical net force, 0.41 N to the right is the entire net force. And then $a = F_{net}/m$, so acceleration is 0.41 N/0.10 kg = 4.1 m/s per second. That's pretty much everything there is to calculate.

The Mistake
It's tempting to use this equation in all sorts of circumstances. For example, I'm sitting in a chair. Since I'm near the Earth, the force of the Earth on me is equal to my weight of 930 N. I know my mass is 93 kg. Use $a = F_{net}/m$. My acceleration must be 930 N/93 kg, or 10 m/s per second.

Um, no. I'm sitting in a chair. My speed isn't changing, so my acceleration is zero. The value of 10 m/s per second of acceleration means I'm in free fall. What went wrong?

I experience more forces than just the force of the Earth, of course. The chair is pushing up on me. Since I know that my acceleration is zero, the chair pushes up on me with 930 N of force. Now the up force and the down force on me subtract to zero. Phew.

Only the *net* force equals mass times acceleration. Never set a force equal to *ma* unless it's the net force.

What Else Could You Be Asked?

The AP Exam doesn't like to ask for calculations. So what else could be asked relating to Example 2?

[7] On the Earth, 1 kg of mass weighs 10 N. This fact is discussed in more detail in Chapter 15, Gravitation.

Here's one thought: If the dice were to instead hang from a bigger angle than 22° from the vertical, would the tension go up, go down, or stay the same?

The best way to answer this type of question is to make the calculation, and then explain what part of the calculation leads to the correct answer. That's the whole method behind answering questions that involve qualitative-quantitative translation, as discussed in Chapter 8.

Make up a bigger angle: call it 60° from the vertical. (Or choose any number; just make a significant difference in the new situation. Don't choose 23°.) Start back from the beginning: The dice still have a 0.10-kg mass, and the weight of the dice is still 1.0 N. The vertical acceleration is still zero, which means we can set the up force equal to the down force. But now the up force has changed, from $T \sin 68°$ to $T \sin 30°$.[8] Now we set $T \sin 30°$ equal to 1.0 N, giving a tension of 2.0 N.

The answer, then, is that the tension increases. The weight remains the same and the vertical component of tension must stay the same, but since to calculate tension we end up dividing the 1.0-N weight by the sine of the angle from the horizontal, a smaller angle from the horizontal gives a bigger tension.

> **Exam Tip from an AP Physics Veteran**
> If you are asked whether something increases, decreases, or stays the same, you might want to start by making a calculation to see numerically what happens to the answer. Be sure to explain *why* the calculation came out the way it did.

Inclined Planes

Treat objects on inclines the same as any other objects. Draw a free-body diagram, break angled forces into components, and use $a = F_{net}/m$ in each direction. The only major difference is that you don't use horizontal and vertical components for the forces. Instead, you look separately at the forces parallel to the incline and at the forces perpendicular to the incline.

Any normal force will be perpendicular to the incline, and so won't have to be broken into components as long as the object is moving up or down the incline. Any friction force will be parallel to the incline and so won't have to be broken into components. It's the weight—the force of the Earth—that will be broken into components.

KEY IDEA

FACT: On an incline of angle θ (measured from the horizontal), break the weight into components:

- The component of the weight that is parallel to the incline is equal to the weight times $\sin \theta$.
- The component of the weight that is perpendicular to the incline is equal to the weight times $\cos \theta$.

Example problems and extra drills on this frequently tested topic are available in Chapter 18.

[8]Remember we had to measure from the horizontal according to the fact on the previous page.

Multiple Objects

When two masses are connected over a pulley, it's often easiest to start by considering both objects as a single system. Draw the free-body diagram for the entire system, and use $a = F_{net}/m$ to find the acceleration of the system. Then, if you need to find the tension in the connecting rope, or if you need to talk about just one of the two connected objects, draw a new free-body diagram just for that object.

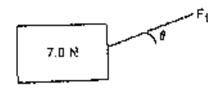
KEY IDEA

FACT: One rope has just one tension.[9]

An alternative approach is to start by drawing two separate free-body diagrams, one for each object. Write $F_{net} = ma$ for each object separately. Then, recognizing that the tension is the same in each equation, solve algebraically for the acceleration and tension.

> Practice Problems

Note: Extra drills on problems including ropes and inclined planes can be found in Chapter 18.



1. A 7.0-N block sits on a rough surface. It is being pulled by a force F_1 at an angle $\theta = 30°$ above the horizontal, as shown above. The block is initially moving to the right with speed 5 m/s. The coefficient of friction between the block and the surface is $\mu = 0.20$. Justify all answers.
   (a) Is it possible for the block to be slowing down? If so, give a possible value of the magnitude of F_1 that would allow the block to slow down. If not, explain why not with reference to Newton's second law.
   (b) In order to double the block's initial speed to 10 m/s, how must the magnitude of the force F_1 change?
      (A) It must double.
      (B) It must quadruple.
      (C) It does not have to change.
2. A drag-racing car speeds up from rest to 22 m/s in 2 s. The car has mass 800 kg; the driver has mass 80 kg.
   (a) Calculate the acceleration of the drag racer.
   (b) Calculate the net force on the drag racer.
   (c) Which experiences a greater net force?
      (A) The driver
      (B) The car
      (C) Both the driver and the car experience the same net force.
3. A car slides up a frictionless inclined plane. How does the normal force of the incline on the car compare with the weight of the car?
   (A) The normal force must be equal to the car's weight.
   (B) The normal force must be less than the car's weight.
   (C) The normal force must be greater than the car's weight.
   (D) The normal force must be zero.
4. Bert, Ernie, and Oscar are discussing the gas mileage of cars. Specifically, they are wondering whether a car gets better mileage on a city street or on a freeway. All agree (correctly) that the gas mileage of a car depends on the force that is produced by the car's engine—the car gets fewer miles per gallon if the engine must produce more force. Whose explanation is completely correct?
   Bert says: Gas mileage is better on the freeway. In town the car is always speeding up and slowing down because of the traffic lights, so because $F_{net} = ma$ and acceleration is large, the engine must produce a lot of force. However, on the freeway, the

[9]This is true unless the rope is tied to or connected over a mass. For example, if the pulley itself had mass, then the rope can have different tensions on each side of the pulley. But that's a rare happening, and that certainly shouldn't require any calculation.

car moves with constant velocity, and acceleration is zero. So the engine produces no force, allowing for better gas mileage.

Ernie says: Gas mileage is better in town. In town, the speed of the car is slower than the speed on the freeway. Acceleration is velocity divided by time, so the acceleration in town is smaller. Because $F_{net} = ma$, then, the force of the engine is smaller in town giving better gas mileage.

Oscar says: Gas mileage is better on the freeway. The force of the engine only has to be enough to equal the force of air resistance—the engine doesn't have to accelerate the car because the car maintains a constant speed. Whereas in town, the force of the engine must often be greater than the force of friction and air resistance in order to let the car speed up.

› Solutions to Practice Problems

1. (a) When an object slows down, its acceleration (and therefore the net force it experiences) is opposite the direction of its motion. Here the motion is to the right, so if the net force is left, it will slow down. The friction force is μF_N, where the normal force F_N is in this case equal to the 7-N weight. The friction force value is 1.4 N. The horizontal component of F_1 is the magnitude of F_1 times the cosine of 30°. Make F_1 equal to 1.4 N. Then the horizontal component of F_1 is something less than 1.4 N; the left force will be bigger than the right force; the net force will be to the left, opposite the motion; and the block will slow down.
   (b) While the net force is related to acceleration, the net force has no effect on an object's speed. Beyond that, no one has said anything about what happens before the problem, about how that initial speed came about. The forces can all be as indicated, and the object can have any initial speed.
2. (a) The car's speed changes by 22 m/s in 2 s. So the car changes its speed by 11 m/s in 1 s, which is what is meant by an acceleration of 11 m/s per second.
   (b) Newton's second law says that the net force on the racer is the drag racer's mass of 800 kg times the 11 m/s per second acceleration. That gives a net force of 8,800 N.
   (c) The driver and the car must experience the same acceleration because they move together; when the car changes its speed by 11 m/s in one second, so does the driver.[10] To calculate the net force on the driver, the driver's 80-kg mass must be used in Newton's second law, $F_{net} = ma$. With the same a and a smaller mass, the driver experiences a smaller net force (and the car experiences a greater net force).
3. (B) The normal force exerted on an object on an inclined plane equals mg (cos θ), where θ is the angle of the incline. If θ is greater than 0, then cos θ is less than 1, so the normal force is less than the object's weight.
4. Although Bert is right that acceleration is zero on the freeway, this means that the *net* force is zero; the engine still must produce a force to counteract air resistance. This is what Oscar says, so his answer is correct. Ernie's answer is way off—acceleration is not velocity/time, acceleration is a *change* in velocity over time.

[10] Otherwise, the driver would fall out of the car.

› Rapid Review

- Only gravitational and electrical forces can act on an object without contact (in AP Physics 1).
- When an object moves along a surface, the acceleration in a direction perpendicular to that surface must be zero. Therefore, the net force perpendicular to the surface is also zero.
- The friction force is equal to the coefficient of friction times the normal force, $F_f = \mu F_n$.
- The force of Object A on Object B is equal in amount and opposite in direction to the force of Object B on Object A. These two forces, which act on different objects, are called Newton's third law companion forces.
- If the net force has both a vertical and a horizontal component, use the Pythagorean theorem to determine the magnitude of the net force, and use the tangent function to determine the direction of the net force.
- When a force acts at an angle θ measured from the horizontal:
  - The vertical component of that force is equal to the amount of the force itself times sin θ.
  - The horizontal component of that force is equal to the amount of the force itself times cos θ.
- The acceleration of an object is F_{net}/m (which is the same thing as saying $F_{net} = ma$).
- On an incline of angle θ (measured from the horizontal), break the weight into components:
  - The component of the weight that is parallel to the incline is equal to the weight times sin θ.
  - The component of the weight that is perpendicular to the incline is equal to the weight times cos θ.
- One rope has just one tension.

CHAPTER

Collisions: Impulse and Momentum

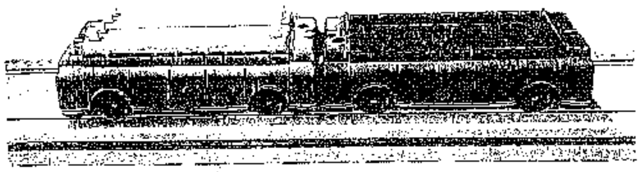
IN THIS CHAPTER

Summary: Whenever you see a collision, the techniques of impulse and momentum are likely to be useful in describing or predicting the result of the collision. In particular, momentum is conserved in all collisions—this means that the total momentum of all objects is the same before and after the collision. When an object (or a system of objects) experiences a net force, the impulse momentum theorem $\Delta p = F \cdot \Delta t$ can be used for predictions and calculations.

Definitions

- A moving object's momentum is its mass times its velocity. Momentum is in the direction of motion.
- Impulse is the change in an object's momentum, which is also equal to the net force on the object multiplied by the time during which the force acts. Impulse is in the direction of the net force.
- A system is made up of several objects that can be treated as a single thing. It's important to define the system you are considering before you treat a set of objects as a system.
- While total momentum is conserved in all collisions, kinetic energy is conserved only in an elastic collision.

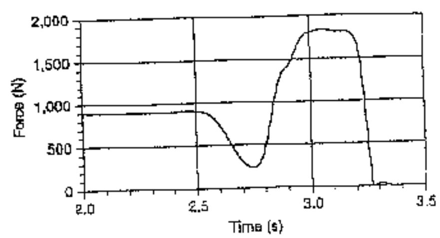
Momentum is a useful quantity to calculate because it is often conserved; that is, the total amount of momentum available in most situations cannot change. Whenever you see a collision, the techniques of impulse and momentum are most likely to be useful. Try impulse and momentum first, before trying to use force or energy approaches.



The Impulse-Momentum Theorem

Here is the impulse-momentum theorem:

$$\Delta p = F \cdot \Delta t$$



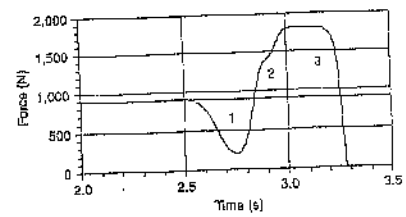
Example 1: A teacher whose weight is 900 N jumps vertically from rest while standing on a platform scale. The scale reading as a function of time is shown in the preceding figure.

A force versus time graph is essentially an invitation to calculate impulse. Since impulse is defined as $F \cdot \Delta t$, from a force versus time graph, impulse is the area under the graph.

Strategy: When you need to take the area of an experimental graph, approximate as best you can with rectangles and triangles.

This graph for Example 1 is tricky—impulse calculations should use the *net* force on an object. The scale reading on the vertical axis of the graph is not the net force; the net force is the scale reading in excess of the person's 900-N weight.

To estimate the impulse given to the jumper in this example, draw a horizontal line at the 900-N mark as a zero point for calculating the net force:



Regions 1 and 2 have somewhat close to the same area, one above and one above the zero net force line; so these areas cancel out. Region 3 looks somewhat like a rectangle, with base something like 0.25 s and height (1,800 N − 900 N) = 900 N. The impulse is then 900 N times 0.25 s, or something like 220 N·s.

> Exam Tip from an AP Physics Veteran
> You must be comfortable with this kind of rough approximation. Sure, the impulse could well be more like 228 N·s, or 210 N·s. Who cares? On a free-response item it will be the reasoning behind your calculation that earns credit, much more so than the answer itself. On a multiple-choice item, the choices might be far separated, like (A) 200 N·s; (B) 2,000 N·s; (C) 20,000 N·s; (D) 200,000 N·s. Isn't the choice obvious?

Impulse by itself doesn't say much. The more interesting question about this jumping teacher in Example 1 is the speed with which he leaves the scale. Since impulse is the change in an object's momentum, you know the teacher changed his momentum by 220 N·s. Since he started from rest, his momentum right after leaving the scale is also 220 N·s. Finally, momentum is mass times speed. The teacher's mass is 90 kg,[1] so plug in to the equation $p = mv$: (220 N·s) = (90 kg)v. His speed is 2.4 m/s, or thereabouts.

Conservation of Momentum

Example 2: Cart A, of mass 0.5 kg, moves to the right at a speed of 60 cm/s. Cart B, of mass 1.0 kg, is at rest. The carts collide.

FACT: In any system in which the only forces acting are between objects in that system, momentum is conserved. This effectively means that momentum is conserved in *all* collisions.

[1] This is because his weight is 900 N, and on Earth 1 kg weighs 10 N.

Define the system for momentum conservation in Example 2—just the two carts. They apply a force to each other in the collision, but that's it, so momentum is conserved.[2]

A common task in a problem with a collision involves calculating the speed of one or both objects after the collision. Even when a collision-between-two-objects question is qualitative or conceptual in nature, it's often a good idea to try calculating speeds after a collision.

To do this, define a positive direction and then make a chart indicating the mass m and speed v of each cart before and after the collision. I use a "prime" mark (′) to indicate when we're dealing with values after a collision rather than before. Indicate the direction of motion with a plus or minus sign on the velocities.

(Note that it's okay to use centimeters per second [cm/s] rather than meters per second [m/s], as long as you are consistent throughout.)

$m_A = 0.5$ kg
$m_B = 1.0$ kg
$v_A = +60$ cm/s
$v_B = 0$
$v_A' = ?$
$v_B' = ?$

Then write the equation for conservation of momentum.

> Uh, where do you get such an equation? I know it's not on the equation sheet on the old or new AP physics exams.

The relevant equation comes from the definition of "conservation," meaning an unchanging quantity. The total change in momentum for the system of the two carts must be zero. Any momentum lost by Cart A is gained by Cart B. Set zero equal to Cart A's change in momentum, plus Cart B's change in momentum:

$$0 = (p_A' - p_A) + (p_B' - p_B)$$

Then, knowing that $p = mv$, plug in what you know. I'm going to leave off the units to make the mathematics clearer; since the table above has values and units, it's clear what units are intended.

$$0 = [(0.5)\,v_A' - (0.5)(60)] + [(1.0)v_B' - 0]$$

Is this solvable? Not yet, because it's only one equation with two variables. The information about this collision is incomplete. The collision in Example 2 could thus have all sorts of results.

One possibility is that the carts stick together. In that case, the carts share the same speed; in the notation above, $v_A' = v_B'$. That makes the calculation solvable; replace the vs with a single variable to get $v = 20$ cm/s.

Perhaps the problem statement continues to tell us the speed of one of the carts after the collision. Then the problem is solvable: plug in the value given, and solve for the other v.

[2]If friction between the track and the carts were significant, then sure, *momentum* wouldn't be conserved—the track wasn't considered part of the system, and it's applying a force to the carts. But *even with friction*, if you consider the moments just before and just after the collision, momentum will be essentially conserved. See the following discussion for a time when the momentum of a system is *not* conserved.

The only tricky part here would be, say, if Cart A rebounded after the collision. Then v_A' would take a negative value. But the solution would be approached the same way.

When Is the Momentum of a System Not Conserved?

The simple answer goes back to the definition of momentum conservation: The momentum of a system is *not* conserved when a force is exerted by an object that's not in the system.

> **Example 3:** Two identical balls are dropped from the same height above the ground, such that they are traveling 50 cm/s just before they hit the ground. Ball A rebounds with speed 50 cm/s; Ball B rebounds with a speed of 10 cm/s. Each is in contact with the ground for the same amount of time.

Define the system here. If the system is just Ball A, say, then is the momentum of Ball A conserved? Of course not! The problem says that Ball A rebounds, which means it changed its direction and thus its momentum.

Mistake: It's tempting to say that since Ball A didn't change its mass, and since its speed was 50 cm/s before *and* after the collision, that Ball A didn't change its momentum. This is not correct; momentum has direction. An object that changes direction loses all its momentum and then gains some more. If Ball A had mass 2 kg, then it lost 1 N·s of momentum in stopping, and then gained another 1 N·s of momentum in order to rebound—for a total change in momentum of 2 N·s.

> Wait. You said in the preceding Fact that momentum is conserved in all collisions. What happened?

Well, yes, momentum *is* conserved in all collisions, if you define the system to include the two (or three) objects that are colliding. In Example 3, Ball A is effectively colliding with the entire Earth. If we consider the system of the Earth and Ball A, then momentum is, in fact, conserved. The change in Ball A's momentum is equal to the change in the Earth's momentum. Since the Earth is so mind-bogglingly massive, its speed won't change in any measurable amount.

Mistake: The total momentum for a system of objects is always the same. So in a single collision, the total momentum cannot change. In a problem like Example 3, though, Balls A and B are involved in two separate collisions. Therefore, they can't be part of the same system! Don't use "conservation of momentum" as a reason for anything about Balls A and B to be equal, when Balls A and B are involved in separate collisions.

The point is that momentum conservation is not an effective approach to consider when a ball collides with the entire Earth.[3] Instead, use the impulse-momentum theorem to find out what you can.

The easy question is as follows: Which ball changes its momentum by a greater amount? That'd be Ball A. Both balls lost the same amount of momentum in coming to a brief rest, then rebounded; since Ball A rebounded faster, and since the balls have the same mass, Ball A changed its momentum by a greater amount.[4]

[3]Or equivalently, this is not an effective approach to consider when a car collides with a concrete pillar, or a bird collides with the window of a building, etc.

[4]If you need to make up a mass of 2 kg for each ball and plug in numbers (including a plus and minus sign for the direction of velocity) to calculate the total momentum change for each ball, feel free. That's not a bad approach if the words are confusing you.

The harder question is this: Which ball exerted a larger force on the ground during its collision? We know that momentum change equals force times time.[5] With the same time of collision, the bigger force is exerted by the ball with the greater momentum change—that's Ball A.

Similar reasoning can explain why airbags make a car safer. You[6] lose all your momentum in a crash regardless of how you come to rest. Airbags extend the time of the collision between you and the car. In the equation $\Delta p = F \cdot \Delta t$ with the same Δp, a bigger Δt gives a smaller F, so the force you experience is less in an airbag collision.

Elastic/Inelastic Collisions

In elastic collisions, the total kinetic energy of both objects combined is the same before and after the collision. A typical AP problem might pose a standard collision problem and then ask, "Is the collision elastic?" To figure that out, add up the kinetic energies ($\frac{1}{2}mv^2$) of both objects before the collision, add up the kinetic energies of both objects after the collision, and compare. If these kinetic energies are essentially the same, the collision is elastic. If the final kinetic energy is less than the initial kinetic energy, the collision was *not* elastic—kinetic energy was converted, generally to work done by nonconservative forces exerted by one colliding object on the other.[7]

Example 4: Two carts of equal mass move toward each other with identical speeds of 30 cm/s. After colliding, the carts bounce off each other, each regaining 30 cm/s of speed, but now moving in the opposite direction.

Mistake: Never start a collision problem writing anything about kinetic energy. Always start with conservation of momentum. Only move on to kinetic energy conservation if you have to, that is, if you don't have enough information to solve with just momentum conservation, *and* if the problem is explicit in saying that the collision is elastic.

Is momentum conserved in this collision? Yes, and you don't have to do any calculations to show it. In a collision, momentum is always conserved because the only forces acting on the carts are exerted by the carts themselves.

Is kinetic energy conserved in this collision? You've got to do the calculation to check.

But the carts bounced off each other. Doesn't that automatically mean the collision is elastic?

No. When carts stick together, the collision cannot be elastic. But when carts bounce off each other, the collision might be elastic, or might not be.

Start with the kinetic energy before the collision. In this case, make up a mass for each cart: they're identical, so call them 1 kg each. Each cart has speed 0.30 m/s, so the kinetic energy of each cart before the collision is $\frac{1}{2}(1\ \text{kg})(0.30\ \text{m/s})^2 = 0.045$ J. The combined kinetic energy before collision is thus 0.090 J.

[5]**Mistake:** The relevant time in the impulse-momentum theorem is always the time of the collision, *not* the time it takes for a ball to fall through the air.

[6]Hopefully not *you*, personally.

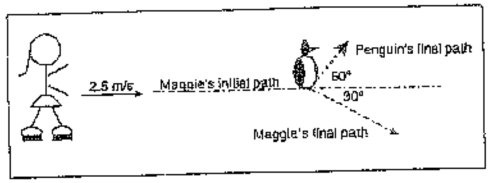
[7]If the final kinetic energy is *greater* than the initial kinetic energy, something weird happened; like a coiled spring was released during the collision, or a firecracker exploded. You'll most often see this sort of "superelastic" collision when the objects are initially at rest and then they are blown apart.

Whoa there—One cart was moving right, the other left; that means the kinetic energies subtract, giving zero total kinetic energy. Right?

Wrong. Kinetic energy is a scalar, which means it has no direction. Kinetic energy can never take on a negative value. Always add the kinetic energies of each object in a system to get the total kinetic energy. After the collision, the calculation is the same: total kinetic energy is still 0.090 J. So the collision is, in fact, elastic.

2-d Collisions

Example 5: Maggie, of mass 50 kg, glides to the right on a frictionless frozen pond with a speed of 2.5 m/s. She collides with a 20-kg penguin. After the collision, the directions of the penguin's and Maggie's motion is shown in the following figure.



Strategy: When objects move in both an x- and a y-direction after a collision, analyze the collision with momentum conservation *separately* in each direction.

You will not likely be asked to do quantitative analysis of a two-dimensional collision, but you do need to understand conceptually how momentum conservation works here. Be able to explain how you would carry out the analysis of momentum conservation in each direction and be able to answer simple qualitative questions.

For example, who has a greater magnitude of momentum in the y-direction after collision? Before the collision, there was no momentum in the y-direction. After the collision, the total y-momentum must also be zero. Since both the penguin and Maggie are moving in the y-direction, their momentums must be equal and opposite so as to subtract to zero. The answer is neither—both the penguin and Maggie have the same amount of y-momentum.

What about the y-component of their velocities? We've already established that they have the same y-momentum, which is equal to mass times y-velocity. Since Maggie has the bigger mass, she must have the smaller y-component of velocity.

Is momentum conserved in the x-direction? Of course it is. The total momentum before collision is all due to Maggie's movement: (50 kg)(2.5 m/s) = 125 N·s, all in the x-direction. After collision, the total x-momentum is also 125 N·s. The x-component of the penguin's momentum after collision is just his momentum mv times the cosine of 60°; Maggie's x-momentum is her momentum times the cosine of 30°. In this problem, the only way to get values for these components is to do some complicated algebra, which is beyond the scope of AP Physics 1. But you should be able to explain everything about this collision in words, as discussed here.

Motion of the Center of Mass

FACT: The center of mass of a system of objects obeys Newton's second law.

Two common examples illustrate this fact:

Example: Imagine that an astronaut on a spacewalk throws a rope around a small asteroid, and then pulls the asteroid toward him. Where will the asteroid and the astronaut collide?

Answer: at the center of mass. Since no forces acted except due to the astronaut and asteroid, the center of mass must have no acceleration. The center of mass started at rest, and stays at rest, all the way until the objects collide.

Example: A toy rocket is in projectile motion, so that it is on track to land 30 m from its launch point. While in the air, the rocket explodes into two identical pieces, one of which lands 35 m from the launch point. Where does the first piece land?

Answer: 25 m from the launch point. Since the only external force acting on the rocket is gravity, the center of mass must stay in projectile motion, and must land 30 m from the launch point. The two pieces are of equal mass, so if one is 5 m beyond the center of mass's landing point, the other piece must be 5 m short of that point.

Finding the Center of Mass

Usually the location of the center of mass (cm) is pretty obvious . . . the formal equation for the cm of several objects is

$$Mx_{cm} = m_1x_1 + m_2x_2 + \cdots$$

Multiply the mass of each object by its position, and divide by the total mass M, and voila, you have the position of the center of mass. What this tells you is that the cm of several equal-mass objects is right in between them; if one mass is heavier than the others, the cm is closer to the heavy mass.

› Practice Problems

1. A 2-kg coconut falls from the top of a tall tree, 30 m above a person's head. The coconut strikes and comes to rest on the person's head. Justify all answers thoroughly.
   (a) Calculate the magnitude momentum of the coconut just before it hits the person in the head.
   (b) Calculate the magnitude and direction of the impulse experienced by the coconut in colliding with the person's head.
   (c) The person's head experienced a force of 10,000 N in the collision. How long was the coconut in contact with the person's head?
   (A) Much more than 10 seconds
   (B) Just a bit more than one second
   (C) Just a bit less than one second
   (D) Much less than 1/10 second
   (d) In a different situation, explain how it could be possible for an identical coconut dropped from the same height to hit the person's head, but produce *less* than 10,000 N of force.

2. A car on a freeway collides with a mosquito, which was initially at rest. Justify all answers thoroughly.
   (a) Did the total momentum of the car-mosquito system increase, decrease, or remain the same after the collision?
   (b) Did the momentum of the mosquito increase, decrease, or remain the same after the collision?
   (c) Did the momentum of the car increase, decrease, or remain the same after the collision?
   (d) Which changed its speed by more in the collision, the car or the mosquito? (Or did they change speed by the same amount?)
   (e) Which changed its momentum by more in the collision, the car or the mosquito? (Or did they change momentum by the same amount?)
   (f) Which experienced a greater impulse in the collision, the car or the mosquito? (Or did they experience the same impulse?)
   (g) Which experienced a greater magnitude of net force during the collision, the car or the mosquito? (Or did they experience the same net force?)

3. Car A has a mass of 1,500 kg and travels to the right with a speed of 20 m/s. Car B initially travels to the left with a speed of 10 m/s. After the vehicles collide, they stick together, moving left with a common speed of 5 m/s. Justify all answers thoroughly.
   (a) Calculate the mass of Car B.
   (b) This collision is not elastic. Explain why not.
   (c) Describe specifically a collision between these two cars with the same initial conditions, but which is *not* elastic, and in which the cars bounce off one another.
   (d) Is the collision elastic when Car B remains at rest after the collision?

› Solutions to Practice Problems

1. (a) Momentum is mass times speed. To find the coconut's speed, use kinematics with $v_0 = 0$, $a = 10$ m/s per second, and $\Delta x = 30$ m. The equation $v_f^2 = v_0^2 + 2a\Delta x$ solved for v_f gives 24 m/s. Multiplying by the 2-kg mass gives a momentum of 48 N·s.[8]
   (b) Impulse is the change in momentum and is in the direction of the net force experienced by an object. The coconut's momentum after colliding with the head is zero—the coconut comes to rest. So its change in momentum, and thus the magnitude of the impulse it experiences, is 48 N·s. The direction of this impulse is upward, because the net force on the coconut must be opposite its speed in order to slow it down.
   (c) Impulse is also equal to force times the time interval of collision. Setting 48 N·s equal to (10,000 N)(Δt), we find the time interval of collision is 48/10,000 of a second—much less than 1/10 second, even without reference to a calculator.
   (d) The impulse-momentum theorem says that $\Delta p = F\Delta t$. Solving for force, $F = \frac{\Delta p}{\Delta t}$. Here the momentum change has to be the same no matter what—the coconut will be traveling 24 m/s and will come to rest on the person's head. But if the person is wearing a soft helmet, or if the coconut has a rotten spot on it somewhere, then the time of collision could be larger than before. Since Δt is in the denominator of the force equation, a bigger time interval of collision leads to a smaller force on the coconut (and therefore on the person's head).

2. (a) Momentum is conserved when no forces are exerted, except for those on and by objects in the system. Here the only forces are of the car on mosquito and mosquito on car. Therefore, momentum was conserved. That means that the total momentum of the car-mosquito system remains the same.

[8] If you used units of kg·m/s, that's fine, too.

(b) The mosquito went from rest to moving freeway speeds after it hit the car. The mosquito's mass didn't change.[9] Momentum is mass times speed, so the mosquito's momentum increased.

(c) Since total momentum of the car-mosquito system doesn't change, and the mosquito gained momentum, the car has to lose that same amount of momentum.

(d) The mosquito's speed went from, say, zero to 60 miles per hour. While the car must lose the same amount of momentum that the mosquito gained, the car's mass is so much larger than the mosquito's that the car's speed will hardly change. And you knew that, because a car hitting a mosquito on the freeway doesn't cause the car to stop.

(e) The momentum change is the same for both, because total momentum remains unchanged. Any momentum gained by the mosquito must be lost by the car.

(f) Impulse is the same thing as momentum change, so the same for both.

(g) Newton's third law says the force of the mosquito on the car is equal to the force of the car on the mosquito. So they're equal.

3. (a) Before the collision, Car A has a momentum of 30,000 N·s to the right. If we call the mass of Car B "M_B," then Car B has momentum of M_B(10 m/s) to the left. Afterward, the total momentum is (M_B + 1,500 kg)(5 m/s) to the left. Let's call right the positive direction. Then the relevant equation for conservation of momentum is $30{,}000 - 10M_B = -5(1{,}500 + M_B)$, where I've left off the units so the algebra is clearer. Solve for M_B to get 7,500 kg. This makes sense—Car B was initially moving slower, yet after the collision the cars moved off together in the direction Car B was going. Car B must therefore have more momentum than Car A initially and more mass because it was going slower.

(b) "Elastic" means that kinetic energy ($= \frac{1}{2}mv^2$) of all objects combined is the same before and after collision. Before collision, Car A had 300 kJ of kinetic energy, and Car B had 375 kJ, for a total of 675 kJ before the collision.[10] After collision, the kinetic energy of the combined cars is 112 kJ. Kinetic energy was lost in the collision. (Note that it's legitimate to remember that collisions in which objects stick together can never be elastic.)

(c) Imagine that Car B keeps moving left, but much slower, say, 1 m/s. Momentum is conserved in a collision, regardless of whether the collision is elastic or not. The total momentum of Car A before collision is 30,000 N·s to the right; the total momentum of Car B before collision is 75,000 N·s to the left. This gives a total momentum of 45,000 N·s to the left before collision. If Car B moves 1 m/s after collision, it has 7,500 N·s of momentum to the left, leaving 37,500 N·s to the left for Car A. Dividing by Car A's 1,500-kg mass, Car A is found to be moving 25 m/s after the collision.

Now check total kinetic energy after collision. Car A has 469 kJ of kinetic energy and Car B has 4 kJ of kinetic energy, for a total of 473 kJ. Before the collision the total kinetic energy was 675 kJ, as calculated in (b). Therefore, kinetic energy is lost and the collision is inelastic. The whole point here is that not all collisions in which cars bounce are elastic.

(d) We need a total of 675 kJ afterward in order to have an elastic collision. Conservation of momentum means that the total momentum after collision is 45,000 N·s to the left. Since Car A is the only moving car, it has all that momentum. Dividing by Car A's 1,500-kg mass, we find Car A moving 30 m/s. Only Car A has kinetic energy, too; its kinetic energy is $\frac{1}{2}mv^2 = 657$ kJ, so the collision is elastic.

[9] . . . though its mass was likely redistributed around the windshield a bit.

[10] No, the total is not −75 kJ. Kinetic energy is a scalar, meaning it cannot have a direction; and kinetic energy cannot be negative. The total kinetic energy of a system is the sum of all the kinetic energies of the constituent objects, regardless of which way the objects are moving.

› Rapid Review

- In any system in which the only forces acting are between objects in that system, momentum is conserved. This effectively means that momentum is conserved in *all* collisions.
- The center of mass of a system of objects obeys Newton's second law.
- The impulse-momentum theorem is $\Delta p = F \cdot \Delta t$.
- The impulse-momentum theorem is always valid, but it is most useful when objects collide.
- The only time when momentum of a system is *not* conserved is when a force is exerted by an object that's not in the system.

CHAPTER 5

Work and Energy

IN THIS CHAPTER

Summary: An object possesses kinetic energy by moving. Interactions with other objects can create potential energy. Work is done when a force acts over a distance parallel to that force. When work is done on an object (or on a system of objects), kinetic energy can change. This chapter shows you how to recognize the different forms of energy and how to use them to make predictions about the behavior of objects.

KEY IDEA

Definitions

- **Kinetic Energy** is possessed by any moving object. It comes in two forms:
  1. *Translational Kinetic Energy* is

     $$\boxed{\tfrac{1}{2}mv^2}$$

     It exists when an object's center of mass is moving.
  2. *Rotational Kinetic Energy* is

     $$\boxed{\tfrac{1}{2}I\omega^2}$$

     It exists when an object rotates.
- **Gravitational potential energy** is energy stored in a gravitational field. Near a planet, the formula is

  $$\boxed{\text{GPE} = mgh}$$

where h is the vertical height above a reference position. A long way from a planet, the formula is

$$GPE = -G\frac{M_1M_2}{d}$$

where d is measured from the planet's center.

- **Elastic potential energy**, also known as spring potential energy, is energy stored by a spring, given by

$$SPE = \tfrac{1}{2}kx^2$$

- **Internal Energy** can refer to two somewhat different ideas. Both refer to the concept that multi-object systems can store energy depending on how the objects are arranged in the system.
  - ***Microscopic internal energy*** is related to the temperature of the object. As the object warms up, energy can be stored by the vibrations of molecules.
  - ***Internal energy of a two-object system*** is just another way of saying "potential energy."
- **Mechanical Energy** refers to the sum of potential and kinetic energies.
- **Work** is done when a force acts on something that moves a distance parallel to that force.
- **Power** is defined as energy used per second, or work done per second.

Energy

The cart pictured above has gravitational potential energy with respect to the location of the motion detector, because the cart is vertically higher than the detector. If the cart were released from rest, it would speed up toward the detector. It would be tempting to try to use the kinematics equations to determine the cart's maximum speed. However, since the track is curved, the cart's acceleration will be changing throughout its motion. Whenever acceleration changes, kinematics as studied in Chapter 10 are invalid. The methods of energy conservation, as described in this chapter, must be used.

The College Board's curriculum guide for AP Physics 1 makes much of the difference between **objects** and **systems**. A system is just a collection of objects. In your class, you might well have talked about an object's kinetic and potential energies. The thing is, the exam development committee doesn't like that language. Sure, an object can have kinetic energy, just by moving. But a single object technically cannot "have" potential energy.

Why not? Potential energy is always the result of an interaction between objects in a system. For example, gravitational potential energy (equation: mgh) exists only if an object is interacting with the Earth. The Earth-object system stores the potential energy, not just the object itself. Similarly, an object attached to a spring *cannot* store potential energy; the spring-object system stores the energy.

Look, I'm going to talk about objects "having" potential energy. It's okay with me if you talk about a block on a spring "having" potential energy because the spring is compressed. You'll still get pretty much everything right on the exam. Just know that the block only "has" potential energy because of its interaction with the spring; and that potential energy is sometimes referred to as the "internal energy of the block-spring system."

Work

KEY IDEA

FACT: Work is done when a force is exerted on an object[1] and that object moves parallel to the direction of the force.

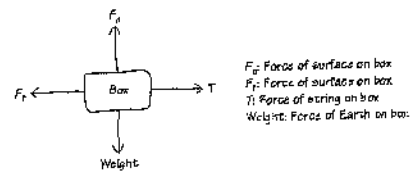
The relevant equation is work = force times parallel displacement:[2]

$$W = F\Delta x_{\parallel}$$

When a force is exerted in the same direction as the object's motion, the work done is considered to be a positive quantity; when a force is exerted in the opposite direction of the object's motion, the work done is considered to be a negative quantity. The *net* work on an object is the algebraic sum of the work done by each force.

Example 1: A string applies a 10-N force to the right on a 2-kg box, dragging it at constant speed across the floor for a distance of 50 cm.

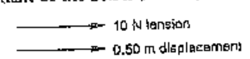
Let's calculate the work done by each force acting, and the net work done on the box. Start by sketching a free-body diagram for the box.



STRATEGY

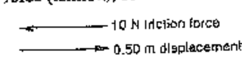
Strategy: Whenever you are calculating work done by a force with the equation $W = F\Delta x_{\parallel}$, always sketch the direction of the force and displacement vectors.

Consider each force separately. Start with the 10-N tension in the string. This force acts to the right. The displacement of the box is 50 cm (i.e., 0.50 m) to the right.



Since the force is in the same direction as the displacement, the work done by the tension is just 10 N times 0.50 m = +5 J.

Now consider the friction force. Since the box moves at constant speed, we know the left force (friction) equals the right force (tension); so the friction force has an amount of 10 N.

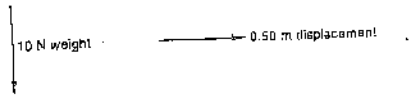


[1] Or on a system of objects—if a force is exerted on a system of objects and the system's center of mass moves parallel to the force, work was done.

[2] Or, equivalently, the equation is displacement times parallel force, which may sometimes be a more convenient expression.

Since the force is in the *opposite* direction of displacement, the work done by the friction force is 10 N times 0.50, with a negative sign. Thus, the work done by friction is −5 J.

What about the force of gravity? The 20-N weight of the box points downward.



Since no component of the weight is parallel to the displacement, the force of the Earth does zero work on the box.

Similarly, the normal force is straight upward while the displacement is to the right; since no component of the normal force is parallel to the displacement, the normal force does no work on the box.

> Exam Tip from an AP Physics Veteran
> If a force acts at an angle to the displacement, just break that force into components. The component perpendicular to the displacement does no work. The component parallel to the displacement can be multiplied by the displacement to get the work done.

Finding the Net Work

You can calculate the net work on the box in two ways:

1. First, determine the net force using a free-body diagram, like we showed in Chapter 11. Then, multiply the component of the net force that's parallel to the displacement by the displacement, just like you would when finding the work done by any force. In Example 1, the net force is zero; so there is no net work done on the box.
2. First, determine the work done by each force separately. Then, add the work done by each force algebraically (i.e., including negative signs). In Example 1, add the +5 J done by the tension to the −5 J done by friction (and the 0 J done by the normal force and the weight) to get zero net work.

Conservative Versus Nonconservative Forces

FACT: A "conservative" force converts potential energy to other forms of mechanical energy when it does work. Thus, a conservative force does not change the mechanical energy of a system.

The amount of work done by a conservative force depends only on the starting and ending positions of the object, i.e., it's "path independent." The only conservative forces that you need to deal with on the AP Physics 1, Algebra-Based Exam are gravity and springs. When a spring does work on an object, energy is stored in the spring that can be recovered and converted back to kinetic energy. The sum of the potential and kinetic energy of the object-spring system is constant.

Conversely, a "nonconservative" force can change the mechanical energy of a system. Friction is the most common example: Work done by friction on an object becomes microscopic internal energy in the object, raising the object's temperature. That microscopic internal energy *cannot* be recovered and converted back to kinetic energy. Other nonconservative forces might include, for example, the propeller of an airplane—it does work on the airplane to increase the airplane's mechanical energy.

The Work-Energy Theorem

In your textbook, you'll see the work-energy theorem written as "net work = change in kinetic energy." That's certainly true—*net* work done on an object must change the object's kinetic energy. The tricky part is, net work must include work done by all forces, conservative and nonconservative.

I think it's easier to separate conservative and nonconservative forces. Work done by a nonconservative force (W_{NC}) changes the total mechanical energy of a system ($KE + PE$). I write the work-energy theorem as follows:

$$\boxed{W_{NC} = (\Delta KE) + (\Delta PE)}$$

Generally, the potential energy involved will be either that due to a spring, or due to a gravitational field. The kinetic energy includes *both* translational kinetic energy ($\frac{1}{2}mv^2$) and rotational kinetic energy[3] ($\frac{1}{2}I\omega^2$).

Example 2: An archer pulls an arrow of mass 0.10 kg attached to a bowstring back 30 cm by exerting a force that increases uniformly from zero to 200 N.

The AP Exam could ask all sorts of questions about this situation. Before you start doing any calculation, categorize the problem. There are only three ways to approach a mechanics problem: kinematics/Newton laws, momentum, and energy. There's no collision, so momentum is unlikely to be useful. The problem talks about a force; but that force is *changing*. A changing force means a changing acceleration, which means that kinematics equations are not valid. Only the work-energy theorem will be useful.

The only types of potential energy used in AP Physics 1 are due to gravity (mgh) and due to a spring ($\frac{1}{2}kx^2$). Which is involved here? The example says that the force of the string varies "uniformly," which means that the force gets bigger as the distance stretched gets bigger, just like a spring. So treat the bowstring just like a spring.

Mistake: An interesting question here might be "how much work does the archer do in pulling back the bowstring?" And you'd be tempted to use the definition of work, $W = F\Delta x_{\parallel}$. But, no, since the force of the archer on the string is changing, this equation for work does not apply. Instead, you must use the entire work-energy theorem.

To find the work done by the archer in pulling back the bowstring, write the work-energy theorem, considering the time from when he starts pulling until the maximum extension. Since the arrow is at rest before the archer starts pulling, and is *still* at rest when the string is pulled all the way back, the change in kinetic energy is zero. The potential energy of a spring is zero at the equilibrium position and is $\frac{1}{2}kx^2$ at full extension. The work done by the archer is a nonconservative force, since it changes the mechanical energy of the string-arrow system. We get

$$W_{NC} = (0) + (\tfrac{1}{2}kx^2 - 0)$$

But this isn't solvable yet—we know the distance x the archer pulled to be 0.30 m (i.e., 30 cm). But what is k, the spring constant of the "spring"? Use the equation $F = kx$ when the bow is fully extended. The problem says the maximum force the archer pulls with is 200 N when the string is extended 0.30. Plugging into $F = kx$ and solving for k gives $k = 670$ N/m.[4]

Now the work done by the archer is $\frac{1}{2}(670\text{ N/m})(0.30\text{ m})^2 = 30$ J.

[3] See Chapter 14 for further discussion of rotational kinetic energy.

[4] No, *not* 666.6666666666 N/m. Use two significant figures, unless you want your 10th-grade chemistry teacher to have heart palpitations.

The AP Exam could certainly ask for this calculation. Or, the exam might ask, "If the archer instead pulls back 60 cm, what will happen to the work done by the archer?" The work-energy theorem still applies, the kinetic energy terms still go away, and the work done is still $\frac{1}{2}kx^2$. The spring constant is a property of a spring (or in this case, a bowstring). So k doesn't change. We doubled the displacement from equilibrium, x. Since the variable x is squared, then we don't multiply the work done by a factor of two when we double x; we multiply the work done by a factor of 2^2 (i.e., by a factor of four). The archer has to do four times as much work.

How about finding out how fast the arrow would be traveling when the archer shoots it? We cannot use kinematics with a varying net force or a varying acceleration. Use the work-energy theorem again; except this time, let the problem start when the archer released the bowstring, and end when the string gets back to the equilibrium position and the arrow is released. Now, since no nonconservative forces act,[5] the equation becomes

$$0 = (0 - \tfrac{1}{2}mv^2) + (\tfrac{1}{2}kx^2 - 0)$$

The kinetic energy goes from zero to something; while we don't know the value of the arrow's final kinetic energy, we know the equation for that kinetic energy is $\frac{1}{2}mv^2$. The potential energy goes from $\frac{1}{2}kx^2$ to zero, because for a spring $x = 0$ is by definition the equilibrium position.

Whenever all the forces acting are conservative, mechanical energy is conserved. This means that energy can be changed from potential to kinetic or back, but the total mechanical energy must remain the same always. Here the initial potential energy was 30 J—that's the same $\frac{1}{2}kx^2$ calculation that we did above. These 30 J of potential energy are entirely converted to kinetic energy. So set $30\text{ J} = \frac{1}{2}mv^2$ and solve for v. (Use the 0.10-kg mass of the arrow for m.) The speed is 24 m/s.

When you are asked to do a calculation, it's worth asking: Is the answer reasonable? One m/s is about 2 miles per hour.[6] This arrow was traveling in the neighborhood of 50 mph—about the speed of a car, but less than the speed of a professional baseball pitch.[7]

> **Exam Tip from an AP Physics Veteran**
> If the spring were instead hanging vertically instead of vibrating horizontally, this problem would be *solved the same way*. When you have a vertical spring, define the equilibrium position $x = 0$ as the place where the mass would hang at permanent rest. Then, $F = kx$ gives the *net* force on the hanging object, rather than just the force applied by the spring; and you can use $PE = \frac{1}{2}kx^2$ (not mgh) to calculate the object's potential energy.

Power

Whether you walk up a mountain or whether a car drives you up the mountain, the same amount of work has to be done on you. (You weigh a certain number of newtons, and you have to be lifted up the same distance either way!) But clearly there's something different

[5] The force of the bowstring is a conservative force, because the potential energy it stores is part of the mechanical energy of the bowstring-arrow system.

[6] For those of you who didn't grow up in America, 1 m/s is a bit less than 4 km/hr.

[7] For those of you who are uncomfortable with the World's Greatest Sport, this is less than the speed of a tennis player's serve.

about walking up over the course of several hours and driving up over several minutes. That difference is power.

Power: energy/time

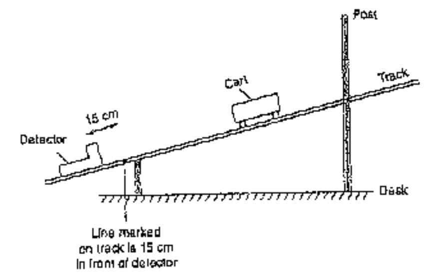
Power is, thus, measured in units of joules/second, also known as watts. A car engine puts out hundreds of horsepower, equivalent to maybe 100 kilowatts; whereas, you'd work hard just to put out a power of a few hundred watts.

〉 Practice Problems

Note: An additional drill involving graphical analysis of a mass on a spring is available in Chapter 18.

1. A 0.5-kg cart released from rest at the top of a smooth incline has gravitational energy of 6 J relative to the base of the incline.
   (a) Calculate the cart's speed at the bottom of the incline.
   (b) When the cart has rolled halfway down the incline, the cart's gravitational potential energy will be:
   (A) Greater than 3 J
   (B) Less than 3 J
   (C) Equal to 3 J
   Justify your answer.
   (c) When the cart has rolled halfway down the incline, the cart's kinetic energy will be
   (A) Greater than 3 J
   (B) Less than 3 J
   (C) Equal to 3 J
   (D) Unknown without knowledge of the cart's speed
   Justify your answer.
   (d) When the cart has rolled halfway down the incline, the cart's speed will be:
   (A) Half of its speed at the bottom
   (B) Greater than half of its speed at the bottom
   (C) Less than half of its speed at the bottom
   (e) A 1.0-kg cart is released from rest at the top of the same incline. At the bottom, it will be moving
   (A) Faster than the 0.5-kg cart
   (B) Slower than the 0.5-kg cart
   (C) The same speed as the 0.5-kg cart

2. A compressed vertical spring stores 40 J of potential energy. The spring has a 0.1-kg stone resting on it. The spring is released, throwing the stone straight up into the air.
   (a) How much kinetic energy will the stone have when it first leaves the spring?
   (b) How much gravitational energy, relative to the spot where the stone was released, will the stone have when it reaches the peak of its flight?
   (c) Calculate the height above the release point to which the stone travels.
   (d) Suggest something we could change about this situation that would cause the stone to reach a height double that calculated in Part (c).



3. In the laboratory, a motion detector records the speed of a cart as a function of time, stopping its reading when the cart is 15 cm in front of the detector at the line marked on the track. The cart is released from rest at the position shown.
   (a) The kinetic energy of the cart at the line marked on the track is equal to the gravitational energy mgh of the cart at its initial position.

On the preceding diagram, draw and label the distance you would measure for the height h of the cart.

(b) Explain in some detail how commonly available laboratory equipment could be used to measure the labeled height h.

(c) If the height h were doubled in a second trial, the motion detector would read
 (A) The same speed as in the first trial
 (B) Two times the speed in the first trial
 (C) Four times the speed in the first trial
 (D) $\sqrt{2}$ times the speed in the first trial

Justify your answer.

4. Student A lifts a 50-N box from the floor straight up to a height of 40 cm in 2 s. Student B lifts a 40-N box straight up from the floor to a height of 50 cm in 1 s.

(a) Compared to Student A, Student B does
 (A) The same work but develops more power
 (B) The same work but develops less power
 (C) More work but develops less power
 (D) Less work but develops more power

Justify your answer.

(b) Now Student A instead lifts the 50-N box from the floor diagonally, moving the box 40 cm to the right and 40 cm upward in the same 2 s.
 (A) Compared to the work he did originally, does Student A do more, less, or the same work?
 (B) Compared to the power he developed originally, does Student A develop more, less, or the same power?

〉 Solutions to Practice Problems

1. (a) Here gravitational energy is converted to kinetic energy. ("Smooth" generally means friction is negligible.) Kinetic energy at the bottom will be 6 J, which is equal to $\frac{1}{2}mv^2$. Plug in the 0.5-kg mass of the cart and solve for v to get 4.9 m/s.

 (b) Gravitational potential energy is mgh. Since the h term is in the numerator and not squared or square rooted, cutting h in half cuts the whole equation in half as well; so the cart's gravitational potential energy will be 3 J.

 (c) Any gravitational potential energy lost by the cart must be converted to kinetic energy. The cart lost 3 J of gravitational energy, so the cart now has 3 J of kinetic energy.

 (d) At the bottom, to find the speed we set 6 J = $\frac{1}{2}mv^2$. Solving for the cart's speed, we get $v = \sqrt{\frac{2\cdot(6\text{ J})}{m}}$. We're going to cut that 6 J term in half. Since the 6 J is under the square root, though, we don't cut the speed in half; instead, we multiply the speed by $\frac{1}{\sqrt{2}}$. If you don't see why that gives a speed greater than half the speed at the bottom, try carrying out the entire calculation—you should get 3.5 m/s.

 (c) The energy conversion is the same—we're setting gravitational energy (mgh) at the top equal to kinetic energy ($\frac{1}{2}mv^2$) at the bottom. Notice the mass on both sides—the mass cancels, and so doesn't affect the result. Thus, the speed is the same for either cart. Again, feel free to do the calculation with m = 1.0 kg to get 4.9 m/s again.

2. (a) The potential energy stored in the spring will be converted to the kinetic energy of the stone. So 40 J.

 (b) Again, with no nonconservative forces acting, the stone's kinetic energy is entirely converted to potential energy—40 J.

 (c) That 40 J of gravitational energy at the peak can be set equal to mgh. Solve for h to get 40 m.

 (d) The energy conversion here is spring energy → gravitational energy. Mathematically, that's $\frac{1}{2}kx^2 = mgh$. Solving for h, $h = \frac{kx^2}{2mg}$. To double the height, we could use a spring with double the spring constant of the original spring, because k is unexponented[8] in the numerator. We could compress the spring 1.4 times its original compression, since when the x in the numerator is squared that would multiply the whole expression by 2. We could use a rock of mass 0.05 kg; with m in the denominator, halving the mass doubles the entire expression. (Okay, I suppose we could go to some new planet where g is 5 N/kg. If you will fund that trip, I'll give you credit for that answer.)

3. (a) In the equation mgh, h represents the vertical distance above the lowest position or some reference point. Here the reference point is the line on the track. The motion detector reads the front of the cart, so h must be measured to the front of the cart, not the middle or back. See above for the answer.

 (b) Use a meterstick, obviously, but it's not an easy measurement to make. First, measure the vertical distance from the desk to the line on the track 15 cm in front of the detector. Then, measure the vertical distance from the desk to the front of the cart; then subtract the two distance measurements. You can get more accurate measurements if you use a bubble level and plumb bob to ensure the table is horizontal and the measurements are vertical. (If you want to measure along the track the distance from the front of the cart to the line, use an angle measurer to get the angle of the track, then use trigonometry.)

 (c) The energy conversion here is gravitational energy → kinetic energy. In equations, that's $mgh = \frac{1}{2}mv^2$. Solve for v to get $v = \sqrt{2gh}$. The variable h is in the numerator but under the square root, so doubling h multiplies the speed by the square root of 2, choice D.

4. (a) The work done by the student is equal to the change in the box's gravitational potential energy—that's mgh. The time it takes the student to lift the box doesn't depend on time at all. Plugging in, we find that Student A does (5 kg)(10 N/kg)(0.40 m) = 20 J of work on the box. Student B does (4 kg)(10 N/kg)(0.50 m) = 20 J of work, also. Now, power is work divided by the time it takes to do that work. Since they do the same amount of work, whoever takes less time to do the work develops more power. That's Student B. So the answer is choice A.

 (b) (A) As above, the work done by Student A on the box is mgh. Here h represents the vertical height above the lowest position. Since that vertical height is still 40 cm, Student A has done the same work. (You could also recognize that the horizontal displacement is not parallel to the box's weight, or to the force Student A applied to lift the box.[7])
 (B) Since the work done by Student A is the same as before, and it took the same amount of time, the power (= work/time) is the same.

〉 Rapid Review

- Work is done when a force is exerted on an object, and that object moves parallel to the direction of the force.
- A "conservative" force converts potential energy to other forms of mechanical energy when it does work. Thus, a conservative force does not change the mechanical energy of a system.
- The only types of potential energy used in AP Physics 1 are due to gravity (mgh) and due to a spring ($\frac{1}{2}kx^2$).
- The work-energy theorem can be written as $W_{NC} = (\Delta KE) + (\Delta PE)$, where W_{NC} is the work done by a nonconservative force.
- Whenever the force on an object is not steady, energy conservation methods must be used to solve the problem. The most common of these situations are curved tracks, springs, and pendulums.

[8] If "unexponented" is a word...

[7] The force Student A applies on the box is straight up, at least while the box is moving at constant speed. Once the box starts moving horizontally, no force in that direction is necessary to continue its motion—that's Newton's first law.

CHAPTER

Rotation

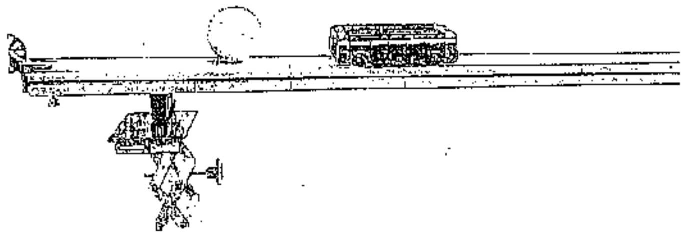
IN THIS CHAPTER

Summary: When an object rotates, the rotation obeys rules similar to those of a moving object. Most equations and concepts have analogues for rotation. Whereas mass describes an object's resistance to a change in speed, *rotational inertia* describes an object's resistance to a change in rotational speed. Rotational inertia depends on mass as well as on the distribution of that mass.

KEY IDEA

Definitions

- **Centripetal acceleration** is the name given to an object's acceleration toward the center of a circle. "Centripetal" simply means "toward the center."
- **Torque** occurs when a force applied to an object could cause the object to rotate.
- The **lever arm** for a force is the closest distance from the fulcrum, pivot, or axis of rotation to the line on which that force acts.
- Everything covered in the previous review chapters is sufficient to describe "translational" motion. When an object rotates around a central point, or when an object is itself rotating as it moves, then we need some additional concepts. Just know that each of these rotational quantities is not truly new. Each rotational quantity should be treated exactly the same way as its translational analogue: for example, if you know how to deal with linear momentum, then angular momentum applies the same ideas to rotating objects.



The ball rolling down the ramp in the preceding figure has translational kinetic energy because it is moving. It also has rotational kinetic energy because it is spinning. The ball is losing gravitational energy because it is changing its vertical height; that loss of potential energy is converted into a gain in kinetic energy because no nonconservative forces act on the ball.

Circular Motion

Example 1: A car of mass 1,000 kg travels at constant speed around a flat curve that has a radius of curvature of 100 m. The car is going as fast as it can go without skidding.

Does this car have an acceleration? Why yes, it does, even though it moves at constant speed. Its acceleration along its direction of motion is zero, because the car isn't speeding up or slowing down. However, its direction of motion is always changing; acceleration is technically the change in an object's vector *velocity* each second, and a change in the direction of motion is a change in velocity.

KEY IDEA

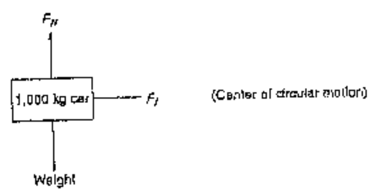
FACT: When an object moves in a circle, it has an acceleration directed toward the center of the circle. The amount of that acceleration is

$$\frac{v^2}{r}$$

STRATEGY

Strategy: When an object is moving in a circle, often a standard Newton's second law approach is correct. Draw a free-body diagram, and then write $F_{net} = ma$ in each direction.

Draw a free-body diagram of this car as it moves. It's most useful to view this car from behind; let's say the car is turning to the right, so that the center of the circular motion is in the place indicated.



The forces are as follows:

F_N is the normal force of the road on the car.
Weight is the force of the Earth on the car.
F_f is the frictional force of the road on the car.

Wait, why is friction acting to the right? Shouldn't friction act opposite the direction of motion?

Ah. There certainly could be friction or air drag acting backwards, opposite the direction of motion; but with the car moving at constant speed, that would have to be canceled by a forward engine force.[1] Since we know the car moves at constant speed, that's probably not particularly relevant to the problem.

When a car goes around a flat curve, some sort of force must act toward the center of the circle—otherwise, the centripetal acceleration couldn't exist. How do we know it's friction in this case? Imagine the car were moving forward on a slick, flat sheet of ice. The car couldn't go around a curve at all, then; turning the wheels would do nothing. On an asphalt road, it's the static frictional force of the asphalt on the tires that pushes the car toward the center of the circle.

No matter what kind of question you're asked about this situation, the next step is to use Newton's second law in both the vertical and horizontal directions. Vertically, the car's acceleration is zero; the car isn't burrowing into the road or lifting off the road. Horizontally, we don't have a numerical value for the acceleration, but we know its equation: $\frac{v^2}{r}$

$$F_N - weight = 0$$
$$F_f = m\frac{v^2}{r}$$

The first equation tells us that the normal force on the car is equal to the car's weight of 10,000 N. In order to calculate the friction force on the car, we'd need to know one of two pieces of additional information. On one hand, if we know the car's speed, we can use the second equation to calculate F_f. On the other hand, since we know the normal force, if we knew the coefficient of static friction between the car's tires and the road, we could calculate the friction force using $F_f = \mu F_n$.

STRATEGY

Strategy: You cannot allow yourself to become angry or frustrated when you don't have enough information to complete a calculation. Sometimes, an AP Physics 1 problem will be deliberately concocted to ask, "What additional information would you need to solve this problem?" (Remember, actual calculation on the exam will be rare.) Or, often, some seemingly necessary information will be omitted, because it will turn out that the omitted information is irrelevant.

For example, an excellent problem using this situation might *not* give the mass of the car but instead give the car's speed and ask what minimum coefficient of friction would be necessary for the car to round the curve. Pretend, say, that the car's speed is 20 m/s.

[1] Technically, this would be a force of static friction between the tires and the road, but that's a different day's lesson.

I can't do that. I need the mass to calculate the force of friction.

Well, true, if you needed a value for the force of friction, but that's not the question. We want the coefficient of friction.

Yeah, I know. The equation is $F_f = \mu F_n$. I need the mass to calculate the friction force, and since the normal force is equal to the weight, I need the mass to calculate that, too. This isn't possible.

When you're stuck with a calculation that you think needs a value that wasn't given, try just making up that value. It's likely that the unknown value will cancel out. Or, if you want to be more elegant, assign a variable to the unknown value.

Let the mass be m. The equations we wrote from the free-body diagram show that the friction force is $m\frac{v^2}{r}$. The normal force is the weight of the car, or the car's mass times the gravitational field g of 10 N/kg. Now use the equation for friction force:

$$F_f = \mu F_n$$

$$m\frac{v^2}{r} = \mu\,(mg)$$

Look at that: solve for μ, and the masses cancel. You can plug in the 20-m/s speed and the 100-m radius to get $\mu = 0.4$. A car of *any* mass can go around this curve, as long as the coefficient of friction is at least 0.4.[2]

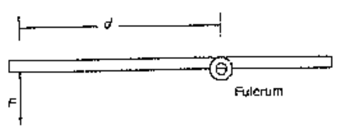
Torque

KEY IDEA

FACT: The torque τ provided by a force is given by the equation

$$\boxed{\tau = Fd_{\perp}}$$

I write the symbol "⊥" by the d to emphasize that the distance we want is the perpendicular distance from the line of the force to the fulcrum.[3] Usually that's an easy distance to visualize.

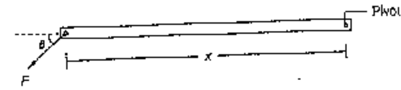


[2]That makes perfect sense—ever see those yellow signs warning of the appropriate speed for going around a curve? They just say, "Curve: 40 mph." They certainly *don't* say something silly like "Curve: go 10 mph for every 500 kg in your vehicle."

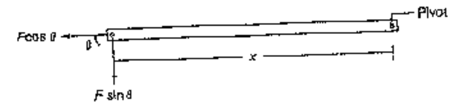
[3]The "fulcrum" is the point about which an object rotates, or could rotate.

What if a force isn't acting perpendicular to an extended object, like the force F on the pivoted bar that follows?

Example 2:



The easiest way to find the torque applied by this force is to break the force F into vertical and horizontal components.

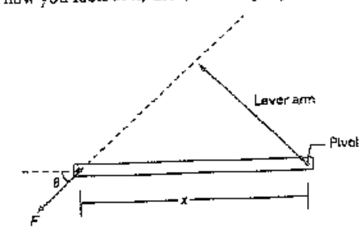


The vertical component of F applies a torque of $(F\sin\theta)x$. The horizontal component of F does not apply any torque, because it could not cause the bar to rotate. The total torque provided by the force F is just $(F\sin\theta)x$.

Lever Arm

This distance $d_{\perp}$ is sometimes referred to as the "lever arm" for a force. By definition, the lever arm for a force is the closest distance from the fulcrum to the line on which that force acts.

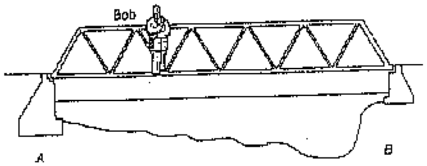
An alternate method of determining the torque applied by the force in Example 2 would be to find the lever arm instead of breaking F into components. Extend the line of the force in the diagram—now it's easy to label the lever arm as the closest distance from the pivot to the line of the force. By trigonometry, you can figure out that the lever arm distance is equal to $x\sin\theta$. No matter how you look at it, then, the torque provided by F is still $(F\sin\theta)x$.



Calculations with Torque

You may be asked to calculate a force or a torque when an extended object experiences multiple forces, but generally only when that object is in equilibrium—that is, when up forces equal down forces, left forces equal right forces, and counterclockwise torques equal clockwise torques.

Example 3: Bob is standing on a bridge. The bridge itself weighs 10,000 N. The span between pillars A and B is 80 m. Bob, whose mass is 100 kg, stands 20 m from the center of the bridge as shown.



Generally, a problem with a bridge, plank, or some sort of extended object will ask you to describe or solve for the forces supporting the bridge. The approach is to make a list of torques acting in each direction, clockwise and counterclockwise, and then set the counterclockwise torques equal to the clockwise torques.

Aargh. Where's the fulcrum? This bridge isn't rotating anywhere!

Exactly. Since the bridge is not actually rotating, you can choose anywhere you like as the fulcrum. It's easiest in this case, to choose one of the supports as the fulcrum, because then that support provides zero torque, and the lever arm for that force would be zero.

Let's choose support A as the fulcrum. What torques do we see?

The force of Support B (I'll call it F_B) provides a torque equal to F_B(80 m), because support B is 80 m from support A. This torque is *counterclockwise*, because pushing up on the bridge pivoted at A would rotate the bridge this way ↺.

The weight of Bob provides a clockwise torque of (1,000 N)(20 m) = 20,000 m·N. (We don't use 100 kg, because that's a mass, not a force; the force acting on the bridge is due to Bob's weight.)

> **Exam Tip from an AP Physics Veteran**
> In a torque problem with a heavy extended object, just pretend that the object's weight is all hanging at the object's center of mass.

The 10,000-N weight of the bridge itself provides a torque. Pretend that all 10,000 N act at the center of the bridge, 50 m away from each support. A weight pulling down at the bridge's center would tend to rotate the bridge clockwise. The torque we want here is (10,000 N)(50 m) = 500,000 m·N, clockwise.

Now, set counterclockwise torques equal to clockwise torques:

$$F_B(80 \text{ m}) = 20{,}000 \text{ m}\cdot\text{N} + 500{,}000 \text{ m}\cdot\text{N}$$

Solve for F_B to get 5,300 N. This is reasonable because pillar B is supporting *less* than half of the 11,000-N weight of the bridge and Bob. Because Bob is closer to pillar A, and otherwise the bridge is symmetric, A should bear the majority of the weight—and it does.

Rotational Kinematics

An object's "rotational speed" says how fast the object rotates—that is, how many degrees or radians it rotates through per second. Rotational speed is generally given by the lowercase Greek variable omega, ω. An object's "rotational acceleration" α describes how much the rotational speed changes in one second. The variable θ represents the total angle through which an object rotates in some time period.

Just like position x, speed v, and acceleration a are related through the kinematics formulas given in Chapter 1, rotational angle, speed, and acceleration are related by the same formulas:

1. $\omega_f = \omega_0 + \alpha t$
2. $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

It is highly unlikely that you'll be asked to actually make much of a calculation with these equations. Rather, you might be asked to rank rotating objects by their angular speed or acceleration; or you might be asked, "Is it possible to calculate...." The general approach to a calculation should be identical to that for nonrotational kinematics: Make a chart with the five variables in it. If you can identify a value for three of the five variables, the problem is solvable.

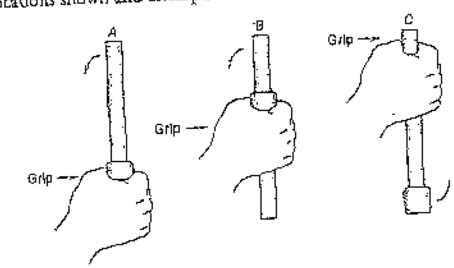
Rotational Inertia

Just like "inertia" refers to an object's ability to resist changes in its motion, "rotational inertia"[4] refers to an object's ability to resist changes in its rotational motion. Two things affect an object's ability to resist rotational motion changes: the object's mass and how far away that mass is from the center of rotation.

There are three ways to figure out something's rotational inertia:

1. For a single "point" particle that is moving in a circle around an axis, its rotational inertia is given by the equation $I = mr^2$. Here, m is the mass of the particle, and r is the radius of the circle.
2. For an object with some kind of structure, like a spinning ball, a disk, or a rod, a formula for its rotational inertia will generally be given if you need it.
3. For a system consisting of several objects, you can add together the rotational inertia of each of the objects to find the total rotational inertia of a system.

Example 4: Three meter-long, uniform 200-g bars each have a small 200-g mass attached to them in the positions shown in the diagram. A person grips the bars in the locations shown and attempts to rotate the bars in the directions shown.



[4]The AP Physics 1, Algebra-Based Exam will always use the term "rotational inertia," usually represented by the variable I. Many textbooks and teachers will use the older term "moment of inertia" to refer to the same quantity. Don't be confused.

It's possible that you might be asked to calculate the rotational inertia of one of these gripped rods. If so, in the problem statement you'd be given the formula to calculate the rotational inertia of a rod: $\frac{1}{12}ML^2$ when pivoted in the center, and $\frac{1}{3}ML^2$ when pivoted at the end.[5] You know that the rotational inertia of the small mass is mr^2, where r is the distance from the mass to the grip. In each situation, add the rotational inertia of the rod to the rotational inertia of the small mass.

For Grip A, the rod's rotational inertia is $\frac{1}{3}ML^2 = \frac{1}{3}(0.2\ \text{kg})(1\ \text{m})^2 = 0.067\ \text{kg}\cdot\text{m}^2$. The small mass contributes nothing to the rotational inertia, because it is not rotating—the r term in $I = mr^2$ is zero. Thus, the total rotational inertia is $0.067\ \text{kg}\cdot\text{m}^2$.

For Grip B, the rod is pivoted in the center, so its rotational inertia is $\frac{1}{12}ML^2 = 0.017\ \text{kg}\cdot\text{m}^2$. The small mass isn't rotating. So the total rotational inertia is just $0.017\ \text{kg}\cdot\text{m}^2$.

Finally, for Grip C, the rod's rotational inertia is $\frac{1}{3}ML^2 = \frac{1}{3}(0.2\ \text{kg})(1\ \text{m})^2 = 0.067\ \text{kg}\cdot\text{m}^2$. The small mass's rotational inertia is $(0.2\ \text{kg})(1\ \text{m})^2 = 0.20\ \text{kg}\cdot\text{m}^2$. Thus, the total rotational inertia is the sum of both contributions, $0.27\ \text{kg}\cdot\text{m}^2$.

Calculations aren't usually the point, though. This situation is just begging to become a ranking task: without any specific values for the masses or lengths of the items, rank the grips by their rotational inertia. As long as we know that the small mass is equal to the mass of the rod, and that the rods are equal in length, then the ranking can be done. You can see that the small mass contributes nothing to the rotational inertia in A and B without calculation.

You can see that Grip A provides a greater rotational inertia than Grip B, *even without knowing the formulas $\frac{1}{12}ML^2$ and $\frac{1}{3}ML^2$*. Reason from the properties of an object that contribute to its rotational inertia: mass, and how far away that mass is from the axis of rotation. The rods have the same mass in A and B. But Rod A has much more mass that is far away from the grip; Rod B has more mass closer to the grip. Therefore, Rod B will be easier to rotate, and Rod A will have more rotational inertia.

Then, of course, Grip C combines the "worst" of both worlds: just the rod by itself provides the same rotational inertia as in Grip A, but the mass is also contributing to the rotational inertia. The final ranking would be $I_C > I_A > I_B$.[6]

Newton's Second Law for Rotation

Just as linear acceleration is caused by a net force, ***angular*** acceleration is caused by a net ***torque***:

$$\boxed{\tau_{net} = I\alpha}$$

Only the net torque can cause an angular acceleration. If more than one force is applying a torque, then use the sum[7] of the torques to find the angular acceleration.

[5] Here, L represents the length of the bar.

[6] Okay, you *do* need to get comfortable with this sort of verbal explanation of concepts that refers to equations and facts but doesn't make direct calculation. If you are confused on this sort of problem, it is okay to make up values for whatever you need, and calculate. I don't at all recommend memorizing all the different formulas for rotational inertia of a rod, sphere, hoop, disk, etc. But if you happen to remember them, it's fine to use them.

[7] Or, if the torques are acting in opposite directions, use the difference.

Example 5: A turntable of known mass and radius is attached to a motor that provides a known torque. Using the torque of the motor and the rotational inertia of the turntable in Newton's second law for rotation, then, using rotational kinematics, a student predicts that it should take 5.0 s for the turntable to speed up from rest to its maximum rotational speed. When the student measures the necessary time, though, he discovers that it takes 6.8 s to reach maximum rotational speed.

First, you should be able to describe how to perform such a measurement in your laboratory. There's a bazillion ways of doing so: the idea is to make many measurements of rotational speed until that rotational speed doesn't change. Rotational speed could be measured with a video camera and a protractor, by running the video frame-by-frame to see how many degrees the turntable advances per frame. Or tape a tiny piece of paper to the edge of the turntable, and have that paper trigger a few photogates; you can measure the angle between the photogates, and the photogates will tell you how much time it took for the turntable to traverse that angle.

This problem is setting up for you to figure out why the prediction didn't match the measurement.[8] The most obvious issue is that the torque provided by the motor might not be the *net* torque on the turntable. Friction in the bearings of the turntable could easily provide a torque in the opposite direction to that provided by the motor. Thus, the real value for net torque will be *lower* than the value the student used. And, by $\tau_{net} = I\alpha$, the real angular acceleration will be lower; finally, by $\omega_f = \omega_0 + \alpha t$, a smaller angular acceleration to get to the same final speed means that the time t will be longer than predicted.

Angular Momentum

The rotational analogue of the impulse-momentum theorem involves torque and angular momentum rather than force and linear momentum:

$$\boxed{\Delta L = \tau \cdot \Delta t}$$

Here τ is the net torque acting on an object, and Δt is the time during which that torque acts. The change in the object's angular momentum is ΔL.

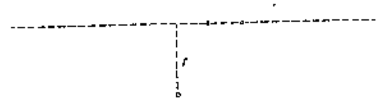
An object's angular momentum can be calculated using three methods:

1. For a single "point" particle that is moving in a circle around an axis, its angular momentum is given by $L = mvr$. Here, r represents the radius of that circle.
2. For a single "point" particle that is moving in a straight line,[9] its angular momentum is also given by $L = mvr$, but in this case r represents the "distance of closest approach"

[8] Please don't automatically say "human error." There's no such thing as "human error," and using that phrase is basically an automatic wrong on the AP Exam.

[9] Angular momentum must always be defined with respect to some central axis of rotation. For most rotating objects, that axis is obvious. For a particle moving in a straight line, you have to say what position you're calculating angular momentum for, but the particle can still have angular momentum.

from the line of the particle's motion to the position about which angular momentum is calculated, as shown below.



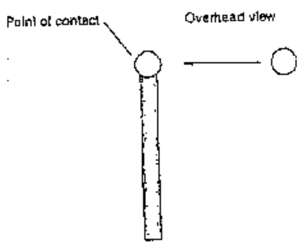
3. For an extended object with known rotational inertia I, angular momentum is given by $L = I\omega$.

For most problems, you can reason by analogy to the linear impulse-momentum theorem. Just as a force applied for some time will change an object's linear momentum, a torque applied for some time will change an object's angular momentum. Just as the area under a force versus time graph gives the change in an object's linear momentum, the area under a torque versus time graph gives the change in an object's angular momentum.

Conservation of Angular Momentum

FACT: In any system in which the only torques acting are between objects in that system, angular momentum is conserved. This effectively means that angular momentum is conserved in *all* collisions, but also in numerous other situations.

Example 6: A uniform rod is at rest on a frictionless table. A ball of putty, whose mass is half that of the rod, is moving to the left, as shown. The ball of putty collides with and sticks to the rod.



Exam Tip from an AP Physics Veteran
Usually, the fulcrum or axis of rotation is obvious. But when an object is not forced to pivot at some specific position, if it rotates it will most likely rotate about its center of mass.

Start with what quantities are conserved for the putty-rod system. It's a collision, in which the only forces involved are the force of the putty-on-rod and rod-on-putty. Therefore, linear momentum is conserved. Similarly, the putty applies a torque to the rod because it pushes on the rod at a position away from its center of mass. But the only torques involved are provided by objects in the system, so angular momentum is also conserved.

This collision cannot be elastic because the putty sticks to the rod; so kinetic energy was not conserved. Mechanical energy was also not conserved because the kinetic energy of the putty was not stored as potential energy in a spring or gravitational field. Of course, the sum of *all* forms of energy was conserved, because whatever kinetic energy was lost by the putty-rod system was converted to microscopic internal energy, and thus the temperature of the putty-rod will increase.

Where's the center of mass? Using the equation from Chapter 12, call $x = 0$ the top of the rod. Pretend the rod is 1-m long and 1 kg in mass. Then the putty is 0.5 kg in mass. So $m_{putty}(0) + m_{rod}(0.5\text{ m}) = m_{total}(x_{cm})$.[10] Plugging in the masses, you get $x = 0.33$ m.

But the problem emphatically did not say that the rod was 1-m long.

Right. Whatever the rod's length, its center of mass is one-third of the way down the rod.

Exam Tip from an AP Physics Veteran
When you're asked about the center of mass speed, you can ignore all angular stuff. In that case, just treat the collision as if these were carts colliding.

Again, you can make up values to find the speed of the center of mass after collision. If the putty's initial speed were 1 m/s, then the total momentum before collision is 0.5 N·s. By conservation of momentum, that's also the total momentum after collision, but the mass of the combined objects after collision is 1.5 kg. The speed of the center of mass would be 0.33 m/s. But since the initial speed wasn't given, you can only definitively say that the speed of the center of mass after collision will be one-third of the speed of the putty before collision.

Conservation of Angular Momentum Without Collisions

FACT: Angular momentum is conserved any time an object, or system of objects, experiences no net torque.

Example 7: A person stands on a frictionless turntable. She and the turntable are spinning at one revolution every two seconds.

Sure, the person can wiggle and exert a torque on the turntable. But this torque is internal to the person-turntable system. The person thus cannot change the angular momentum of the person-turntable system.

This doesn't mean that she can't change her angular speed. What if she throws her arms way out away from her body?[11] Her rotational inertia would change, because she'd have the same total mass but more of that mass would be far away from the center of rotation. Her angular momentum can't change. By $L = I\omega$, to keep a constant L with a bigger I, the angular speed ω must decrease. This is the physical basis for how figure skaters can control their spinning.

[10] It's (0.5 m) because the center of mass of the rod by itself must be halfway down the rod.

[11] AP reader Matt Sckalor quite reasonably asks, "After she throws the first arm, what part of her body does she use to throw the other arm?" Perhaps I should say she "extends" her arms.

Example 8: A planet orbits a sun in an elliptical orbit.

You won't have to deal with elliptical orbits in the sense of making calculations, or using Kepler's Laws and Physics C-style calculations. But you can understand that angular momentum of a planet orbiting a sun must be conserved. Since the force of the sun on a planet is always on a line toward the sun itself, this force cannot provide any torque—there's no lever arm. With no torque exerted on a planet, that planet cannot change its angular momentum. Treat the planet as a point particle; its angular momentum is $L = mvr$, where r is the distance from the sun. Whenever r is big, v must be small to keep L constant. The farther away the planet is from the sun, the slower it moves.

Rotational Kinetic Energy

KEY IDEA

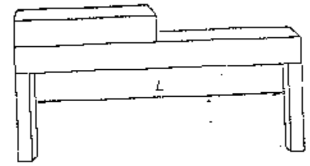
FACT: When an object is rotating, its rotational kinetic energy is

$$\tfrac{1}{2}I\omega^2$$

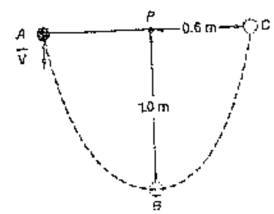
In Example 6, the putty-rod system has *both* rotational *and* linear kinetic energy after the collision. The total kinetic energy is then the sum of $\frac{1}{2}mv^2$, where v is the speed of the center of mass, plus the rotational kinetic energy.

When an object rotates, the work-energy theorem still applies. The kinetic energy terms each include the addition of rotational and linear kinetic energy. In Example 6, work is done by a nonconservative force (the force of the rod on the putty). To find out how much work was done, set $W_{NC} = \Delta KE + \Delta PE$. This situation doesn't involve a spring or a changing vertical height, so $\Delta PE = 0$. The kinetic energy was originally just $\frac{1}{2}mv^2$ for the putty; after the collision, the kinetic energy is as discussed in the previous paragraph, the speed of the center of mass plus the rotational kinetic energy.

> Practice Problems



1. A uniform wooden block has a mass m. On it is resting half of an identical block, as shown above. The blocks are supported by two table legs, as shown.
   (a) Which table leg, if either, should provide a larger force on the bottom block? Answer with specific reference to the torque equation.
   (b) In terms of given variables and fundamental constants, what is the force of the right-hand table leg on the bottom mass?



2. A small ball of mass m moving on a frictionless horizontal surface is attached to a rubber band whose other end is fixed at point P. The ball moves along the dotted line in the preceding figure, stretching the rubber band. When it passes Point A, its velocity is v directed as shown.
   (a) Is the angular momentum of the ball about Point P conserved between positions A and B?
   (b) Is the linear momentum of the ball conserved between positions A and B?
   (c) Describe a system in this problem for which mechanical energy is conserved as the ball moves from A to B.
   (d) Explain why the net force on the ball at Point B is not $\frac{mv^2}{(1.0\text{ m})}$.

3. A smooth, solid ball is released from rest from the top of an incline, whose surface is very rough. The ball rolls down the incline without slipping.
   (a) Describe in words the energy conversion for the ball from its release until it reaches the bottom of the incline.
   (b) Explain why work done by the friction force should not be included in the energy conservation statement in Part (a), even though the surface of the incline is rough.
   (c) Is the mechanical energy of the ball-Earth system conserved during its roll?
   (d) This ball is replaced by a new ball, whose surface and mass are identical to the first ball, but which is predominantly hollow inside. Describe any differences in its roll down the incline without slipping, with explicit reference to forms of energy.

> Solutions to Practice Problems

1. (a) Call the force of the left support F_L and the force of the right support F_R. Consider the middle of the bottom block as the fulcrum. Then one clockwise torque acts: $F_L\cdot(L/2)$. Two counterclockwise torques act, though: $F_R\cdot(L/2)$ and $(1/2)m\cdot(1/4)L$. The point is that if you have to add something to the torque provided by the right support to get the torque provided by the left support, the left support thus provides more torque. Because the supports are the same distance from the center, the left support provides more force, too.
   (b) You certainly could use the reasoning in Part (a) with the fulcrum in the center, along with the total support force equaling $1.5Mg$ (vertical equilibrium of forces). However, it's much easier mathematically to just call the left end of the rod the fulcrum. Then the counterclockwise torque is $F_R\cdot L$. The clockwise torque is $(1/2)mg\cdot(L/4) + mg\cdot(L/2)$. Set these equal and play with the fractions to get $F_R = (\frac{1}{4}+\frac{1}{2})mg = \frac{3}{4}mg$.

2. (a) Angular momentum is conserved when no torques external to the system act. Here the system is just the ball. The only force acting on the ball is the rubber band, which is attached to Point P. The force applied by the rubber band can't have any lever arm with respect to P and thus provides no torque about point P, so the ball's angular momentum about point P can't change. Angular momentum is conserved.

(b) Linear momentum is conserved when no forces external to the system act. Here the system is just the ball. The rubber band is external to the system and applies a force; therefore, linear momentum is *not* conserved.

(c) Mechanical energy is conserved when no force external to the system does work. The rubber band does work on the ball, because it applies a force and stretches in a direction parallel to the force it produces. Consider the rubber band part of the system. The post at Point P still applies a force to the ball–rubber band system, but since the post doesn't move, that force does no work on the ball–rubber band system. Any kinetic energy lost by the ball will be stored as elastic energy in the rubber band. The mechanical energy of the ball–rubber band system is conserved.

(d) The general form of this equation is fine—the ball's path at Point P is, at least in the neighborhood of P, approximately circular. The ball experiences a centripetal acceleration at Point B, and centripetal acceleration is v^2/r. The problem is that if the r term is 1.0 m, then the v term must represent the speed at Point B. With angular momentum conserved, the total of mvr must always be the same. The ball's mass doesn't change. The distance r from Point P gets bigger from A to B, so the speed must get smaller. The equation given uses the given variable v which represents the speed of the ball at Point A, not the speed at B, and so is invalid.

3. (a) Gravitational energy at the top (because the ball is some vertical height above its lowest position) is converted to both rotational and translational kinetic energy at the bottom—rotational because the ball will be spinning, and translational because the ball's center of mass will move down the incline.

(b) The ball rolls without slipping. The force of the very rough surface on the ball is tangent to the ball's surface, but the ball's surface never slips parallel to this force. Work is force times displacement parallel to the force. It's the force of *static* friction that must be acting on the ball, and since static friction doesn't move the ball's surface, the displacement term is zero, and static friction can do no work.

(c) Mechanical energy is conserved when no nonconservative forces act. Here the Earth's gravitational field can give the ball kinetic energy, but since the Earth is part of the system and since the gravitational force is conservative, that still allows for conservation of mechanical energy. Friction is a nonconservative force, but here friction does no work.

(d) The hollow ball of the same mass will have greater rotational inertia, because the mass is concentrated farther from the center of rotation. The ball's gravitational energy before the rolling begins is the same as the previous scenario, because the height of the incline is the same. The total kinetic energy at the bottom will not change; the question is how much of that kinetic energy will be rotational, and how much will be translational.

Rotational KE is $\frac{1}{2}I\omega^2$; the angular speed ω depends on the translational speed v. (The faster the ball is moving, the more it's rotating, too.) Therefore, rotational kinetic energy depends on v^2. Translational kinetic energy also depends on v^2 in the formula $\frac{1}{2}mv^2$. The hollow ball has bigger I. The speed v must be lower for the hollow ball so that $\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ adds to the same value for both balls.

› Rapid Review

- When an object moves in a circle, it has an acceleration directed toward the center of the circle. The amount of that acceleration is $\frac{v^2}{r}$.
- The torque τ provided by a force is given by the equation $\tau = Fd_{\perp}$.
- In any system in which the only torques acting are between objects in that system, angular momentum is conserved. This effectively means that angular momentum is conserved in *all* collisions, but also in numerous other situations.
- Angular momentum is conserved any time an object, or system of objects, experiences no net torque.
- When an object is rotating, its rotational kinetic energy is $\frac{1}{2}I\omega^2$.

CHAPTER 3

Gravitation

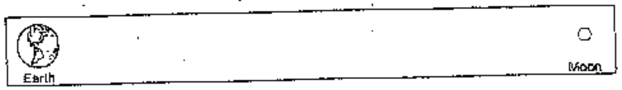
IN THIS CHAPTER

Summary: The force on an object due to gravity is *mg*, where *m* is mass and *g* is the gravitational field. The gravitational field produced by an object of mass *m* is $G\frac{M}{d^2}$, where d is the distance from the object's center. Want to be sure you know the difference between a gravitational field, a gravitational force, and the universal gravitation constant? This chapter explains these concepts.

Definitions

- The **gravitational field** *g* near a planet tells how much 1 kg of mass weighs at a location. Near Earth's surface, the gravitational field is 10 N/kg.
- The **gravitational force** of a planet on any other object in the planet's gravitational field is *mg*, where *m* is the mass of the object experiencing the force.
- **Newton's gravitation constant** is the universal constant $G = 6 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.
- The **free-fall acceleration** (sometimes imprecisely called the **acceleration due to gravity**) near a planet is, by an amazing coincidence of the universe, equal to the gravitational field near that planet. Near Earth, then, the free-fall acceleration is 10 m/s per second because the gravitational field is 10 N/kg.

The gravitational force is the weakest of the fundamental forces in nature. However, when enormous amounts of mass congregate—as in a star or a planet—the gravitational force becomes dominant. The picture as follows gives a hint of the scales involved in studying stars and planets—it's the Earth and the Moon, but drawn to approximately the proper scale. Sure, the Earth seems humongous, especially when you're caught in traffic. But *all* planets and stars seem small when the distances between them are considered.



The word *gravity* by itself is an ambiguous term. Begin this chapter by carefully reading the differences between all the various things that could be referred to by the word *gravity*. See the four preceding definitions, each of which relate to the word.

Determining the Gravitational Field

The gravitational field is a vector quantity—this means it has an amount and a direction. The direction is always toward the center of the Earth (or whatever is creating the gravitational field).

FACT: The amount[1] of gravitational field depends on two things: the mass of the planet creating the field (M) and the distance you are from that planet's center (d). The relevant equation for the gravitation field g produced by a planet is

$$g = G\frac{M}{d^2}$$

Some books, and probably even the AP Exam, will use the variable r for the distance from the planet's center. That's fine, but know that this r does *not* necessarily stand for the radius of the planet—it means the distance from the planet's center.

Example 1: A 20-kg rover sits on newly discovered Planet Z, which has twice the mass of Earth and twice the diameter of Earth.

You do not get a table of astronomical information on the AP Physics 1, Algebra-Based Exam. Nevertheless, you might well be asked to calculate the gravitational field near the surface of Planet Z. How can that be done without knowing the mass of Planet Z? You're expected to be fluent in semiquantitative reasoning.

Even though you don't know the value of the mass of the Earth, you know that the mass of Planet Z is twice Earth's mass. Whatever the exact mass, the numerator in the gravitational field equation will double for Planet Z.

The surface of a planet is one planet-radius away from the planet's center; so here, d means the radius of the planet. Planet Z's diameter is twice Earth's, which also means Z's radius is twice Earth's. The d term in the denominator is doubled; because d is squared, the entire denominator is multiplied by 2^2, which is 4.

[1]The exam will refer not to the "amount," but to the "magnitude" of a vector quantity. Just translate in your head.

Combining these effects of mass and radius, the numerator is multiplied by 2, the denominator multiplied by 4, so the entire gravitational field of the Earth is multiplied by one-half. We know Earth's gravitational field—that's 10 N/kg. Planet Z produces a gravitational field of 5 N/kg at the surface. No calculator is necessary.

The formula shows that the gravitational field produced by a planet drops off rapidly as you get far from the planet's center: if you double your distance from the planet's center, then you cut in one-fourth the value of the gravitational field.

This seems easy enough, but think about reality for a moment. Just how often do you double your distance from the center of the Earth? The radius of the Earth is about 4,000 miles. Even when you fly in an airplane, you're no more than about seven miles above the surface; so your distance from the center of the Earth is *still* about 4,000 miles.

The point is, unless you're an astronaut, the gravitational field near the surface of a planet is a constant value. Don't be tricked by the d^2 in the denominator—that only matters when you're considering objects in space.

Determining Gravitational Force

FACT: The weight of an object—that is, the gravitational force of a planet on that object—is given by mg.

That 20-kg rover would weigh 200 N on Earth (20 kg times 10 N/kg). But on Planet Z, the rover weighs only 100 N (that's 20 kg times 5 N/kg).

A weight of 100 N means that Planet Z pulls the rover downward with 100 N of force. What about the force of the rover on Planet Z? That's got to be so small it's negligible, right?

Wrong. Newton's Third Law says that the force of Planet Z on the rover is equal to the force of the rover on Planet Z. The rover pulls up on Planet Z with a force of 100 N.

Now, as you might suspect, you'd never notice or measure any effect from the rover's 100-N force on Planet Z. Planet Z is enormously massive—in the neighborhood of 10^{25} kg. By $F_{net} = ma$, you can calculate that the *acceleration* provided to the planet is immeasurably small.[2] It's the force that's the same, and the acceleration that's different.

Force of Two Planets on One Another—Order of Magnitude Estimates

Example 2: The Earth has a mass of 6.0×10^{24} kg. The Sun has a mass of 2.0×10^{30} kg. The Earth orbits the sun in a circle of radius 1.5×10^{11} m.

FACT: The gravitational force of one object on another is given by

$$F = \frac{Gm_1m_2}{d^2}$$

[2]And the rover's force on the planet is certainly not the net force on the planet, so this $F_{net} = ma$ calculation is silly anyway.

So it seems straightforward, if calculator intensive, to calculate the force the Sun exerts on the Earth (or vice versa). Just plug in the numbers. But that's *not* a likely AP Physics 1 exercise! No one cares whether you can use the buttons on your calculator correctly.

Instead, you might be asked, "Which of the following is closest to the force of the Sun on the Earth?"

(A) 10^{12} N
(B) 10^{22} N
(C) 10^{32} N
(D) 10^{42} N

Look how far apart these answer choices are. Don't use a calculator—instead make an *order of magnitude estimate*. Plug in just the powers of 10, and the answer will leap off the page. Leaving out the units of each individual term for simplicity, in the equation $F = \frac{Gm_1m_2}{d^2}$ we have

$$\frac{(10^{-11})(10^{24})(10^{30})}{(10^{11})^2}$$

That's easy to simplify without a calculator—add exponents in the numerator, and then subtract the exponents in the denominator.

$$\frac{(10^{43})}{(10^{22})} = 10^{21}\,\text{N}.$$

But that's not one of the choices.

The only reasonable choice, though, is (B) 10^{22} N. The others are at least a factor of a billion too big or too small. Sure, you could have spent five minutes plugging in the more precise values into your calculator,[3] getting 3.3×10^{22} N as the answer. The order of magnitude estimate is as precise as would ever be necessary on the AP Physics 1 exam, and it's a lot easier, too.

Gravitational Potential Energy

Near the surface of the Earth, the potential energy provided by the gravitational force is

$$PE_{gravity} = mgh$$

That's plenty good enough for calculations with everyday objects.

However, if you're talking about objects way out in space, the gravitational potential energy possessed by two objects near one another is given by

$$PE_{gravity} = G\frac{M_1M_2}{d}$$

[3]And then spend another 10 minutes swearing at the calculator because you left out a parenthesis, or you forgot to type a negative sign, or you forgot a decimal.

This equation is usually written with a negative sign; that negative sign simply indicates that this potential energy is *less* than zero; and in outer-space situations, zero potential energy means the objects are infinitely far away from each other.

This gravitational potential energy can be converted into kinetic energy—if two planets move toward one another due to their mutual gravitational attraction, you might be able to figure out how fast they move by calculating the gravitational potential energy possessed before and after they move. Any lost potential energy was converted into kinetic energy.

Gravitational and Inertial Mass

Example 3: Neil Armstrong had a mass of 77 kg when he went to the moon. The gravitational field on the Moon is one-sixth that on Earth.

The term "mass" is often colloquially defined as the amount of "stuff" in an object. You're likely to see an AP question of the form, "What was Neil Armstrong's mass on the Moon?" The answer is, still 77 kg. Sure, Neil's weight on the Moon was smaller than his weight on Earth, because the gravitational field on the Moon is smaller, but since he didn't cut off his leg or go on a starvation diet, his mass didn't change.

FACT: Gravitational mass indicates how an object responds to a gravitational field.

FACT: Inertial mass indicates how an object accelerates in response to a net force.

FACT: In every experiment ever conducted, an object's gravitational mass is equal to its inertial mass.

The AP Exam requires you to distinguish between the two meanings of mass. Simply put, if there's acceleration involved, you're talking about inertial mass. You'll be asked to design experiments to measure each type of mass.

So let's say you want to figure out who has more *gravitational* mass, you or Neil Armstrong. Just put each of you on a balance scale—the one with the bigger scale reading has more gravitational mass. You could use a spring scale, too—whoever compresses the springs more experiences more gravitational force in the same gravitational field, and so has more gravitational mass.

But to figure out experimentally who has more *inertial* mass, you'd put each of you in an identical buggy. Speed up each buggy using the same net force for the same amount of time. Whichever of you has sped up by more—that is, whichever experienced the greater acceleration under the same net force—has the smaller inertial mass.

And if your experiment gives contradictory results—say, that Neil has more inertial mass but that you have more gravitational mass—then you should reject that result as ridiculous. An exam question might pose just this type of situation in which you have to reject impossible results.[4]

Fundamental Forces: Gravity Versus Electricity

The gravitational force is one of several "fundamental" forces in nature. The only other fundamental force on the AP Exam is the electrical force. In this context, "fundamental" means that all forces on the AP Exam are manifestations of one of these two forces.

[4]Well, if you really have repeatable and unassailable results of this nature, you should publish. I guarantee you'll win a Nobel Prize for your work, which is a bit more likely to earn college credit than a 5 on the AP Exam.

Oh yeah? What about, say, friction?

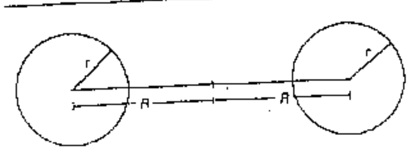
All of the everyday forces that result from "contact" between two objects are fundamentally electrical. The electron shells in the outermost atomic layers of the objects interact, causing friction, normal forces, tensions, punches, etc. Really. But AP Physics 1 doesn't care at all about the details of these interactions; just know that they're electrical.

As discussed in Chapter 16, the electrical force between two particles is generally much, much stronger than the gravitational force between them. The AP Exam is likely to ask you to explain why gravity is such a dominant everyday force, then.

The answer is twofold. First of all, consider the overwhelmingly massive scale of the Earth. It's tens of thousands of miles in circumference, and a trillion trillion kilograms in mass. And even with that much stuff in it, the gravitational force it exerts on you is equal to the (electrical) contact force of the ground pushing up on you.

The bigger issue is that masses always cause a gravitational field that attracts other masses—there's no such thing as a repulsive gravitational force. Electrical forces, though, can be both repulsive and attractive, depending on the sign of the charges interacting. Since in general the Earth is electrically neutral, all that unimaginably immense stuff in the Earth has very little net electrical effect, leaving gravity to keep you firmly attached to the ground.

> Practice Problems



1. Two stars, each of mass M, form a binary system. The stars orbit about a point a distance R from the center of each star, as shown in the diagram above. The stars themselves each have radius r.
   (a) In terms of given variables and fundamental constants, what is the force each star exerts on the other?
   (b) In terms of given variables and fundamental constants, what is the magnitude of the gravitational field at the surface of one of the stars due only to its own mass?
   (c) In terms of given variables and fundamental constants, what is the magnitude of the gravitational field at the midpoint between the stars?
   (d) Explain why the stars don't crash into each other due to the gravitational force between them.

2. A space shuttle orbits Earth 300 km above the equator.
   (a) Explain why it would be impractical for the shuttle to orbit 10 km above the Earth's surface (about 1 km higher than the top of Mount Everest).
   (b) A "geosynchronous orbit" means that the shuttle will always remain over the same spot on Earth. Explain and describe the calculations you would perform in order to determine whether this orbit is geosynchronous. You should not actually carry out the numerical calculations, just describe them in words and show them in symbols.
   (c) The radius of Earth is 6,400 km. At the altitude of the space shuttle, what fraction of the surface gravitational field g does the shuttle experience?
   (d) When the shuttle was on Earth before launch, the shuttle's mass (not including any fuel) was 2×10^6 kg. At the orbiting altitude, what is the shuttle's mass, not including fuel?

3. A satellite is in circular orbit around an unknown planet. A second, different satellite also travels in a circular orbit around this same planet, but with an orbital radius four times larger than the first satellite.
   (a) Explain what information must be known in order to calculate the speed the first satellite travels in its orbit.
   (b) Compared to the first satellite, how many times faster or slower is the second satellite's speed?
   (c) Bob the bad physics student says:
   "The gravitational force of the planet on a satellite in circular orbit depends inversely on the orbital radius squared. Since the second satellite's orbital radius is four times that of the first satellite, the second satellite experiences one-sixteenth the gravitational force that is exerted on the first satellite."
   Explain what is wrong with Bob's explanation.

4. A spacecraft is positioned between the Earth and the Moon such that the gravitational forces on the spacecraft exerted by the Earth and the Moon cancel.
   (a) Is this position closer to the Moon, closer to the Earth, or halfway in between?
   (b) Are the gravitational forces on the spaceship (the force exerted by the Moon, and the force exerted by the Earth) a Newton's third law force pair?

> Solutions to Practice Problems

1. (a) The term in the denominator of Newton's law of gravitation refers to the distance between the centers of the two stars. That distance is given as $2R$. So the answer is $G\frac{M^2}{(2R)^2}$ or $G\frac{M^2}{4R^2}$. Notice you must use the notation given in the problem—this means a capital R here. (The M is squared because the equation for gravitational force multiplies the masses of the stars applying and experiencing the force. Since the masses of both stars are the same, you get $M \cdot M = M^2$.)
   (b) When calculating gravitational field at the surface of a star, the term in the denominator is the star's radius. That's r here. So $G\frac{M}{r^2}$. (The M is *not* squared here because the field is produced by a single star, not an interaction between two stars.)
   (c) The left-hand star provides a field of $G\frac{M}{R^2}$ at the location of the midpoint, pointing toward the left-hand star. The right-hand star also provides a field of $G\frac{M}{R^2}$, pointing toward the right-hand star. These fields with the same amount but opposite direction cancel out when they add as vectors, producing a net gravitational field of zero at the midpoint.
   (d) The direction of a force is not the same as the direction of an object's motion. Here, at any time a star is moving along an orbit, tangent to the radius of the orbit. The force applied by the other star will always be toward the center of the orbit, perpendicular to the direction of the star's motion. When a force is applied perpendicular to an object's velocity, the result is circular motion. The centripetal force changes the direction (not the amount) of the star's speed, but the force itself always changes direction so that it is pulling toward the center of the circular motion.

2. (a) The shuttle must be above the atmosphere in order to maintain a circular orbit without continually burning fuel. If the Earth had no atmosphere, then a satellite could orbit at any distance from the surface that's greater than the tallest mountain. But air resistance in the atmosphere does work on the shuttle, reducing its mechanical energy. At 300 km above the surface, though, the shuttle is above the atmosphere and experiences no forces in or against the direction of travel.
   (b) This question requires a calculation: the force on the shuttle is $G\frac{M_{earth}M_{shuttle}}{d^2}$, where d is the distance from the center of the Earth to the location of the shuttle. Since this force

is a centripetal force, we can set it equal to $\frac{M_{shuttle}v^2}{d}$, where v is the speed of the shuttle in orbit. If the orbit is geosynchronous, that doesn't mean that the shuttle goes the same speed as a position on Earth, it means that the *period* of the orbit is 24 hours—it goes around the Earth in the same time as a location on the surface does.

To determine the period of the orbit, set the speed equal to the orbit's circumference ($2\pi d$) divided by the period, which I'll call T. Here's how our equation looks now:

$$G\frac{M_{earth}M_{shuttle}}{d^2} = \frac{M_{shuttle}\left(\frac{2\pi d}{T}\right)^2}{d}.$$

There is lots of algebra here, but notice that all values are things that can be looked up: G, the mass of the Earth, the mass of the shuttle (which cancels anyway), and d, which is the radius of the Earth plus 300 km. Solve this equation for T. If the value of T turns out to be 24 hours, then the orbit is geosynchronous; if not, the orbit is not geosynchronous.[5]

(c) Your first instinct might be that you need the mass of the Earth to answer this question, because the equation for the gravitational field is $G\frac{M}{R^2}$. On one hand, we could calculate the mass of Earth knowing that the gravitational field near the surface is 10 N/kg—just plug in the given value of R and G from the table of information. That will work. It's more elegant to solve in variables: The fraction we want is

$$\frac{G\frac{M}{(6700\text{ km})^2}}{G\frac{M}{(6400\text{ km})^2}}.$$

The G and M cancel, whatever their value. Work the improper fraction to get that the gravitational field 300 km above the surface is $\frac{(6400\text{ km})^2}{(6700\text{ km})^2} = 91\%$ of g at the surface.

(d) Mass is the amount of "stuff" in an object. It doesn't matter where that object is in the universe, 1 kg of mass is 1 kg of mass. Unless the shuttle loses a wing, its mass is still 2×10^6 kg.

3. (a) Set the gravitational force on the satellite equal to the formula for centripetal force:

$$\frac{GM_{planet}M_{satellite}}{d^2} = \frac{M_{satellite}v^2}{d}.$$

We'll want to be able to solve for v. The mass of the satellite cancels, and G is a universal constant. But we'll need to know d, the distance of the satellite's orbit from the center of the planet. And we'll need to know the mass of the planet or the period of the orbit.

(b) When we solve for v, we get $v = \sqrt{\frac{GM_{planet}}{d}}$. The second satellite has four times the orbital radius, which is represented by d. Multiplying by four in the denominator under the square root multiplies the whole expression by one-half. The second satellite's speed is one-half as large.

(c) The equation for the force of the planet on the satellite is $F = G\frac{M_{planet}M_{satellite}}{d^2}$. Sure, Bob is right about the inverse square dependence on d, but he's assumed that the satellites have the same masses as each other. If they do, Bob is correct; if not, then, the mass of the satellite shows up in the numerator of the force equation.

4. (a) The relevant equation here comes from setting the forces on the spacecraft (of mass m) equal:

$$G\frac{M_{earth}m}{d_{earth}^{\ 2}} = G\frac{M_{moon}m}{d_{moon}^{\ 2}}.$$

The mass of the spacecraft cancels, as does the G. Since the mass of the Earth is bigger than the mass of the Moon, the equation shows that the distance of this location from Earth d_{earth} must be larger than the distance of the location from the Moon d_{moon}. The location is closer to the Moon.

(b) A Newton's third law force pair cannot act on the same object. The force of the Moon on the spacecraft is paired with the force of the spacecraft on the Moon; the force of the Earth on the spacecraft is paired with the force of the spacecraft on the Earth.

[5] If you do all the plugging and chugging—which are *not* necessary—you'll find that the orbit is not geosynchronous. Geosynchronous orbits are somewhere in the 35,000-km range above Earth's surface.

› Rapid Review

- The amount of gravitational field depends on two things: the mass of the planet creating the field (M) and the distance you are from that planet's center (d). The relevant equation for the gravitation field g produced by a planet is $g = G\frac{M}{d^2}$.
- The weight of an object—that is, the gravitational force of a planet on that object—is given by mg.
- The gravitational force of one object on another is given by $F = \frac{Gm_1m_2}{d^2}$.
- Gravitational mass indicates how an object responds to a gravitational field.
- Inertial mass indicates how an object accelerates in response to a net force.
- In every experiment ever conducted, an object's gravitational mass is equal to its inertial mass.

CHAPTER

Electricity: Coulomb's Law and Circuits

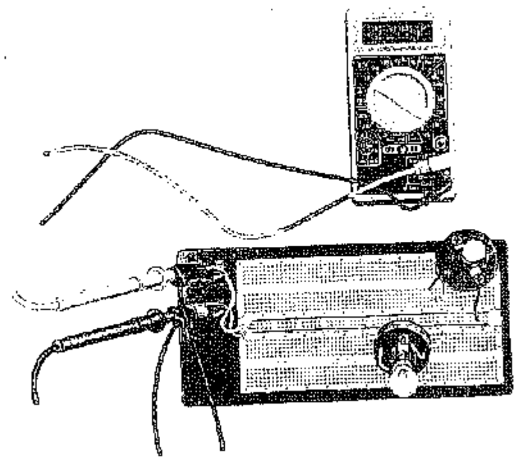
IN THIS CHAPTER

Summary: Coulomb's law says that the force of one charged particle on another is $k\frac{q_1q_2}{d^2}$. AP Physics 1 deals only with electrical forces between exactly two charges. But AP Physics 1 does require an understanding of direct-current circuits, including series resistors, parallel resistors, and combinations thereof. This chapter shows you how to deal with circuits, both using conceptual descriptions and using calculations.

Definitions

- **Electric charge** (Q) exists due to excess or deficient electrons on an object. Charge comes in two kinds: positive and negative. The unit of charge is the coulomb.
- **Electric current** (I) is the flow of (positive) charge per second. The units of current are amperes. One ampere means one coulomb of charge flowing per second.
- **Resistance** (R), measured in ohms (Ω), tells how difficult it is for charge to flow through a circuit element.
- **Resistivity** (ρ) is a property of a material, which implies what the resistance would be of a meter-cube bit of that material.
- **Voltage** is electrical potential energy per coulomb of charge.
- Resistors are connected in **series** if they are connected in a single path.
- Resistors are connected in **parallel** if the path for current divides, then comes immediately back together.

The AP Physics 1, Algebra-Based Exam requires you to learn about two aspects of electricity. First, you must understand how charged objects apply forces to each other in isolation, as when a balloon sticks to the wall. Next, you need to know about circuits, in which gazillions of submicroscopic flowing charges produce effects that can be measured and observed. The following picture shows such a circuit, in which flowing charge causes two bulbs to light up. The meter is positioned to read the voltage across one of the bulbs.



Electric Charge

All matter is made up of three types of particles: protons, neutrons, and electrons. Protons have an intrinsic property called "positive charge." Neutrons don't contain any charge, and electrons have a property called "negative charge."

The unit of charge is the coulomb, abbreviated C. One proton has a charge of 1.6×10^{-19} coulombs.

Most objects that we encounter in our daily lives are electrically neutral—things like couches, for instance, or trees, or bison. These objects contain as many positive charges as negative charges. In other words, they contain as many protons as electrons.

When an object has more protons than electrons, though, it is described as "positively charged"; and when it has more electrons than protons, it is described as "negatively charged." The reason that big objects like couches and trees and bison don't behave like charged particles is because they contain so many bazillions of protons and electrons that an extra few here or there won't really make much of a difference. So even though they might have a slight electric charge, that charge would be much too small, relatively speaking, to detect.

Tiny objects, like atoms, more commonly carry a measurable electric charge, because they have so few protons and electrons that an extra electron, for example, would make a big difference. Of course, you can have very large charged objects. When you walk across a carpeted floor in the winter, you pick up lots of extra charges and become a charged object yourself . . . until you touch a doorknob, at which point all the excess charge in your body travels through your finger and into the doorknob, causing you to feel a mild electric shock.

Electric charges follow a simple rule: *Like charges repel; opposite charges attract.* Two positively charged particles will try to get as far away from each other as possible, while a positively charged particle and a negatively charged particle will try to get as close as possible.

Only two types of charge exist. If a question on the exam purports to give evidence of a third type of charge, reject that evidence; if a question suggests a kind of charge that repels *both* positive and negative charges, reject that suggestion as silly.

Coulomb's Law

Just *how much* do charged objects attract and repel? Coulomb's law tells how much.

KEY IDEA

FACT: Coulomb's law is an equation for the force exerted by one electrical charge on another:

$$F = k\frac{Q_1Q_2}{d^2}$$

The Qs represent the amount of charge on each object; the d represents the distance between the two objects. The variable k is the Coulomb's law constant, $9.0 \times 10^9\ N{\cdot}m^2/C^2$.

As with Newton's law of universal gravitation for the force of one planet on another, you're not going to be asked to calculate much with Coulomb's law. Rather, the questions will be qualitative and semiquantitative. For example, if the amount of charge A is doubled, what happens to the force of charge A on charge B? (It doubles.) Or, if you double the distance between two charges, what happens to the force of one on the other? (It is cut by one-fourth.)

Conservation of Charge

KEY IDEA

FACT: The total amount of charge in a system (or in the universe itself) is always the same.

Equal amounts of positive and negative charge can cancel out to make an object neutral, but those charges still exist on the object in the form of protons and electrons. Charge can be transferred from one object to another, for example by touching two charged metal spheres together or scuffing your feet on the carpet, but the total amount of charge stays the same.

Conservation of charge is usually discussed in conjunction with circuits and Kirchoff's junction rule, as discussed below.

Circuits

A circuit is any wire path that allows charge to flow. Technically, a current is defined as the flow of positive charge.[1] Under what conditions would this charge flow through a wire? This would occur when a coulomb of charge has a potential energy that's higher at one position in the wire than the other. We call this difference in potential energy per coulomb a "voltage," and a battery's job is to provide this voltage that allows current to flow. Current flows out of a battery from the positive side of the battery to the negative side.

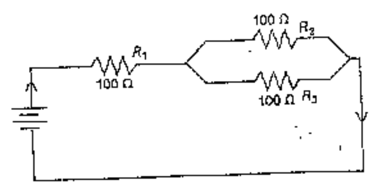
Resistance Versus Resistivity

Resistance tells how difficult it is for charge to flow through something. Usually that "something" is a resistor, or a light bulb, or something that has a known or determinable resistance; usually the resistance of the wires connecting the "somethings" together is nothing, at least compared to the resistance of the things.

FACT: The resistance R of a wire of known dimensions is given by

$$R = \frac{\rho L}{A}$$

Sometimes, though, it's important to know how the properties of a wire affect its resistance. The longer the wire is, the more its resistance. The "wider" the wire is—that is, the bigger its cross-sectional area—the less its resistance. Two wires with the same shapes can have different resistances if they are made of different materials. Assuming the same shape, the wire with more resistance has a greater resistivity, represented by the variable ρ.



[1] No, not the flow of protons . . . okay, look, you really want to know? It's the flow of "holes in the electron sea." Understand? No? Neither do I. For the AP Exam, who cares. Current is the flow of positive charge, *capish*?

Example 1: The preceding circuit diagram contains a battery and three identical 100 Ω resistors.

Questions about circuits will occasionally ask for calculation: Find the voltage across each resistor and the current through each resistor. More often, though, you'll be asked qualitative questions, like which bulb takes the greatest current, or rank resistors from largest to smallest voltage across.

The Four Key Facts About Circuits

FACT: Series resistors each carry the same current, which is equal to the total current through the series combination.

FACT: The voltage across series resistors is different for each but adds to the total voltage across the series combination.

FACT: The voltage across parallel resistors is the same for each and equal to the total voltage across the parallel combination.

FACT: Parallel resistors each carry different currents, which add to the total current through the parallel combination.

Exam Tip from an AP Physics Veteran

Many first-year physics students are more comfortable making calculations with circuit problems than with explaining effects in words. If you are confused by a qualitative circuit question, try answering with a calculation: "Well, with a 150-V battery here's a calculation showing that I get 1 A of current in the circuit, but with a 75-V battery I only get 0.5 A. Thus, cutting the battery's voltage in half also cuts the current in half."

When you see a circuit, regardless of what questions about it are asked, it's worth making a V-I-R chart listing the voltage, current, and resistance for each resistor. Then the four facts above and Ohm's Law—the equation $V = IR$—can be used to find the missing value on any row of the chart. The easiest way to understand the V-I-R chart is to see it in action. Watch.

Start by sketching a chart and filling in known values. Right now, we know the resistance of each resistor. The voltage of the battery isn't given, so make it up. Try 100 V.[2]

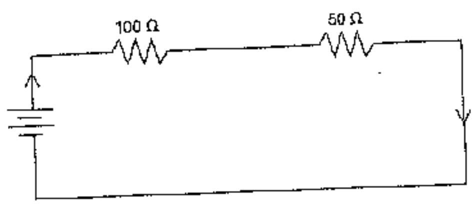
[2] Why is it okay to make up a voltage that wasn't given? Say you're asked to rank the currents in the resistors. It doesn't matter how much current you calculate in each resistor, all that matters is whether R_1 or R_2 takes a bigger current. Any reasonable value will do for answering qualitative questions.

	V	I	R
R_1			100 Ω
R_2			100 Ω
R_3			100 Ω
total	100 V		

Next, simplify the circuit, collapsing sets of parallel and series resistors into their equivalent resistors.

FACT: The equivalent resistance of series resistors is the sum of all of the individual resistors. The equivalent resistance of parallel resistors is less than any individual resistor.

Strategy: When two *identical* resistors are connected in parallel, their equivalent resistance is half of either resistor. Here, then, the parallel combination of resistors has equivalent resistance 50 Ω. Even with nonidentical parallel resistors, you can usually estimate their equivalent resistance enough to do qualitative problems. If you need to make the calculation, the equivalent resistance of parallel resistors is given by $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.



Since the other 100 Ω resistor is in series with the 50 Ω equivalent resistance, the equivalent resistance of the whole circuit is 150 Ω. We can put that equivalent resistance into the chart.

	V	I	R
R_1			100 Ω
R_2			100 Ω
R_3			100 Ω
total	100 V		150 Ω

Mistake: The V-I-R chart is not a magic square—it's merely a tool for organizing your calculations for a complicated circuit. You can *not* just add values up and down the columns.

Ah, progress: Two of the three entries in the "total" row are complete. Therefore, we can use Ohm's Law to calculate the total current in this circuit.

FACT: Ohm's law says that voltage across a circuit element equals that element's current times its resistance:

$$V = IR$$

This equation can *only* be used across a single row in a *V-I-R* chart.

In the total row, (100 V) = I (150 Ω), giving a current in the circuit of $I = 0.67$ A.

Now what? We can't use Ohm's law because we don't have any rows missing just one entry. We go back to the Four Key Facts.

Resistor R_1 is in series with the battery; since current through series resistors is equal to the total current, R_1 must take the entire current flowing from the battery, all 0.67 A. Aha! Put 0.67 A in the chart for the current through R_1, and we can use Ohm's Law to calculate the voltage across R_1: that's 67 V. The chart appears as follows.

	V	I	R
R_1	67 V	0.67 A	100 Ω
R_2			100 Ω
R_3			100 Ω
total	100 V	0.67 A	150 Ω

Now to figure out R_2 and R_3. Look at their 50 Ω equivalent resistor in the redrawn diagram. It's in series with the 100 Ω resistor. Therefore, the 50 Ω resistor must add its voltage to 67 V to get the total voltage of 100 V and the voltage across the 50 Ω equivalent resistor is 33 V.

Then use the facts. Parallel resistors take the same voltage across each that is equal to the total voltage across the combination. Both R_2 and R_3 take 33 V across them. Fill in the chart.

	V	I	R
R_1	67 V	0.67 A	100 Ω
R_2	33 V		100 Ω
R_3	33 V		100 Ω
total	100 V	0.67 A	150 Ω

Now use Ohm's law across the rows for R_2 and R_3 to finish the chart.[3]

	V	I	R
R_1	67 V	0.67 A	100 Ω
R_2	33 V	0.33 A	100 Ω
R_3	33 V	0.33 A	100 Ω
total	100 V	0.67 A	150 Ω

This chart can now be used to answer any qualitative question. Sure, you should give more justification than just "look at my chart here." The chart will ensure you get the *right* answers, and that you have a clue about how to approach the qualitative questions.

For example, the exam might ask the following: Rank the voltage across each resistor from largest to smallest. Easy: $R_1 > R_2 = R_3$. Justify that with a verbal description of why you decided to use the calculations that you did: "Look at the simplification of the circuit to two series resistors. The voltage across these two series resistors must add to the voltage of the battery, but the current through them must be the same. By $V = IR$, the bigger resistor must take the larger voltage; this is R_1. Then R_2 and R_3 take equal voltage because they are in parallel with one another."

[3] Um, is that really right? The current through parallel resistors should add to the total current. But 0.33 A plus 0.33 A gives 0.66 A, and we said the total current was 0.67 A. This is *fine*. Expect that rounding in a V-I-R chart will not allow for ten-figure accuracy. Who cares—the whole point is generally to help answer conceptual questions, anyway.

Kirchoff's Laws: Conservation of Charge and Energy

FACT: Kirchoff's junction rule says that the current entering a wire junction equals the current leaving the junction.

This fact is a statement of conservation of charge: Since charge can't be created or destroyed, if 1 C of charge enters each second, the same amount each second must leave.

FACT: Kirchoff's loop rule says that the sum of voltage changes around a circuit loop is zero.

This fact is a statement of conservation of energy because voltage is a change in the electrical potential energy of 1 C of charge. A battery can raise the electrical potential energy of some charge that flows; a resistor will lower the potential energy of that charge.[4] But the sum of all these energy changes must be zero.

Look back at the Four Key Facts: These are just restatements of Kirchoff's laws and thus of conservation of energy and charge.

The junction rule applies to the facts about current. Series resistors take the same current through each, because there's no junction. The current through parallel resistors adds to the total because of the junction before and after the parallel combination.

The loop rule applies to the facts about voltage. The voltage across series resistors adds to the total voltage because the resistors can only drop the potential energy of 1 C of charge as much as the battery raised the charge's potential energy. The voltage across parallel resistors must be the same because Kirchoff's loop rule applies to all loops of the circuit. No matter which parallel path you look at, the sum of voltage changes must still be zero.

Power in a Circuit

Power is still defined as energy per second, just as it was in the chapter about energy. Resistors generally convert electrical energy to other forms of energy—the amount of power says how quickly that conversion occurs.

FACT: To determine the power dissipated by a resistor, use the equation

$$P = IV$$

Of course, using Ohm's law, you can show that IV is equivalent to I^2R as well as $\frac{V^2}{R}$.

Exam Tip from an AP Physics Veteran

If an AP question asks about power, or equivalently, "the energy dissipated by a resistor per unit time," make a fourth column in your V-I-R chart. Use whichever of the power equations you can to calculate power.

[4] The potential energy can be converted by the resistor to internal energy of the resistor or the surrounding air, raising the temperature, as in a toaster oven; or could be converted to light, as in a lamp; or could even be converted to mechanical energy, as in an electric motor.

Power doesn't obey the Four Key Facts. The total power dissipated by a bunch of resistors is just the sum of the power dissipated by each, whether the resistors are in series, parallel, or whatever.

Circuits from an Experimental Point of View

When a real circuit is set up in the laboratory, it usually consists of more than just resistors—light bulbs and motors are common devices to hook to a battery, for example. For the purposes of computation, though, we can consider pretty much any electronic device to act like a resistor.

But what if your purpose is *not* computation? Often on the AP Exam, as in the laboratory, you are asked about observational and measurable effects. The most common questions involve the brightness of light bulbs and the measurement (not just computation) of current and voltage.

Brightness of a Bulb

The brightness of a bulb depends solely on the power dissipated by the bulb. (Remember, power is given by any of the equations I^2R, IV, or V^2/R). You can remember that from your own experience—when you go to the store to buy a light bulb, you don't ask for a 400-ohm bulb, but for a 100-watt bulb. And a 100-watt bulb is brighter than a 25-watt bulb. But be careful—a bulb's power can change depending on the current and voltage it's hooked up to. Consider this problem.

> A light bulb is rated at 100 W in the United States, where the standard wall outlet voltage is 120 V. If this bulb were plugged in in Europe, where the standard wall outlet voltage is 240 V, which of the following would be true?
> (A) The bulb would be one-quarter as bright.
> (B) The bulb would be one-half as bright.
> (C) The bulb's brightness would be the same.
> (D) The bulb would be twice as bright.
> (E) The bulb would be four times as bright.

Your first instinct might be to say that because brightness depends on power, the bulb is exactly as bright. But that's not right! The power of a bulb can change.

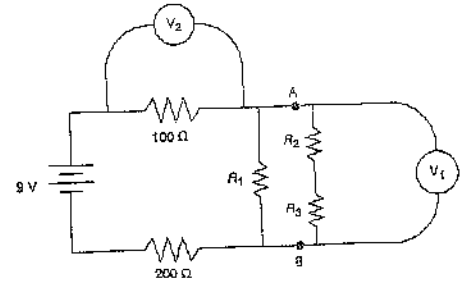
> The resistance of a light bulb is a property of the bulb itself and so will not change no matter what the bulb is hooked to.

Since the resistance of the bulb stays the same while the voltage changes, by V^2/R, the power goes up, and the bulb will be brighter. How much brighter? Since the voltage in Europe is doubled, and because voltage is squared in the equation, the power is multiplied by 4—choice E.

Ammeters and Voltmeters

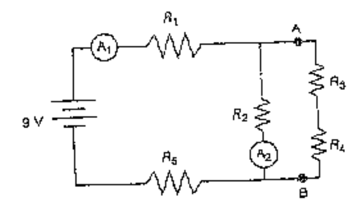
Ammeters measure current, and voltmeters measure voltage. This is pretty obvious, because current is measured in amps, voltage in volts. It is *not* necessarily obvious, though, how to connect these meters into a circuit.

Remind yourself of the properties of series and parallel resistors—voltage is the same for any resistors in parallel with each other. So if you're going to measure the voltage across a resistor, you must put the voltmeter in *parallel* with the resistor. In the following figure, the meter labeled V_2 measures the voltage across the 100 Ω resistor, while the meter labeled V_1 measures the potential difference between points *A* and *B* (which is also the voltage across R_1).



Measuring voltage with a voltmeter.

Current is the same for any resistors in *series* with one another. So, if you're going to measure the current through a resistor, the ammeter must be in series with that resistor. In the following figure, ammeter A_1 measures the current through resistor R_1, while ammeter A_2 measures the current through resistor R_2.



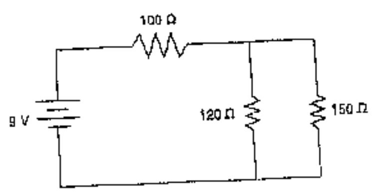
Measuring current with an ammeter.

As an exercise, ask yourself, is there a way to figure out the current in the other three resistors based only on the readings in these two ammeters? Answer is in the footnote.[5]

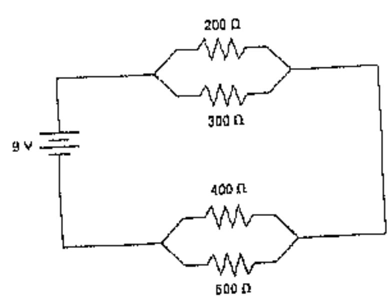
[5]The current through R_5 must be the same as through R_1, because both resistors carry whatever current came directly from the battery. The current through R_3 and R_4 can be determined from Kirchoff's junction rule: subtract the current in R_2 from the current in R_1 and that's what's left over for the right-hand branch of the circuit.

> Practice Problems

Note: Additional drills on circuit calculation can be found in Chapter 18.

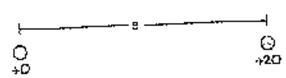


1. Three resistors are connected to a 9-V battery in the circuit shown in the preceding figure. Justify all answers thoroughly.
   (a) Rank the resistors, from greatest to least, by the voltage across each.
   (b) Rank the resistors, from greatest to least, by the current through each.
   (c) Rank the resistors, from greatest to least, by the power dissipated by each.
   (d) Is it possible to replace the 120 Ω resistor with a different resistor and change the voltage ranking? Answer thoroughly in a short paragraph.
   (e) Is it possible to replace the 120 Ω resistor with a different resistor and change the current ranking? Answer thoroughly in a short paragraph.



2. Four resistors are connected to a 9-V battery in the circuit in the diagram.
   (a) Calculate the equivalent resistance of the circuit.
   (b) Calculate the voltage across each resistor.
   (c) Calculate the current through each resistor.
   (d) The 500 Ω resistor is now removed from the circuit. Describe in words, without using calculations, what effect this would have on the circuit. Be specific about each resistor.

3. Justify your answers to the following in short paragraphs.
   (a) Should an ammeter be connected in series or parallel with the resistor it measures?
   (b) Should a voltmeter be connected in series or parallel with the resistor it measures?
   (c) Does an ideal ammeter have large or small resistance?
   (d) Does an ideal voltmeter have large or small resistance?



4. Two positive charges $+Q$ and $+2Q$ are separated by a distance a, as shown above.
   (a) Which is greater, the force of the $+Q$ charge on the $+2Q$ charge, or the force of the $+2Q$ charge on the $+Q$ charge?
   (b) In terms of given variables and fundamental constants, determine the magnitude and direction of the force of the $+2Q$ charge on the $+Q$ charge.
   (c) By what factor would the force calculated in (b) change if the distance between the charges were increased to $3a$?
   (d) Now the $+Q$ charge is replaced by a negative charge of the same magnitude, and the distance between the charges is returned to a. Describe how the magnitude and direction of the force exerted by each charge on the other will change from the original situation.

> Solutions to Practice Problems

1. (a) The circuit can be redrawn, simplifying the two parallel resistors to their equivalent resistance of 67 Ω. Then what's left is two resistors—100 Ω and 67 Ω—in series with the 9-V battery. Since series resistors take the same current through each, by $V = IR$ with I constant, the larger resistance takes the larger voltage. That's the 100 Ω resistor. Parallel resistors take the same voltage across each, equal to the voltage across the equivalent resistor. Ranking by voltage gives $V_{100\,\Omega} > V_{120\,\Omega} = V_{150\,\Omega}$.
   (b) Current only runs through a wire. All the current in the circuit must run through the 100 Ω resistor and then split at the junction with the parallel resistors. The parallel resistors each take the same voltage; by $V = IR$, with constant V, the smaller resistor takes the larger current. That's the 120 Ω resistor. The ranking by current is $I_{100\,\Omega} > I_{120\,\Omega} > I_{150\,\Omega}$.
   (c) Power is current times voltage. The 100 Ω resistor has the largest current *and* the largest voltage, so it has the greatest power. The other two resistors take the same voltage, but the 120 Ω resistor takes more current; thus, it has more power than the 150 Ω resistor. Ranking by power is $P_{100\,\Omega} > P_{120\,\Omega} > P_{150\,\Omega}$.
   (d) On one hand, it's not possible for the two parallel resistors to take different voltages, by definition (or by Kirchoff's loop rule, if you will). But it's totally possible for the parallel combination to take more voltage than the 100 Ω resistor. We need the equivalent resistance of the parallel combination to be greater than 100 Ω; then by $V = IR$ with I constant, the equivalent resistors would take higher voltage. Replace the 120 Ω resistor with, say, a 1,000 Ω resistor. Then the equivalent resistance of the parallel combination would be 130 Ω and would take more voltage than the 100 Ω resistor.
   (e) There's no way to avoid having the largest current go through the 100 Ω resistor; by Kirchoff's junction rule, it must take all the current in the circuit, while current splits between the other two resistors. Now, nothing says that the parallel resistor that comes first in the diagram has to take the larger current. Replace the 120 Ω resistor with, say, a 200 Ω resistor—the voltage will still be the same for both parallel resistors, but now the 150 Ω resistor is the smaller one, taking the larger current.

2. (a) First simplify the two parallel combinations using $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$. This gives 120 Ω for the top branch, and 220 Ω for the bottom branch. These two equivalent resistors are in series with one another, so their resistances add to the equivalent to give 340 Ω.
   (b) Each pair of parallel resistors takes the same voltage which is equal to that across the equivalent resistance of the pair. Treat this as series resistors of 120 Ω and 220 Ω connected to 9 V. The total current in the circuit is found with Ohm's law with the battery and the equivalent resistance of the whole circuit: $I = \frac{9\ V}{340\ \Omega} = 0.026$ A. (That's easier to understand as 26 mA.) The same 26 mA goes through each of the 120 Ω and 220 Ω resistors. Use Ohm's law with 0.026 A and each of these resistances to get voltages of 3.2 V and 5.8 V. The final answer:
   200 Ω resistor: 3.2 V
   300 Ω resistor: 3.2 V
   400 Ω resistor: 5.8 V
   500 Ω resistor: 5.8 V
   (c) We know the voltage and resistance for each individual resistor. Ohm's law can be used to get the current through each by just dividing V/R. The answers are as follows (don't worry if you rounded slightly differently than I did):
   200 Ω resistor: 16 mA
   300 Ω resistor: 11 mA
   400 Ω resistor: 15 mA
   500 Ω resistor: 12 mA
   (d) Start with the entire equivalent circuit. The 9-V battery is unchanged. The total resistance increases—the bottom branch previously had a resistance of 220 Ω but now is just the remaining 400 Ω. By $V = IR$ with constant V,

a larger total resistance causes a smaller total current. And the 400 Ω resistor is now in series in the battery, and so it would take this entire current. The 200 Ω and 300 Ω resistors would still split the current, and in the same proportion; but the total current is smaller than before, so both will take smaller current now. As for voltage, the equivalent series circuit has more resistance in the second branch, so the 400 Ω resistor takes more of the 9 V than before, and the first parallel combination takes less voltage than before.

3. (a) Series resistors take the same current as each other, which is equal to the total—that's Kirchoff's junction rule. An ammeter measures current; it should be connected in series so that all current that passes through the resistor also passes through the ammeter.
   (b) Parallel resistors take the same voltage as each other, which is equal to the total—that's Kirchoff's loop rule. A voltmeter measures voltage; it should be connected in parallel so that it also takes the same voltage as the resistor it's measuring.
   (c) An ammeter is in series with a resistor. If the ammeter has a large resistance, then it also takes a large voltage, leaving less voltage to go across the resistor, and affecting the circuit. If the ammeter has a very *small* resistance, then it takes hardly any of the total voltage, leaving the resistor to have the same voltage and current as in a circuit without the ammeter.
   (d) A voltmeter is in parallel with a resistor. If the voltmeter has a small resistance, then more current will choose the parallel path with the voltmeter than with the resistor, affecting the circuit. But if the voltmeter has a very *large* resistance, then almost none of the current would take the parallel path with the voltmeter in it, leaving the resistor to have the same voltage and current as in a circuit without the voltmeter.

4. (a) These forces are the same—Newton's third law applies to all forces, even electrical forces.
   (b) Coulomb's law is the relevant equation. Using the variables as given, the force of one charge on the other is $k\frac{(Q)(2Q)}{a^2}$. You can rearrange this to $\frac{2kQ^2}{a^2}$ if you want, but it's not mandatory for AP-style credit, but notice that you should *not* be including a plus sign anywhere, or a minus sign if the charges were negative. The magnitude of a force should not have any signs on it. The direction is repulsive, so the $+2Q$ charge pushes the $+Q$ charge to the left.
   (c) The distance between charges is in the denominator, so increasing the distance decreases the force. The distance term is squared, so that multiplying a by 3 multiplies the whole denominator by $3^2 = 9$. Therefore, the magnitude of the force is reduced by a factor of 9.
   (d) Now the force between charges is attractive, meaning that the $-Q$ charge will be pulled to the right. The magnitude of the force will not change from Part (b), since the amount of charge of each item and the distance between charges will not change.

Rapid Review

- Coulomb's law is an equation for the force exerted by one electrical charge on another:

$$F = k\frac{Q_1Q_2}{d^2}$$

- The total amount of charge in a system (or in the universe itself) is always the same.
- The resistance R of a wire of known dimensions is given by $R = \frac{\rho L}{A}$.

- Series resistors each carry the same current, which is equal to the total current through the series combination.
- The voltage across series resistors is different for each but adds to the total voltage across the series combination.
- The voltage across parallel resistors is the same for each and equal to the total voltage across the parallel combination.
- Parallel resistors each carry different currents, which add to the total current through the parallel combination.
- The equivalent resistance of series resistors is the sum of all of the individual resistors. The equivalent resistance of parallel resistors is less than any individual resistor (and, if you have to do the calculation, is given by $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$).
- Ohm's law says that voltage across a circuit element equals that element's current times its resistance: $V = IR$. This equation can *only* be used across a single row in a *V-I-R* chart.
- Kirchoff's junction rule says that the current entering a wire junction equals the current leaving the junction.
- Kirchoff's loop rule says that the sum of voltage changes around a circuit loop is zero.
- To determine the power dissipated by a resistor, use the equation $P = IV$.

CHAPTER

Waves and Simple Harmonic Motion

IN THIS CHAPTER

Summary: This chapter introduces basic properties of waves, especially of sound waves. You'll define wave speed, frequency, and wavelength, and relate them through $v = \lambda f$. Although a wave moves through a material, the pieces of the material themselves do not move. Rather, they tend to oscillate in simple harmonic motion. This chapter describes exactly what that means.

KEY IDEA

Definitions

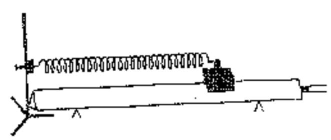
- The **period** is the time for one cycle of simple harmonic motion, or the time for a full wavelength to pass a position.
- The **frequency** is the number of cycles, or the number of wavelengths passing a position, in one second.
- The unit of frequency is the Hz, which means "per second."
- The **amplitude** is the distance from the midpoint of a wave to its crest, or the distance from the midpoint of simple harmonic motion to the maximum displacement.
- The **wavelength** is measured from peak to peak, or between any successive identical points on a wave.
- The **spring constant** k, measured in units of newtons per meter (N/m), is related to the stiffness of a spring.
- A **restoring force** is any force that always pushes an object toward an equilibrium position.

- Nodes are the stationary points on a standing wave.
- Antinodes are the positions on standing waves with the largest amplitudes.

You're probably most familiar with waves on the surface of a lake or pond. Those are transverse waves, which look like the waves on the machine in the following figure. The wavelength of the wave can be measured with a ruler from peak to peak.

Simple Harmonic Motion

Simple harmonic motion refers to a back-and-forth oscillation whose position-time graph looks like a sine function. The typical examples are a mass vibrating on a spring, and a pendulum.



Example: A cart of mass 0.5 kg is attached to a spring of spring constant 30 N/m on a frictionless air track, as shown. The cart is stretched 10 cm from the equilibrium position and released from rest.

KEY IDEA

FACT: The period of a mass on a spring in simple harmonic motion is given by

$$2\pi\frac{\sqrt{m}}{\sqrt{k}}.$$

Of course, the AP Exam will not likely ask, "What is the period of this oscillation?" Rather, it might ask for the specific effect that doubling the mass of the cart would have on the period. Since the m term is in the numerator of the period equation, a bigger mass means a larger (longer) period of oscillation. Since the m is under a square root, doubling the mass multiplies the period by the square root of 2.

What if the amplitude of the motion were doubled? How would that affect the period? Since you don't see an A in the equation for the period, the period would not change. This is a general result for all simple harmonic motion and wave problems: The amplitude does not affect the period.[3]

KEY IDEA

FACT: The frequency and period are inverses of one another.

Once you know by being told or by doing the calculation that the period of this mass on a spring is 0.81 s, you can use your calculator to do 1 divided by 0.81 s, giving a frequency of 1.2 s.

KEY IDEA

FACT: The amount of restoring force exerted by a spring is given by

$$F = kx.$$

The force of the spring on the cart is therefore greatest when the spring is most stretched, but zero at the equilibrium position. And since $a = F_{net}/m$, the acceleration likewise changes from lots at the endpoints, to nothing at the middle.

STRATEGY

This means that you cannot use kinematics equations with harmonic motion. A kinematics approach requires constant acceleration. Instead, when a problem asks for the speed of a cart somewhere, use conservation of energy.

KEY IDEA

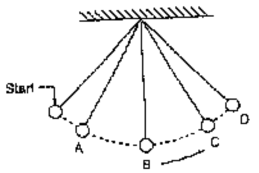
FACT: The spring potential energy is given by

$$PE_{spring} = \tfrac{1}{2}kx^2.$$

[3]There are exceptions for simple and physical pendulums once the amplitude reaches large enough values, but the AP will not likely ask much about these situations.

The energy stored by the spring is thus largest at the endpoints and zero at the equilibrium position. There, the spring energy is completely converted to the kinetic energy of the cart. Where is the cart's speed greatest, then? At the equilibrium position, of course, because kinetic energy is $\frac{1}{2}mv^2$—largest kinetic energy means largest speed.

To calculate the value of the maximum speed, write out the energy conversion from the endpoint of the motion to the midpoint of the motion: spring potential energy is converted to kinetic energy. Translated into equations, you get $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Plug in values, and solve for the speed. Here, you get 0.77 m/s (i.e., 77 cm/s).[2]



Example 2: The pendulum shown in the preceding figure is released from rest at the start position. It oscillates through the labeled positions A, B, C, and D.

Treat a pendulum pretty much the same way as a spring. It's still in harmonic motion; it still requires an energy approach, not a kinematics approach, to determine its speed at any position.

FACT: The period of a pendulum is given by

$$2\pi\frac{\sqrt{L}}{\sqrt{g}}.$$

As always, it's unlikely you're going to plug in numbers to calculate a period. Among the gazillion possibilities, you might well be asked to rank the listed positions in terms of some quantity or other. Here are some ideas:

Rank the lettered positions from greatest to least by the bob's gravitational potential energy. Gravitational potential energy is mgh; the bob always has the same mass and g can't change, so the highest vertical height has the greatest gravitational potential energy. Ranking: D > C = A > B.

Rank the lettered positions from greatest to least by the bob's total mechanical energy. Total mechanical energy means the sum of potential and kinetic energies. Here, with no nonconservative force like friction acting, and no internal structure to allow for internal energy, the total mechanical energy doesn't change. The ranking is as follows: (A = B = C = D).

Rank the lettered positions from greatest to least by the bob's speed. Since gravitational potential energy is converted to kinetic energy, the bob moves fastest when the gravitational potential energy is smallest. Ranking: B > C = A > D.

The gravitational field at the surface of Jupiter is 26 N/kg and on the surface of the Moon, 1.6 N/kg. Rank this pendulum's period near these two planets and earth. Since g is in the denominator of the period equation, the lowest gravitational field will have the greatest period; so $T_{Moon} > T_{Earth} > T_{Jupiter}$. The ranking by frequency would be just the opposite—because frequency is the inverse of period, a bigger g leads to a smaller period but a bigger frequency.

[2]Don't forget to convert the maximum distance from equilibrium to 0.1 meters before plugging into the equation.

Waves

The AP Physics 1 Exam covers only "mechanical" waves, such as sound, or waves on the surface of the ocean. Light waves (i.e., electromagnetic waves) are not covered in detail.

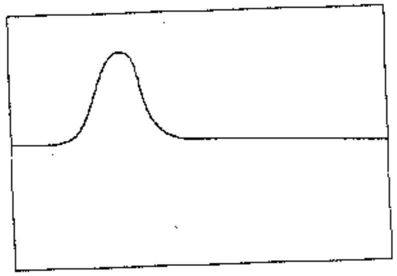
FACT: Whenever the motion of a material is at right angles to the direction in which the wave travels, the wave is a transverse wave.

Example 3: A wave pulse travels to the right through a spring and then extends into a second spring in the preceding figure. The speed of the waves is faster on the right-hand spring.

This is a transverse wave pulse—the coils of the spring travel up and down the page, while the wave itself moves to the right.

FACT: The energy carried by a wave depends on the wave's amplitude.

A good AP-style question might ask you to resketch the diagram so that a pulse of about the same wavelength carries more energy. Make the amplitude—the maximum displacement of the coil above the resting position—bigger, then, because amplitude is related to energy carried by a wave. Keep the pulse about the same length.



FACT: When a wave changes materials, its frequency remains the same.

When this waves moves into the new spring, the wave speeds up, but the frequency remains the same. By $v = \lambda f$, the wavelengths will also increase, because multiplying the same frequency by the wavelength has to give a bigger value for v. The wave will look wider, then, in the new spring.

FACT: When a material vibrates parallel to the direction of the wave, the wave is a **longitudinal wave.**

Example 4: A wave travels through a spring. A picture of the spring is shown in the preceding figure, with point *C* labeled.

This wave is a longitudinal wave—the disturbance is traveling through the spring, so it is traveling right or left. The coils of the spring itself are spread out and compressed. Since the motion of the spring's coils is also left-right, parallel to the way the disturbance is traveling, this is a longitudinal wave.

Try drawing a wavelength on the picture. A wavelength is defined as the distance between two identical positions on the wave. From position C to the next spot where the coils are all stretched out is one wavelength.

Superposition and Interference

When two waves collide, they don't bounce or stick like objects do. Rather, the waves interfere—they form one single wave for just a moment, and then the waves continue on their merry way.

FACT: In constructive interference the crest of one wave overlaps the crest of another. The result is a wave of increased amplitude.

Two wave pulses about to interfere constructively.

The two wave pulses on a string moving toward each other in the preceding figure will interfere constructively, since their amplitudes are on the same side of the string's resting position. When the pulses meet, the wave will look like the dark line in the following figure.

Two wave pulses interfering constructively.

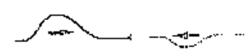
Then the waves continue on in the direction they were originally traveling (see the following figure).

Two wave pulses after interfering constructively.

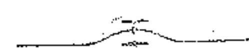
FACT: In destructive interference the crest of one wave overlaps the trough of another. The result is a wave of reduced amplitude.

The same principle applies to waves with amplitudes on opposite sides of the string's resting position, producing destructive interference.

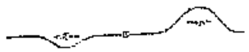
If we send the two wave pulses in the following figure toward each other, they will interfere destructively (next figure following) and then continue along their ways (last figure following).



Two wave pulses about to interfere destructively.



Two wave pulses interfering destructively.



Two wave pulses after interfering destructively.

Standing Waves

A standing wave by definition is a wave that appears to stay in one place. In some positions, the strings vibrate with large amplitude—these are called antinodes. In other positions, the strings don't vibrate at all—these are called nodes.

The reason that a standing wave exists revolves around interference. Waves are traveling back and forth on the string, reflecting off of each end and interfering with each other all willy-nilly. The net effect of all this interference is a pattern of nodes and antinodes.

Example 5: A guitar string of length 1 m is plucked.

Generally when you pluck a string, you produce a standing wave of the longest possible wavelength, and thus the smallest possible frequency. The string will look like the preceding picture if you watch it carefully.

FACT: The wavelength of a standing wave is twice the node-to-node distance.

In this case, the node-to-node distance is equal to the length of the string: 1 m. So the wavelength here is 2 m.

But what if you put your finger very lightly on the middle of the string, forcing a node to exist there? Then you create a harmonic, like the one shown in the following figure. Now the wavelength of this standing wave is 1 m.

And you can put not just one or two, but any whole number of antinodes on a string. (See the following figure.)

FACT: For a string fixed at both ends, or for a pipe open at both ends, the smallest frequency of standing waves is given by

$$f_1 = \frac{v}{2L}$$

where v is the speed of the waves on the string or in the pipe. Other harmonic frequencies for this string or pipe must be whole-number multiples of the fundamental frequency.

When we're dealing with an open pipe, the speed v is generally the speed of sound in air, or about 340 m/s. For a string, the wave speed depends on the tension in the string and the mass of 1 meter worth of string—usually you're talking a few hundred meters per second (m/s), but that can vary.

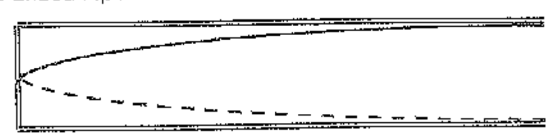
So if this guitar string produces a fundamental frequency of, say, 100 Hz, then the harmonics that can be played are 200 Hz, 300 Hz, 400 Hz, etc. This string *cannot* play a frequency of 150 Hz or 370 Hz, at least unless the length of the string or the tension in the string is changed.

FACT: The pitch of a musical note depends on the sound wave's frequency; the loudness of a note depends on the sound wave's amplitude.

A guitar can generally play any note in a musical scale. But how can that be, if the harmonics are restricted to multiples of the fundamental frequency? Producing harmonics is generally not the way to play a guitar.[3] Rather, the frets on the neck are used to shorten or lengthen the string; since the speed of waves on the string is unchanged, shortening the string lowers the L in $f_1 = \frac{v}{2L}$. The fundamental frequency of the shorter string will be higher, and thus the note played will be higher pitched.

The guitar can be tuned by tightening or loosening the string. A tighter string, for example, will produce a standing wave with higher wave speed. For the same length of string and thus the same wavelength, the frequency will be higher by $v = \lambda f$. A higher frequency means a higher-pitched note.

Closed-Ended Pipe



Example 6: A 1-m-long PVC pipe is covered at one end and open at the other.

When a pipe is open at one end and closed at the other, the standing wave in this pipe must have a node at one end and an antinode at the other. The wave with the longest possible

[3]Harmonics (other than the fundamental) are not the notes you hear. The *overtones* (the harmonics higher than the fundamental) are what give the note its *quality*, which makes a guitar sound like a guitar or a violin sound like a violin. After all, the strings themselves are about the same, so did you ever wonder how it was you are able to tell the difference between the sounds of stringed instruments?

wavelength is shown in the preceding figure. The wavelength of a standing wave is always twice the node-to-node distance. But the wave is so long that we don't even see a second node. It turns out that the wavelength of this wave is 4 m (four times the length of the pipe).

FACT: For a pipe closed at one end, or for a string fixed at one end but free at the other, the smallest frequency of standing waves is given by

$$\boxed{f_1 = \frac{v}{4L}}$$

where v is the speed of the waves on the string or in the pipe. Other harmonic frequencies for this string or pipe must be *odd* multiples of the fundamental frequency.

The speed of waves in this pipe is the speed of sound, or about 340 m/s. The fundamental frequency is 85 Hz. The other frequencies that this pipe can produce are only the *odd* multiples of 85 Hz: 255 Hz, 425 Hz, etc.

Beats

FACT: Beats are rhythmic interference that occurs when two notes of unequal but close frequencies are played.

If you have a couple of tuning forks of similar—but not identical—frequency to play with, or if you have a couple of tone generators at your disposal, you might enjoy generating some beats of your own. They make a wonderful "wa-wa" sound, which is due to a periodic increase and decrease in intensity, or loudness. The frequency of the "wa-wa" is equal to the *difference* between the two frequency generators.

Doppler Effect

Whenever a fire engine or ambulance races by you with its sirens blaring, you experience the Doppler effect. Similarly, if you enjoy watching auto racing, that "Neeee-yeeeer" you hear as the cars scream past the TV camera is also attributable to the Doppler effect.

FACT: The Doppler effect is the apparent change in a wave's frequency that you observe whenever the source of the wave is moving toward or away from you.

To understand the Doppler effect, let's look at what happens as a fire truck travels past you (see the following two figures).

When the fire truck moves toward you, the sound waves get squished together, increasing the frequency you hear.

As the fire truck moves away from you, the sound waves spead apart, and you hear a lower frequency.

As the fire truck moves toward you, its sirens are emitting sound waves. Let's say that the sirens emit one wave pulse when the truck is 50 meters away from you. It then emits another pulse when the truck is 49.99 meters away from you. And so on. Because the truck keeps moving toward you as it emits sound waves, it appears to you that these waves are getting scrunched together.

Then, once the truck passes you and begins to move away from you, it appears as if the waves are stretched out.

Now, imagine that you could record the instant that each sound wave hits you. When the truck is moving toward you, you would observe that the time between when one wave hits and when the next wave hits is very small. However, when the truck is moving away from you, you would have to wait a while between when one wave hits you and when the next one does. In other words, when the truck is moving toward you, you register that the sirens are making a higher frequency noise; and when the truck is moving away, you register that the sirens are making a lower frequency noise.

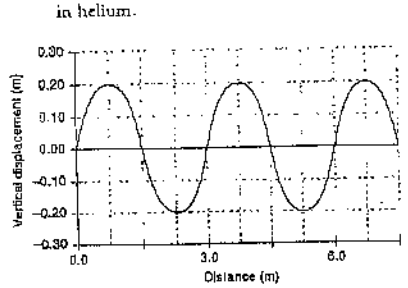
That's all you really need to know about the Doppler effect. Just remember, the effect is rather small—at normal speeds, the frequency of, say, a 200 Hz note will only change by a few tens of Hz, not hundreds of Hz.

〉 Practice Problems

Note: Additional drills regarding simple harmonic motion graphs are included in Chapter 18.

1. In a pipe closed at one end and filled with air, a 384-Hz tuning fork resonates when the pipe is 22-cm long; this tuning fork does not resonate for any smaller pipes.

   (a) State three other lengths at which this pipe will resonate with the 384-Hz tuning fork.
   (b) The end of the pipe that was closed is now opened, so that the pipe is open at both ends. Describe any changes in the lengths of pipe that will resonate with the 384-Hz tuning fork.
   (c) The air in the closed pipe is replaced with helium. Describe an experiment that would use the pipe to determine the speed of sound in helium.



2. The wave shown in the previous figure travels in a material in which its speed is 30 m/s.

   (a) What is the wavelength of this wave?
   (b) Calculate the frequency of this wave.
   (c) On the diagram, draw a different wave that has a larger frequency but carries less energy than the one pictured.

3. Consider the following questions about electromagnetic and mechanical waves. Justify your answer to each.

   (a) Which of the following types of wave can be transmitted through space, where there is no air? Choose all that apply. Justify your answer briefly.
      (i) Visible light
      (ii) Radio waves
      (iii) Gamma rays
      (iv) Sound waves
   (b) (multiple choice) A tuning fork vibrating in air produces sound waves. These waves are best classified as
      (A) Transverse, because the air molecules are vibrating parallel to the direction of wave motion
      (B) Transverse, because the air molecules are vibrating perpendicular to the direction of wave motion
      (C) Longitudinal, because the air molecules are vibrating parallel to the direction of wave motion
      (D) Longitudinal, because the air molecules are vibrating perpendicular to the direction of wave motion
   (c) (multiple choice) Radio waves and gamma rays traveling in space have the same
      (A) Frequency
      (B) Wavelength
      (C) Period
      (D) Speed
   (d) Which type of wave exhibits the Doppler effect? Choose all that apply. Justify your answer briefly.
      (i) Visible light
      (ii) Radio waves
      (iii) Gamma rays
      (iv) Sound waves

4. The driver of a car blows the horn as the car approaches you.

   (a) Compared to the horn's pitch heard by the driver, will the pitch observed by you be higher, lower, or the same?
   (b) The car passes you, while the driver continues to blow the horn. After the car passes, you notice that the horn doesn't sound as loud as it did when it was near you. Is this observation a result of the Doppler effect?
   (c) The car recedes from you after passing you, still producing sound waves from the horn. Discuss how the amplitude, period, and frequency of the sound waves that you would measure compare to the amplitude, period, and frequency of the sound waves that the driver would measure.

5. The period of a mass-on-a-spring system is doubled, while still using the same spring.

   (a) By what factor does the frequency of the mass-on-a-spring system increase or decrease?
   (b) By what factor does the mass attached to the spring increase or decrease?

〉 Solutions to Practice Problems

1. (a) Because 22 cm is the shortest length of pipe that resonates, 384 Hz is the fundamental frequency, the one that produces waves without any nodes inside the pipe. There must be an antinode at one end and a node at the other (because it's closed at one end and open at the other). As the pipe length is increased, the wavelength of the sound wave doesn't change, because the frequency of the tuning fork and the speed of sound don't change. The pipe will next resonate when once again there is an antinode at one end and a node at the other. (This time, though, there will be another node inside somewhere, too.) Since the antinode-to-node distance was 22 cm, we need to add that distance twice to get another full "hump" of a standing wave inside the pipe. Add 44 cm to the pipe to get resonance at a pipe length of 66 cm; add another 44 cm to get resonance at 110 cm; and add another 44 cm to get resonance at 154 cm.
   (b) Now only standing waves with nodes at both ends of the pipe will resonate. The wave of 22 cm has a node at one end and an antinode at the other—it will no longer resonate. But 44 cm is the node-to-node distance, so 44 cm will resonate. Whereas 66 cm used to resonate in the closed pipe, it will not in the open pipe because an antinode is at one end. Instead, any multiple of 44 cm will resonate because adding 44 cm adds a full node-to-node distance, ensuring a node at each end.
   (c) We know the frequency of the tuning fork. To use the equation $v = \lambda f$, we need the wavelength of the wave as well. Play the tuning fork and shorten the pipe until once again we find the shortest pipe length that resonates with the tuning fork. That's the fundamental, with an antinode at one end, a node at the other, and no nodes in between. The wavelength of the sound wave is four times this shortest resonating pipe length.

2. (a) The wavelength is measured from peak to peak or trough to trough. That's 3.0 m.
   (b) Use $v = \lambda f$... (30 m/s) = (3.0 m)f... f = 10 Hz.
   (c) The energy carried by a wave depends on the wave's amplitude; this wave should have a smaller amplitude. Since the wave speed doesn't change, a bigger frequency means a smaller wavelength by $v = \lambda f$.

3. (a) Correct answers: (i), (ii), and (iii). Only electromagnetic waves can be transmitted through a vacuum. That includes gamma rays, visible light, and radio waves, which are all parts of the electromagnetic spectrum. Sound waves require a material to be transmitted.
   (b) Correct answer: (C). Sound waves are longitudinal, by definition. Also by definition, "longitudinal" means that the particles of the material are vibrating parallel to the direction that the wave travels.
   (c) Correct answer: (D). The speed of light—or any electromagnetic wave—in a vacuum is 300 million m/s. Radio and gamma rays have different frequencies, and thus different wavelengths and periods.
   (d) Correct answers: (i), (ii), (iii), (iv). All waves exhibit the Doppler effect. When the source of the wave moves toward an observer, the observer observes waves of higher frequency. For sound waves, this means you'd hear a higher pitch; for visible light, this means you'd see a "blue shift." Gamma and radio waves would also be observed to have a higher frequency.

4. (a) By definition, when a source of waves approaches an observer, the observer observes waves of a higher frequency. Pitch of a sound is related to the sound wave's frequency.
   (b) The Doppler effect says nothing about loudness, only about frequency. The horn sounds less loud because the energy created by the wave source spreads out over a larger and larger space as the wave gets farther from the source. Since the energy carried by a wave is related to its amplitude, and since amplitude is related to loudness for a sound wave, you hear a softer noise.
   (c) The horn won't seem as loud (as discussed in [b]), so the amplitude is smaller than what the driver hears. The frequency you hear will be lower based on the Doppler effect. Since period is the inverse of frequency, a smaller frequency means a larger period.

5. (a) Frequency is the inverse of the period. When the period doubles, the frequency is cut in half.
   (b) The relevant equation is $T = 2\pi\frac{\sqrt{m}}{\sqrt{k}}$. The spring constant doesn't change because it's the same spring. Since the mass term is in the numerator and square rooted, the mass should quadruple. Then, square rooting the factor of four increases the whole fraction by a factor of two.

〉 Rapid Review

- The period of a mass on a spring in simple harmonic motion is given by $2\pi\frac{\sqrt{m}}{\sqrt{k}}$.
- The frequency and period are inverses of one another.
- The amount of restoring force exerted by a spring is given by $F = kx$.
- The spring potential energy is given by $PE_{spring} = \frac{1}{2}kx^2$.
- The period of a pendulum is given by $2\pi\frac{\sqrt{L}}{\sqrt{g}}$.
- Whenever the motion of a material is at right angles to the direction in which the wave travels, the wave is a **transverse wave.**
- The energy carried by a wave depends on the wave's amplitude.
- When a wave changes materials, its frequency remains the same.
- When a material vibrates parallel to the direction of the wave, the wave is a **longitudinal wave.**
- In **constructive interference** the crest of one wave overlaps the crest of another. The result is a wave of increased amplitude.
- In **destructive interference** the crest of one wave overlaps the trough of another. The result is a wave of reduced amplitude.
- The wavelength of a standing wave is twice the node-to-node distance.
- For a string fixed at both ends, or for a pipe open at both ends, the smallest frequency of standing waves is given by $f_1 = \frac{v}{2L}$, where v is the speed of the waves on the string or in the pipe. Other harmonic frequencies for this string or pipe must be whole-number multiples of the fundamental frequency.
- The pitch of a musical note depends on the sound wave's frequency; the loudness of a note depends on the sound wave's amplitude.

- For a pipe closed at one end, or for a string fixed at one end but free at the other, the smallest frequency of standing waves is given by $f_1 = \frac{v}{4L}$, where v is the speed of the waves on the string or in the pipe. Other harmonic frequencies for this string or pipe must be *odd* multiples of the fundamental frequency.
- Beats are rhythmic interference that occurs when two notes of unequal but close frequencies are played.
- The Doppler effect is the apparent change in a wave's frequency that you observe whenever the source of the wave is moving toward or away from you.