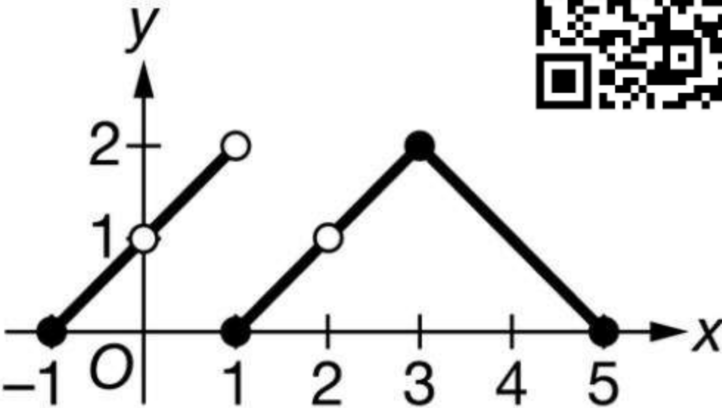
1.



Graph of f

The graph of the function f is shown above. What are all values of x for which f has a removable discontinuity?

(A) 0 only

(B) 1 only

(C) 0 and 2 only

(D) 0, 1, and 2

2. Let f be the function defined by $f(x) = \frac{x^3 - 2x^2 - 3x}{x^3 - 3x^2 + 4}$. Which of the following statements about f at $x = 2$ and $x = -1$ is true?

(A) f has a jump discontinuity at $x = 2$, and f is continuous at $x = -1$.

(B) f has a jump discontinuity at $x = 2$, and f has a removable discontinuity at $x = -1$.

(C) f has a discontinuity due to a vertical asymptote at $x = 2$, and f is continuous at $x = -1$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$, and f has a removable discontinuity at $x = -1$.

4. Let f be the function defined by $f(x) = \frac{3x^3+2x^2}{x^2-x}$. Which of the following statements is true?

(A) f has a discontinuity due to a vertical asymptote at $x = 0$ and at $x = 1$.

(B) f has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 1$.

(C) f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 1$.

(D) f is continuous at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 1$.

$$\frac{3(0)^3 + 2(0)^2}{0^2 - x} = \frac{0}{0}$$ → removable discontinuity

$$\frac{3(1)^3 + 2(1)^2}{1^2 - 1} = \frac{5}{0}$$ ← vertical asymptote undefined

1. $f(x) = \frac{x^3+4x^2+x-6}{3\sin\left(-\frac{\pi}{2}x\right)+3x^2}$

Let f be the function defined above. Which of the following conditions explains why f is not continuous at $x = 1$?

(A) Neither $\lim_{x\to 1} f(x)$ nor $f(1)$ exists.

(B) ✓ $\lim_{x\to 1} f(x)$ exists, but $f(1)$ does not exist.

"hole" → removable discontinuity

(C) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist, but $\lim_{x\to 1} f(x) \neq f(1)$.

(D) Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist, and $\lim_{x\to 1} f(x) = f(1)$.

2. A student attempted to confirm that the function f defined by $f(x) = \frac{x^2+x-6}{x^2-7x+10}$ is continuous at $x = 2$. In which step, if any, does an error first appear?

- Step 1: $f(x) = \frac{x^2+x-6}{x^2-7x+10} = \frac{(x-2)(x+3)}{(x-2)(x-5)}$
- Step 2: $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x+3}{x-5} = \frac{2+3}{2-5} = -\frac{5}{3}$
- Step 3: $f(2) = \frac{2+3}{2-5} = -\frac{5}{3}$
- Step 4: $\lim_{x\to 2} f(x) = f(2)$, so f is continuous at $x = 2$.

(A) Step 2

(B) Step 3

(C) Step 4

(D) There is no error in the confirmation.

3. $$f(x) = \begin{cases} x^2 + 2x & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ x^3 + x^2 + x & \text{for } 1 < x < 3 \\ 0 & \text{for } x = 3 \\ 2x + 1 & \text{for } x > 3 \end{cases}$$

Let f be the piecewise function defined above. Which of the following statements is false?

(A) f is continuous at $x = 1$.

(B) f is continuous at $x = 2$.

(C) f is continuous at $x = 3$.

(D) f is continuous at $x = 4$.

1.

$$f(x)=\begin{cases}\sin x & \text{for } x<0\\ \cos x & \text{for } 0\le x\le \frac{3\pi}{2}\\ \tan x & \text{for } \frac{3\pi}{2}<x\le 2\pi\\ \cot x & \text{for } 2\pi<x\le \frac{5\pi}{2}\end{cases}$$

Let f be the function given above. On which of the following intervals is f continuous?

(A) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(B) $\left(\frac{\pi}{4},\pi\right)$

(C) $\left(\pi,\frac{7\pi}{4}\right)$

(D) $\left(\frac{7\pi}{4},\frac{5\pi}{2}\right)$

2. Which of the following functions is not continuous on the interval $-\infty < x < \infty$?

(A) $f(x) = x^4 + x^3 + x^2 + x + 1$

(B) $g(x) = \frac{1}{x^3+x^2+x+1}$

(C) $h(x) = \frac{\pi}{2}\sin x$

(D) $k(x) = \frac{1}{1+e^{-x}}$

3. Which of the following functions are continuous on the interval $0 < x < 2$?

1. $f(x) = \frac{x-1}{x^2-1}$
2. $g(x) = \frac{x+1}{x^2+1}$
3. $h(x) = \ln(x^2 - 1)$

(A) II only

(B) I and II only

(C) I and III only

(D) II and III only

1. $f(x) = \begin{cases} \frac{\sin(5x)}{8x} & \text{for } x \neq 0 \\ c & \text{for } x = 0 \end{cases}$

The function f is defined above, where c is a constant. For what value of c is f continuous at $x = 0$?

(A) 0

(B) $\frac{5}{8}$

(C) 1

(D) 8

2. $f(x) = \begin{cases} 2 - \sin x & \text{for } x \leq 1 \\ cx\sqrt{x^2+2} + c & \text{for } x > 1 \end{cases}$

Let f be the function defined above, where c is a constant. For what value of c is f continuous for all x ?

(A) 1.159

(B) 0.424

(C) 0.409

(D) There is no such value of c.

3. $f(x) = \begin{cases} x^2 + b^2 & \text{for } x < 2 \\ bx + 2b & \text{for } x \geq 2 \end{cases}$

Let f be the function defined above, where b is a constant. For what values of b, if any, is f continuous at $x = 2$?

(A) 0 only

(B) 2 only

(C) 0 and 2

(D) There is no such b.