CollegeBoard
Advanced Placement Program

AP® Calculus AB

Practice Exam

The questions contained in this AP® Calculus AB Practice Exam are written to the content specifications of AP Exams for this subject. Taking this practice exam should provide students with an idea of their general areas of strengths and weaknesses in preparing for the actual AP Exam. Because this AP Calculus AB Practice Exam has never been administered as an operational AP Exam, statistical data are not available for calculating potential raw scores or conversions into AP grades.

This AP Calculus AB Practice Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location. Teachers may not redistribute the files electronically for any reason.

Contents

AP® Calculus AB
Directions for Administration

The AP Calculus AB Exam is 3 hours and 15 minutes in length and consists of a multiple-choice section and a free-response section.

- The 105-minute two-part multiple-choice section contains 45 questions and accounts for 50 percent of the final grade. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

- The 90-minute two-part free-response section contains 6 questions and accounts for 50 percent of the final grade. Part A of the free-response section (3 questions in 45 minutes) contains some questions or parts of questions for which a graphing calculator is required. Part B of the free-response section (3 questions in 45 minutes) does not allow the use of a calculator. During the timed portion for Part B, students are permitted to continue work on questions in Part A, but they are not allowed to use a calculator during this time.

A 10-minute break should be provided after Section I is completed. Students should not have access to their graphing calculators during the break.

The actual AP Exam is administered in one session. Students will have the most realistic experience if a complete morning or afternoon is available to administer this practice exam. If a schedule does not permit one time period for the entire practice exam administration, it would be acceptable to administer Section I one day and Section II on a subsequent day.

Many students wonder whether or not to guess the answers to the multiple-choice questions about which they are not certain. It is improbable that mere guessing will improve a score. However, if a student has some knowledge of the question and is able to eliminate one or more answer choices as wrong, it may be to the student's advantage to answer such a question.

- Graphing calculators are required to answer some of the questions on the AP Calculus AB Exam. Before starting the exam administration, make sure each student has a graphing calculator from the approved list at http://www.collegeboard.com/ap/calculators. During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks; calculators may not be shared. Calculator memories do <u>not</u> need to be cleared before or after the exam. Since graphing calculators can be used to store data, including text, it is important to monitor that students are using their calculators appropriately.

- It is suggested that Section I of the practice exam be completed using a pencil to simulate an actual administration. Students may use a pencil or pen with black or dark blue ink to complete Section II.

- Teachers will need to provide paper for the students to write their free-response answers. Teachers should provide directions to the students indicating how they wish the responses to be labeled so the teacher will be able to associate the response with the question the student intended to answer.

- Instructions for the Section II free-response questions are included. Ask students to read these instructions carefully at the beginning of the administration of Section II. Timing for Section II should begin <u>after</u> you have given students sufficient time to read these instructions.

- Remember that students are not allowed to remove any materials, including scratch work, from the testing site.

Section I

Multiple-Choice Questions

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\int \cos(3x)\, dx =$

(A) $-3\sin(3x) + C$

(B) $-\frac{1}{3}\sin(3x) + C$

(C) $\frac{1}{3}\sin(3x) + C$

(D) $\sin(3x) + C$

(E) $3\sin(3x) + C$

---

2. $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) nonexistent

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \le 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

3. The function f is defined above. For what value of k, if any, is f continuous at $x = 2$?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of k will make f continuous at $x = 2$.

---

4. If $f(x) = \cos^3(4x)$, then $f'(x) =$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3 - \sqrt{105}}{4}$ (E) $\frac{3 + \sqrt{105}}{4}$

---

6. Let f be the function given by $f(x) = (2x - 1)^5 (x + 1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

GO ON TO THE NEXT PAGE.

7. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx =$

(A) $2e^{\sqrt{x}} + C$

(B) $\frac{1}{2}e^{\sqrt{x}} + C$

(C) $e^{\sqrt{x}} + C$

(D) $2\sqrt{x}\, e^{\sqrt{x}} + C$

(E) $\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}} + C$

---

x	0	2	4	6
$f(x)$	4	k	8	12

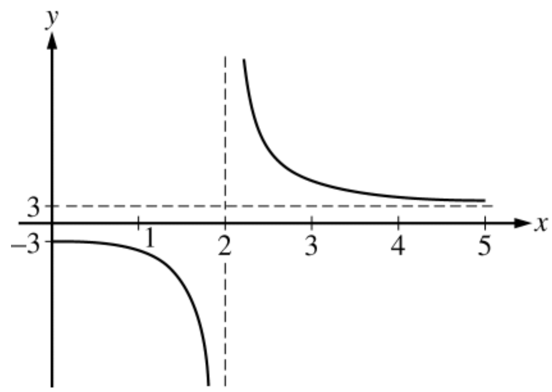
8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\, dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

GO ON TO THE NEXT PAGE.

9. A particle moves along the x-axis so that at any time $t > 0$, its velocity is given by $v(t) = 4 - 6t^2$. If the particle is at position $x = 7$ at time $t = 1$, what is the position of the particle at time $t = 2$?

(A) -10 (B) -5 (C) -3 (D) 3 (E) 17



10. The function f is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of f. Which of the following could be the values of the constants a and b ?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

GO ON TO THE NEXT PAGE.

11. What is the slope of the line tangent to the graph of $y = \frac{e^{-x}}{x + 1}$ at $x = 1$?

(A) $-\frac{1}{e}$ (B) $-\frac{3}{4e}$ (C) $-\frac{1}{4e}$ (D) $\frac{1}{4e}$ (E) $\frac{1}{e}$

---

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) ln 25 (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

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13. $\int \left(x^3 + 1\right)^2 \, dx =$

(A) $\frac{1}{7}x^7 + x + C$

(B) $\frac{1}{7}x^7 + \frac{1}{2}x^4 + x + C$

(C) $6x^2\left(x^3 + 1\right) + C$

(D) $\frac{1}{3}\left(x^3 + 1\right)^3 + C$

(E) $\frac{\left(x^3 + 1\right)^3}{9x^2} + C$

---

14. $\lim_{h \to 0} \frac{e^{(2+h)} - e^2}{h} =$

(A) 0 (B) 1 (C) $2e$ (D) e^2 (E) $2e^2$

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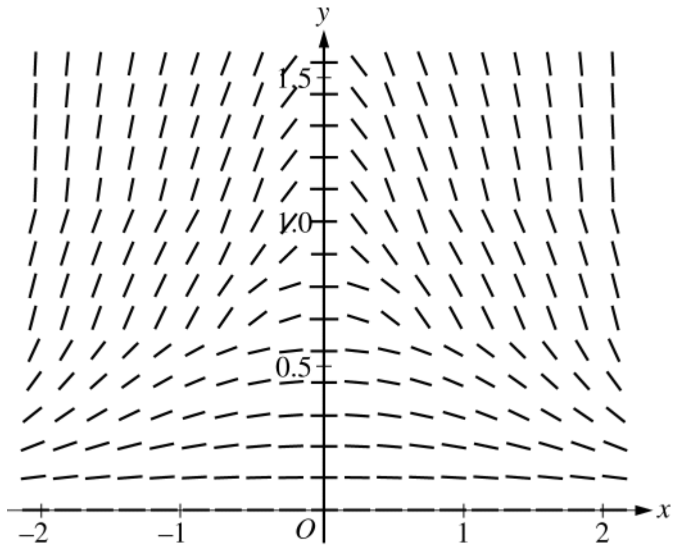


15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

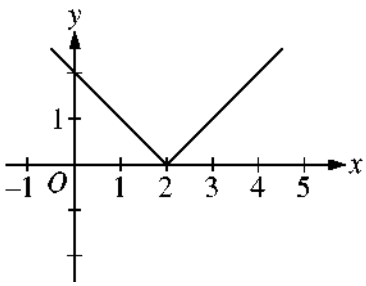
(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

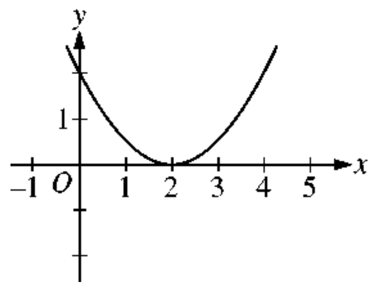
(E) $y = \dfrac{1}{1 + x^2}$

16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)



(B)



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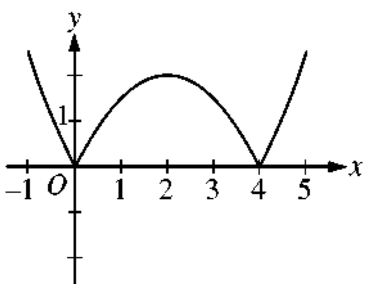

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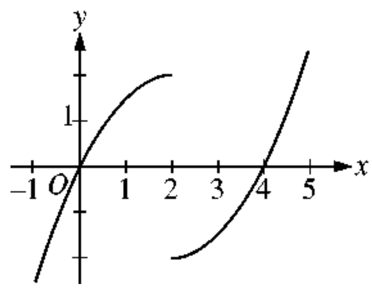
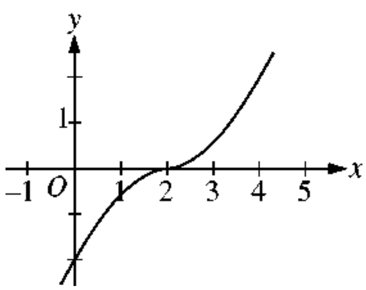


(E)



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17. What is the area of the region enclosed by the graphs of $f(x) = x - 2x^2$ and $g(x) = -5x$?

(A) $\frac{7}{3}$ (B) $\frac{16}{3}$ (C) $\frac{20}{3}$ (D) 9 (E) 36

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18. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

19. Let f be the function given by $f(x) = x^3 - 6x^2$. The graph of f is concave up when

(A) $x > 2$

(B) $x < 2$

(C) $0 < x < 4$

(D) $x < 0$ or $x > 4$ only

(E) $x > 6$ only

---

20. If $g(x) = x^2 - 3x + 4$ and $f(x) = g'(x)$, then $\int_1^3 f(x)\,dx =$

(A) $-\frac{14}{3}$ (B) -2 (C) 2 (D) 4 (E) $\frac{14}{3}$

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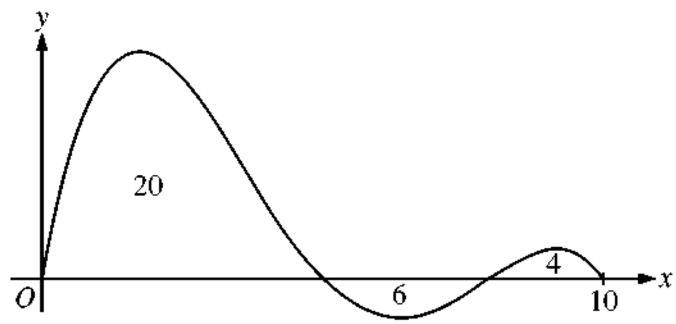

Graph of f'

21. The graph of f', the derivative of the function f, is shown above for $0 \le x \le 10$. The areas of the regions between the graph of f' and the x-axis are 20, 6, and 4, respectively. If $f(0) = 2$, what is the maximum value of f on the closed interval $0 \le x \le 10$?

(A) 16 (B) 20 (C) 22 (D) 30 (E) 32

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22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

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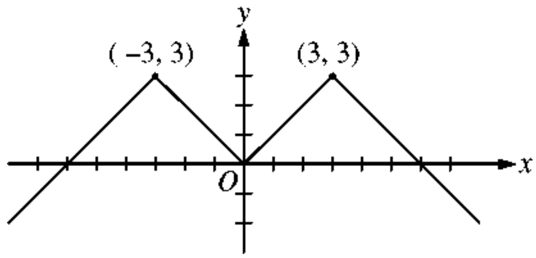

23. The graph of the even function $y = f(x)$ consists of 4 line segments, as shown above. Which of the following statements about f is false?

(A) $\lim_{x\to 0}\left(f(x) - f(0)\right) = 0$

(B) $\lim_{x\to 0}\dfrac{f(x) - f(0)}{x} = 0$

(C) $\lim_{x\to 0}\dfrac{f(x) - f(-x)}{2x} = 0$

(D) $\lim_{x\to 2}\dfrac{f(x) - f(2)}{x - 2} = 1$

(E) $\lim_{x\to 3}\dfrac{f(x) - f(3)}{x - 3}$ does not exist.

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

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25. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$ (B) $-\frac{7}{13}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{14}$ (E) 7

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26. For $x > 0$, f is a function such that $f'(x) = \frac{\ln x}{x}$ and $f''(x) = \frac{1 - \ln x}{x^2}$. Which of the following is true?

(A) f is decreasing for $x > 1$, and the graph of f is concave down for $x > e$.

(B) f is decreasing for $x > 1$, and the graph of f is concave up for $x > e$.

(C) f is increasing for $x > 1$, and the graph of f is concave down for $x > e$.

(D) f is increasing for $x > 1$, and the graph of f is concave up for $x > e$.

(E) f is increasing for $0 < x < e$, and the graph of f is concave down for $0 < x < e^{3/2}$.

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27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

28. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1 + 25x^2}$

(B) $\frac{5}{1 + 25x^2}$

(C) $\frac{-5}{\sqrt{1 - 25x^2}}$

(D) $\frac{1}{\sqrt{1 - 25x^2}}$

(E) $\frac{5}{\sqrt{1 - 25x^2}}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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76. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = t^2 \ln(t + 2)$. What is the acceleration of the particle at time $t = 6$?

(A) 1.500 (B) 20.453 (C) 29.453 (D) 74.860 (E) 133.417

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77. If $\int_0^3 f(x)\,dx = 6$ and $\int_3^5 f(x)\,dx = 4$, then $\int_0^5 (3 + 2f(x))\,dx =$

(A) 10 (B) 20 (C) 23 (D) 35 (E) 50

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78. For $t \geq 0$ hours, H is a differentiable function of t that gives the temperature, in degrees Celsius, at an Arctic weather station. Which of the following is the best interpretation of $H'(24)$?

(A) The change in temperature during the first day

(B) The change in temperature during the 24th hour

(C) The average rate at which the temperature changed during the 24th hour

(D) The rate at which the temperature is changing during the first day

(E) The rate at which the temperature is changing at the end of the 24th hour

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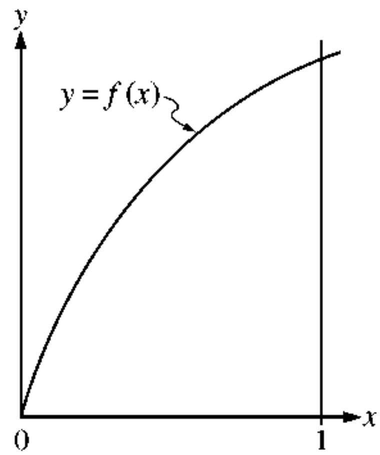
79. A spherical tank contains 81.637 gallons of water at time $t = 0$ minutes. For the next 6 minutes, water flows out of the tank at the rate of $9\sin\left(\sqrt{t+1}\right)$ gallons per minute. How many gallons of water are in the tank at the end of the 6 minutes?

(A) 36.606 (B) 45.031 (C) 68.858 (D) 77.355 (E) 126.668



80. A left Riemann sum, a right Riemann sum, and a trapezoidal sum are used to approximate the value of $\int_0^1 f(x)\,dx,$ each using the same number of subintervals. The graph of the function f is shown in the figure above. Which of the sums give an underestimate of the value of $\int_0^1 f(x)\,dx$?

I. Left sum
II. Right sum
III. Trapezoidal sum

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

81. The first derivative of the function f is given by $f'(x) = x - 4e^{-\sin(2x)}$. How many points of inflection does the graph of f have on the interval $0 < x < 2\pi$?

(A) Three (B) Four (C) Five (D) Six (E) Seven

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82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

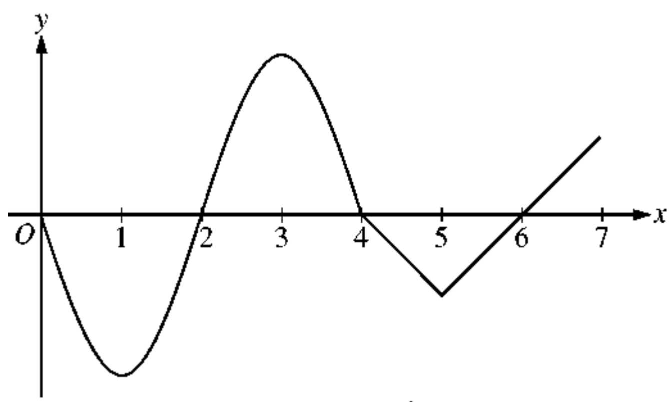
(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \le x \le 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5



Graph of f'

84. The graph of f', the derivative of the function f, is shown above. On which of the following intervals is f decreasing?

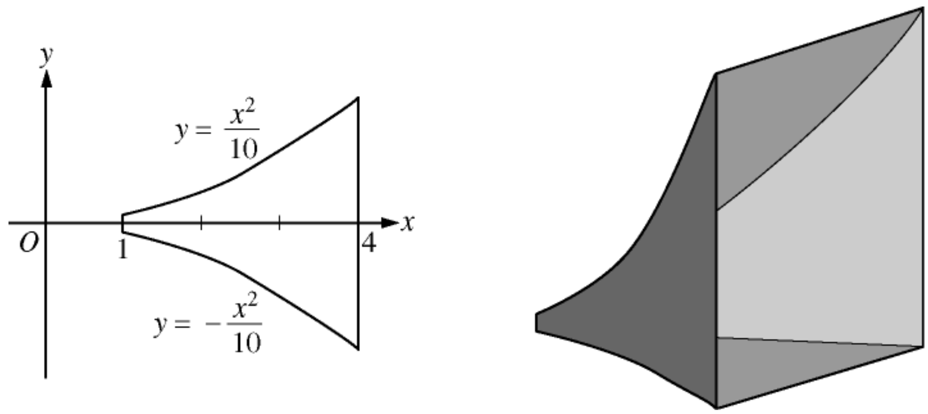
(A) $[2, 4]$ only

(B) $[3, 5]$ only

(C) $[0, 1]$ and $[3, 5]$

(D) $[2, 4]$ and $[6, 7]$

(E) $[0, 2]$ and $[4, 6]$



85. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the x-axis are squares. What is the volume of the loudspeaker, in cubic units?

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

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x	3	4	5	6	7
$f(x)$	20	17	12	16	20

86. The function f is continuous and differentiable on the closed interval $[3, 7]$. The table above gives selected values of f on this interval. Which of the following statements must be true?

I. The minimum value of f on $[3, 7]$ is 12.

II. There exists c, for $3 < c < 7$, such that $f'(c) = 0$.

III. $f'(x) > 0$ for $5 < x < 7$.

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) I, II, and III

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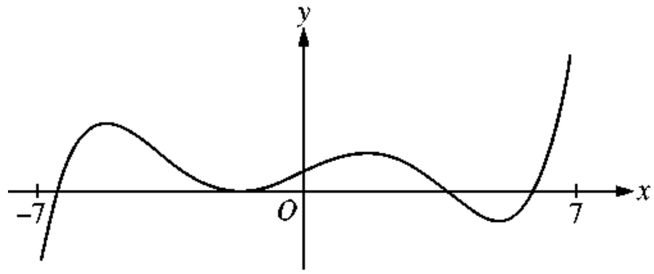

Graph of f'

87. The figure above shows the graph of f', the derivative of the function f, on the open interval $-7 < x < 7$. If f' has four zeros on $-7 < x < 7$, how many relative maxima does f have on $-7 < x < 7$?

(A) One (B) Two (C) Three (D) Four (E) Five

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88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

89. The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) –28 (B) –16 (C) 40 (D) 44 (E) 47

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90. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

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91. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 5te^{-t} - 1$. At $t = 0$, the particle is at position $x = 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

(A) 0.366 (B) 0.542 (C) 1.542 (D) 1.821 (E) 2.821

---

92. Let f be the function with first derivative defined by $f'(x) = \sin(x^3)$ for $0 \leq x \leq 2$. At what value of x does f attain its maximum value on the closed interval $0 \leq x \leq 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

Section II

Free-Response Questions

AP® Calculus
Instructions for Section II Free-Response Questions

Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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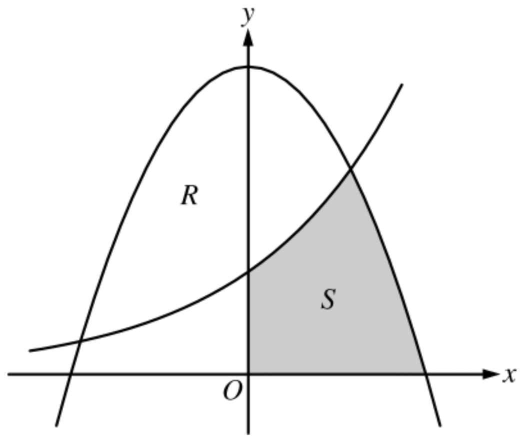
1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?



2. Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.
   (a) Find the area of S.
   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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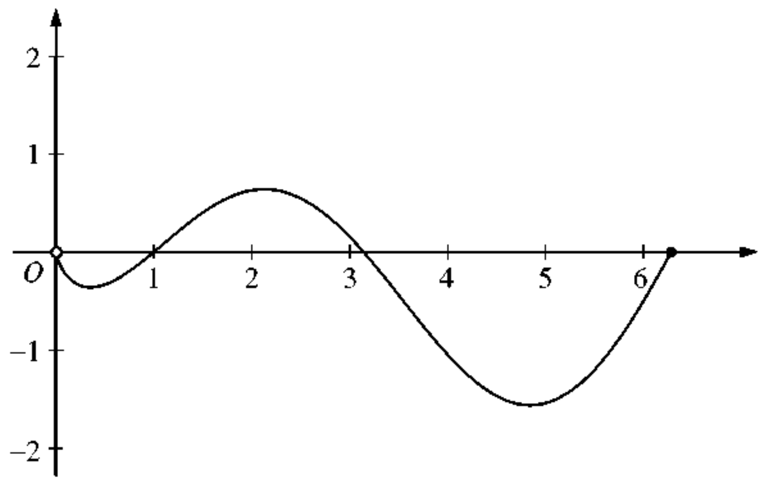
t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.
   (a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.
   (b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.
   (c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.
   (d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

END OF PART A OF SECTION II

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

(a) Find $g(1)$ and $g'(1)$.

(b) On what intervals, if any, is g increasing? Justify your answer.

(c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

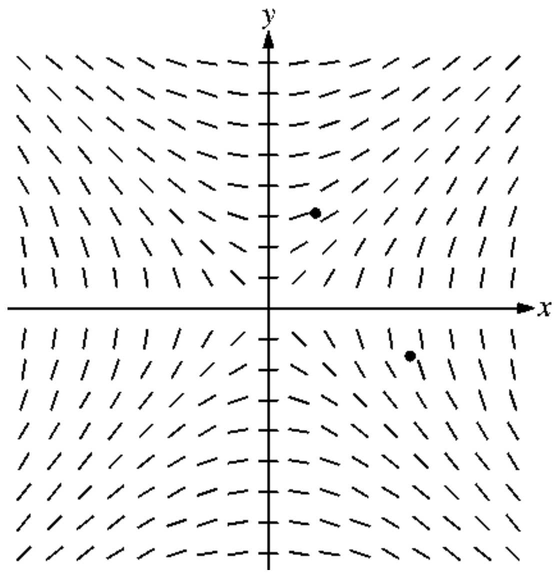
(d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

GO ON TO THE NEXT PAGE.

5. Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$.

(Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)



(b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.

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6. Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.

(a) Find $\lim_{x \to \infty} g(x)$.

(b) For what positive value of b does g have an absolute maximum at $x = \frac{2}{3}$? Justify your answer.

(c) Find all values of b, if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.

STOP

END OF EXAM

Name: ______________________________

AP® Calculus AB
Student Answer Sheet for Multiple-Choice Section

No.	Answer
1	
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No.	Answer
76	
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92	

AP® Calculus AB
Multiple-Choice Answer Key

No.	Correct Answer
1	C
2	D
3	C
4	B
5	C
6	B
7	A
8	D
9	C
10	D
11	B
12	D
13	B
14	D
15	E
16	E
17	D
18	D
19	A
20	C
21	C
22	A
23	B
24	D
25	A
26	C
27	E
28	E

No.	Correct Answer
76	C
77	D
78	E
79	A
80	D
81	B
82	C
83	D
84	E
85	D
86	B
87	A
88	C
89	D
90	E
91	D
92	C

Question 1

The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

(a) $\int_0^4 E(t)\,dt \approx 3981$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Let $S(t)$ be the amount of sewage in the treatment tank at time t. Then $S'(t) = E(t) - 645$ and $S'(t) = 0$ when $E(t) = 645$. On the interval $0 \le t \le 4$, $E(t) = 645$ when $t = 2.309$ and $t = 3.559$.

t (hours)	amount of sewage in treatment tank
0	0
2.309	$\int_0^{2.309} E(t)\,dt - 645(2.309) = 1637.178$
3.559	$\int_0^{3.559} E(t)\,dt - 645(3.559) = 1228.520$
4	$3981.022 - 645(4) = 1401.022$

The amount of sewage in the treatment tank is greatest at $t = 2.309$ hours. At that time, the amount of sewage in the tank, rounded to the nearest gallon, is 1637 gallons.

$4 : \begin{cases} 1 : \text{sets } E(t) = 645 \\ 1 : \text{identifies } t = 2.309 \text{ as a candidate} \\ 1 : \text{amount of sewage at } t = 2.309 \\ 1 : \text{conclusion} \end{cases}$

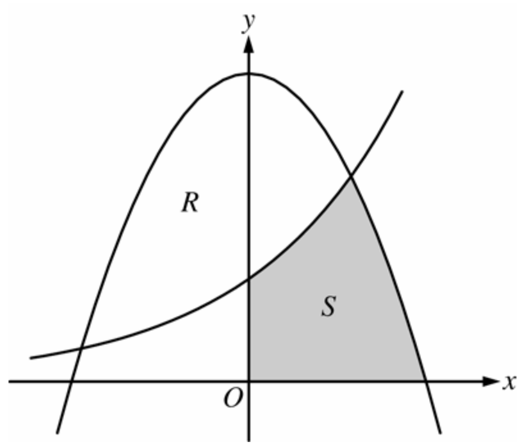
(c) Total cost $= \int_0^4 (0.15 - 0.02t)E(t)\,dt = 474.320$

The total cost of treating the sewage that enters the tank during the time interval $0 \le t \le 4$, to the nearest dollar, is \$474.

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

AP® Calculus AB
Free-Response Scoring Guidelines

Question 2



Let R and S in the figure above be defined as follows: R is the region in the first and second quadrants bounded by the graphs of $y = 3 - x^2$ and $y = 2^x$. S is the shaded region in the first quadrant bounded by the two graphs, the x-axis, and the y-axis.

(a) Find the area of S.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with one leg across the base of the solid. Write, but do not evaluate, an integral expression that gives the volume of the solid.

$3 - x^2 = 2^x$ when $x = -1.63658$ and $x = 1$

Let $a = -1.63658$

(a) Area of $S = \int_0^1 2^x\,dx + \int_1^{\sqrt{3}} \left(3 - x^2\right) dx = 2.240$	3 : { 1 : integrands 1 : limits 1 : answer
(b) Volume $= \pi \int_a^1 \left(\left(3 - x^2 + 1\right)^2 - \left(2^x + 1\right)^2 \right) dx = 63.106$ or 63.107	4 : { 2 : integrand 1 : limits and constant 1 : answer
(c) Volume $= \frac{1}{2} \int_a^1 \left(3 - x^2 - 2^x\right)^2\,dx$	2 : { 1 : integrand 1 : limits and constant

AP® Calculus AB
Free-Response Scoring Guidelines

Question 3

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(a) $H'(10) \approx \dfrac{H(12) - H(8)}{12 - 8} = \dfrac{80 - 73}{4} = \dfrac{7}{4}\,°\text{C}/\text{min}$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer with units} \end{cases}$

(b) Average temperature is $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$

$\displaystyle\int_0^{16} H(t)\,dt \approx 4 \cdot (65 + 68 + 73 + 80)$

Average temperature $\approx \dfrac{4 \cdot 286}{16} = 71.5°\text{C}$

$3 : \begin{cases} 1 : \dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt \\ 1 : \text{left Riemann sum} \\ 1 : \text{answer} \end{cases}$

(c) The left Riemann sum approximation is an underestimate of the integral because the graph of H is increasing. Dividing by 16 will not change the inequality, so 71.5°C is an underestimate of the average temperature.

1 : answer with reason

(d) If a continuous function is increasing at an increasing rate, then the slopes of the secant lines of the graph of the function are increasing. The slopes of the secant lines for the four intervals in the table are $\dfrac{3}{4}$, $\dfrac{5}{4}$, $\dfrac{7}{4}$, and $\dfrac{10}{4}$, respectively.

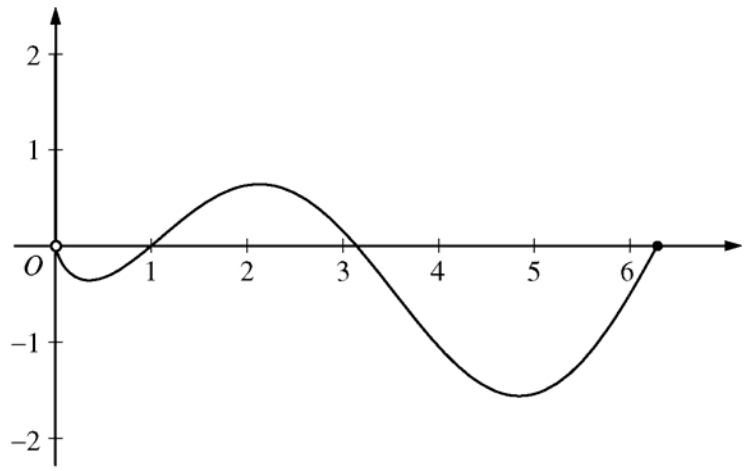
Since the slopes are increasing, the data are consistent with the claim.

OR

By the Mean Value Theorem, the slopes are also the values of $H'(c_k)$ for some times $c_1 < c_2 < c_3 < c_4$, respectively. Since these derivative values are positive and increasing, the data are consistent with the claim.

$3 : \begin{cases} 1 : \text{considers slopes of four secant lines} \\ 1 : \text{explanation} \\ 1 : \text{conclusion consistent with explanation} \end{cases}$

Question 4



Graph of f

Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

(a) Find $g(1)$ and $g'(1)$.

(b) On what intervals, if any, is g increasing? Justify your answer.

(c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

(d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

(a) $g(1) = \int_1^1 f(t)\,dt = 0$ and $g'(1) = f(1) = 0$	$2 : \begin{cases} 1 : g(1) \\ 1 : g'(1) \end{cases}$
(b) Since $g'(x) = f(x)$, g is increasing on the interval $1 \le x \le \pi$ because $f(x) > 0$ for $1 < x < \pi$.	$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$
(c) For $0 < x < 2\pi$, $g'(x) = f(x) = 0$ when $x = 1, \pi$. $g' = f$ changes from negative to positive only at $x = 1$. The absolute minimum must occur at $x = 1$ or at the right endpoint. Since $g(1) = 0$ and $g(2\pi) = \int_1^{2\pi} f(t)\,dt = \int_1^{\pi} f(t)\,dt + \int_{\pi}^{2\pi} f(t)\,dt < 0$ by comparison of the two areas, the absolute minimum occurs at $x = 2\pi$.	$3 : \begin{cases} 1 : \text{identifies 1 and } 2\pi \text{ as candidates} \\ \quad\quad \text{- or -} \\ \quad \text{indicates that the graph of } g \\ \quad \text{decreases, increases, then decreases} \\ 1 : \text{justifies } g(2\pi) < g(1) \\ 1 : \text{answer} \end{cases}$
(d) Yes, the graph of g is tangent to the x-axis at $x = 1$ since $g(1) = 0$ and $g'(1) = 0$.	$2 : \begin{cases} 1 : \text{answer of "yes" with } x = 1 \\ 1 : \text{explanation} \end{cases}$

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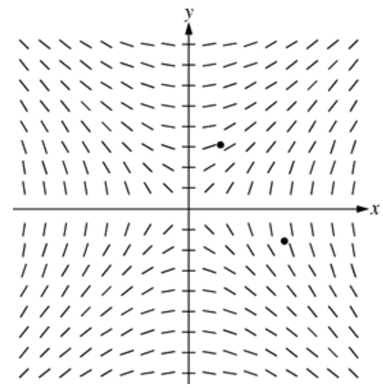
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$, where $y \neq 0$.

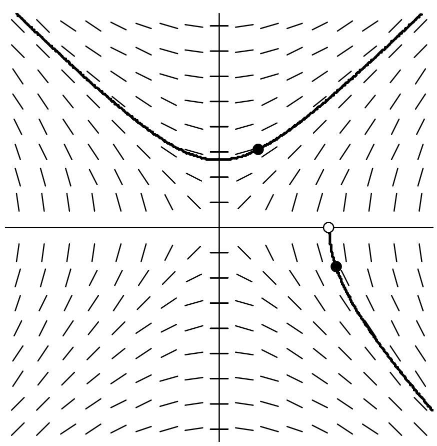
(a) The slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(3, -1)$, and sketch the solution curve that passes through the point $(1, 2)$. (Note: The points $(3, -1)$ and $(1, 2)$ are indicated in the figure.)

(b) Write an equation for the line tangent to the solution curve that passes through the point $(1, 2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(3) = -1$, and state its domain.



(a)



$2 : \begin{cases} 1 : \text{solution curve through } (3, -1) \\ 1 : \text{solution curve through } (1, 2) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field or the x-axis.

(b) $\left.\frac{dy}{dx}\right|_{(1, 2)} = \frac{1}{2}$

An equation for the line tangent to the solution curve is $y - 2 = \frac{1}{2}(x - 1)$.

1 : equation of tangent line

(c) $y\,dy = x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + A$

$y^2 = x^2 + C$

$C = -8$

Since the particular solution goes through $(3, -1)$, y must be negative.

$y = -\sqrt{x^2 - 8}$ for $x > \sqrt{8}$

6 :
- 5 :
  - 1 : separates variables
  - 1 : antiderivatives
  - 1 : constant of integration
  - 1 : uses initial condition
  - 1 : solves for y
- Note: max $2/5$ [1-1-0-0-0] if no constant of integration
- 1 : domain

AP® Calculus AB
Free-Response Scoring Guidelines

Question 6

Let $g(x) = xe^{-x} + be^{-x}$, where b is a positive constant.

(a) Find $\lim_{x \to \infty} g(x)$.

(b) For what positive value of b does g have an absolute maximum at $x = \frac{2}{3}$? Justify your answer.

(c) Find all values of b, if any, for which the graph of g has a point of inflection on the interval $0 < x < \infty$. Justify your answer.

(a) $\lim_{x \to \infty} g(x) = 0$	1 : answer
(b) $g'(x) = e^{-x} - xe^{-x} - be^{-x} = (1 - x - b)e^{-x}$ $g'\left(\frac{2}{3}\right) = \left(\frac{1}{3} - b\right)e^{-2/3} = 0 \Rightarrow b = \frac{1}{3}$ When $b = \frac{1}{3}$, $g'(x) = \left(\frac{2}{3} - x\right)e^{-x}$. For $x < \frac{2}{3}$, $g'(x) > 0$ and for $x > \frac{2}{3}$, $g'(x) < 0$. Therefore, when $b = \frac{1}{3}$, g has an absolute maximum at $x = \frac{2}{3}$.	$4 : \begin{cases} 2 : g'(x) \\ 1 : \text{solves } g'\left(\frac{2}{3}\right) = 0 \text{ for } b \\ 1 : \text{justification} \end{cases}$
(c) $g''(x) = -e^{-x} - (1 - x - b)e^{-x} = (x - 2 + b)e^{-x}$ If $0 < b < 2$, then $g''(x)$ will change sign at $x = 2 - b > 0$. Therefore, the graph of g will have a point of inflection on the interval $0 < x < \infty$ when $0 < b < 2$.	$4 : \begin{cases} 2 : g''(x) \\ 1 : \text{interval for } b \\ 1 : \text{justification} \end{cases}$