2. A student attempted to confirm that the function f defined by $f(x) = \frac{x^2+x-6}{x^2-7x+10}$ is continuous at $x = 2$. In which step, if any, does an error first appear?

- Step 1: $f(x) = \frac{x^2+x-6}{x^2-7x+10} = \frac{(x-2)(x+3)}{(x-2)(x-5)}$
- Step 2: $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \frac{x+3}{x-5} = \frac{2+3}{2-5} = -\frac{5}{3}$
- Step 3: $f(2) = \frac{2+3}{2-5} = -\frac{5}{3}$
- Step 4: $\lim\limits_{x \to 2} f(x) = f(2)$, so f is continuous at $x = 2$.

$$\frac{2^2+2-6}{2^2-7(2)+10}$$

$\frac{0}{0}$

hole at

$x=2$

(A) Step 2

(B) Step 3

(C) Step 4

(D) There is no error in the confirmation.

3.

$$f(x) = \begin{cases} x^2 + 2x & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ x^3 + x^2 + x & \text{for } 1 < x < 3 \\ 0 & \text{for } x = 3 \\ 2x + 1 & \text{for } x > 3 \end{cases}$$

$\lim_{x \to 1} x(x+2)$ $1(1+2) = 3$

limit 7 0

$\lim_{x \to 3} 2x+1 = 2(3)+1 = 7$

Let f be the piecewise function defined above. Which of the following statements is false?

(A) f is continuous at $x = 1$. ✓

(B) f is continuous at $x = 2$. ✓

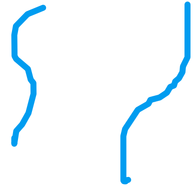
(C) f is continuous at $x = 3$.

Do not match

$f(3) = 0$ $\lim_{x \to 3} f(x) = 7$

(D) f is continuous at $x = 4$. ✓

1. $$f(x) = \begin{cases} \sin x & \text{for } x < 0 \\ \cos x & \text{for } 0 \le x \le \frac{3\pi}{2} \\ \tan x & \text{for } \frac{3\pi}{2} < x \le 2\pi \\ \cot x & \text{for } 2\pi < x \le \frac{5\pi}{2} \end{cases}$$



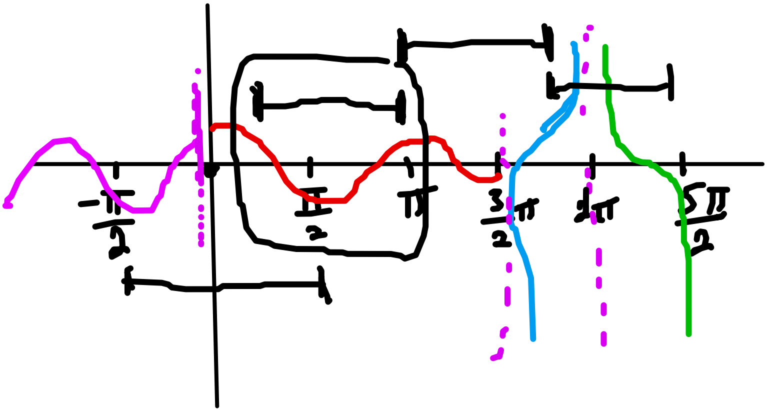
Let f be the function given above. On which of the following intervals is f continuous?

(A) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\left(\frac{\pi}{4}, \pi\right)$

(C) $\left(\pi, \frac{7\pi}{4}\right)$

(D) $\left(\frac{7\pi}{4}, \frac{5\pi}{2}\right)$



2. Which of the following functions is not continuous on the interval $-\infty < x < \infty$?

(A) $f(x) = x^4 + x^3 + x^2 + x + 1$ ✓

(B) $g(x) = \frac{1}{x^3+x^2+x+1}$

Cubic function - must have at least one zero. At least one x value not in domain

(C) $h(x) = \frac{\pi}{2}\sin x$ ✓

(D) $k(x) = \frac{1}{1+e^{-x}}$ ✓

3. Which of the following functions are continuous on the interval $0 < x < 2$?

1. $f(x) = \frac{x-1}{x^2-1}$ X
2. $g(x) = \frac{x+1}{x^2+1}$
3. $h(x) = \ln(x^2 - 1)$ X

$$\lim_{x \to 1} \frac{x-1}{x^2-1} \qquad \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}$$

$$\frac{1}{1+1} = \frac{1}{2}$$

$\cancel{g(x)}$

$$\lim_{x \to 1} \frac{x+1}{x^2+1} \qquad \frac{1+1}{1^2+1} = \frac{2}{2} = 1$$

$g(1) = 1$ ← match

1. $f(x) = \begin{cases} \frac{\sin(5x)}{8x} & \text{for } x \neq 0 \\ c & \text{for } x = 0 \end{cases}$

$\frac{x}{\sin x} = 1$ $\frac{\sin x}{x} = 1$

The function f is defined above, where c is a constant. For what value of c is f continuous at $x = 0$?

(A) 0

(B) $\boxed{\frac{5}{8}}$

(C) 1

(D) 8

$$\lim_{x \to 0} \frac{\sin(5x)}{8x} = \frac{5}{8} \cdot \frac{\sin x}{x}$$

$$\frac{5}{8} \cdot 1 = \boxed{\frac{5}{8}}$$

2. $f(x) = \begin{cases} 2 - \sin x & \text{for } x \le 1 \\ cx\sqrt{x^2+2} + c & \text{for } x > 1 \end{cases}$

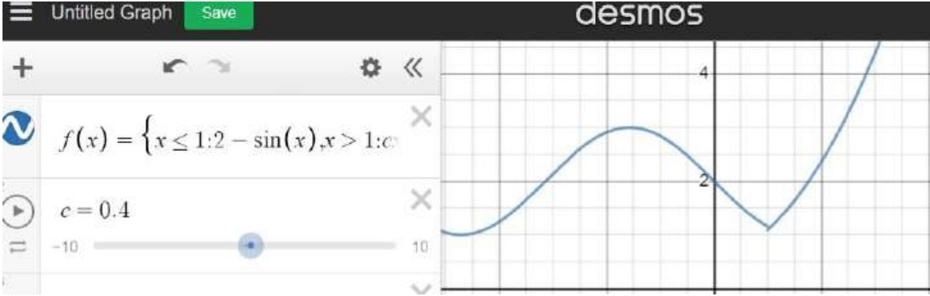
Let f be the function defined above, where c is a constant. For what value of c is f continuous for all x ?

(A) 1.159

(B) 0.424

(C) 0.409

(D) There is no such value of c.



You must use correct notation in Desmos. Then, use the slider to find the point where the "pieces" meet.

3. $f(x) = \begin{cases} x^2 + b^2 & \text{for } x < 2 \\ bx + 2b & \text{for } x \geq 2 \end{cases}$

Let f be the function defined above, where b is a constant. For what values of b, if any, is f continuous at $x = 2$?

(A) 0 only

(B) 2 only

(C) 0 and 2

(D) There is no such b.

$\lim_{x \to 2^-} x^2 + b^2 \quad 2^2 + b^2 = \underline{4 + b^2}$

$\lim_{x \to 2} bx + 2b = 2b + 2b = \underline{4b}$

$4 + b^2 = 4b$

$b^2 - 4b + 4$

$(b-2)(b-2)$

$b - 2 = 0$

$\boxed{b = 2}$