4-2 Angle Relationships in Triangles

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

1. Find the measure of exterior $\angle DBA$ of $\triangle BCD$, if $m\angle DBC = 30°$, $m\angle C = 70°$, and $m\angle D = 80°$. 150°

73°

2. What is the complement of an angle with measure 17°?

Objectives

Find the measures of interior and exterior angles of triangles.

Apply theorems about the interior and exterior angles of triangles.

Vocabulary

auxiliary line

corollary

interior

exterior

interior angle

exterior angle

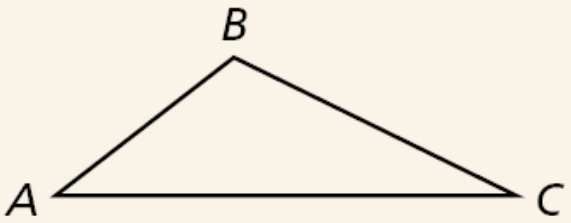
remote interior angle

4-2 Angle Relationships in Triangles

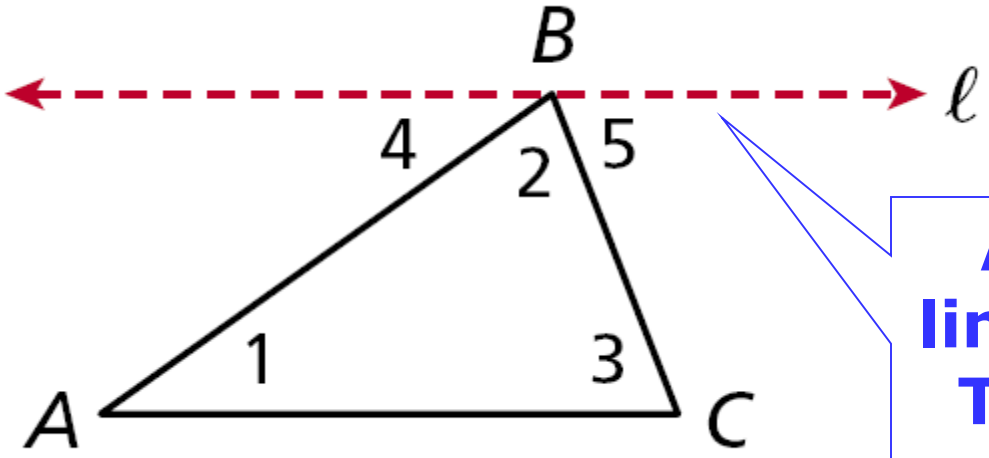
Theorem 4-2-1 **Triangle Sum Theorem**

The sum of the angle measures of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$



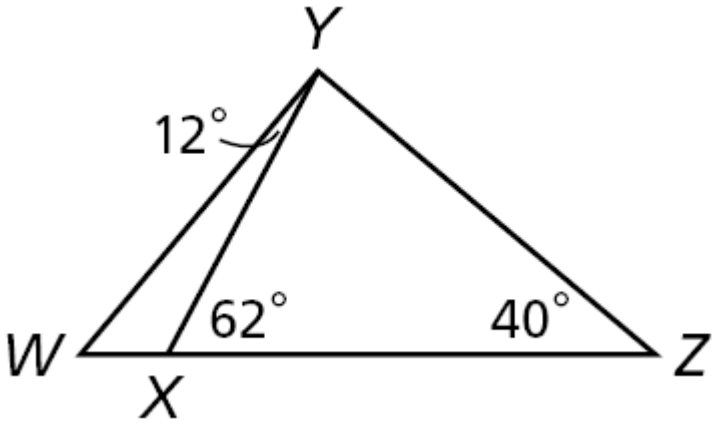
An **auxiliary line** is a line that is added to a figure to aid in a proof.



An auxiliary line used in the Triangle Sum Theorem

Example 1A: Application

After an accident, the positions of cars are measured by law enforcement to investigate the collision. Use the diagram drawn from the information collected to find m∠*XYZ*.



$m\angle XYZ + m\angle YZX + m\angle ZXY = 180°$ △ *Sum. Thm*

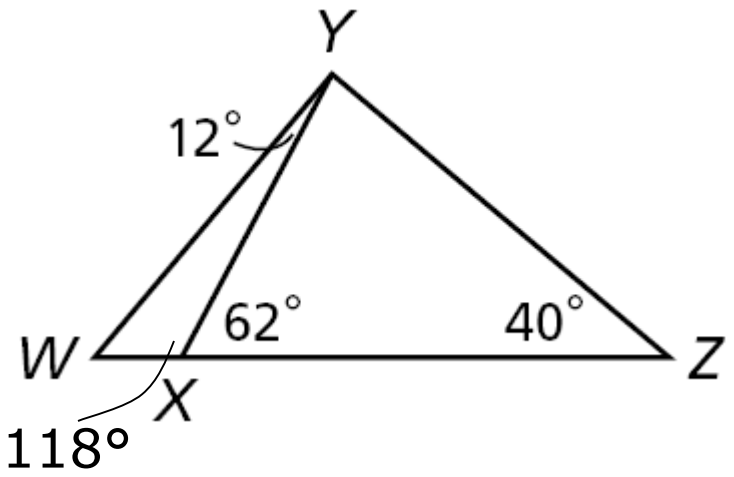
$m\angle XYZ + 40 + 62 = 180$ *Substitute 40 for m∠YZX and 62 for m∠ZXY.*

$m\angle XYZ + 102 = 180$ *Simplify.*

$m\angle XYZ = 78°$ *Subtract 102 from both sides.*

Example 1B: Application

After an accident, the positions of cars are measured by law enforcement to investigate the collision. Use the diagram drawn from the information collected to find m∠*YWZ*.



Step 1 Find m∠*WXY*.

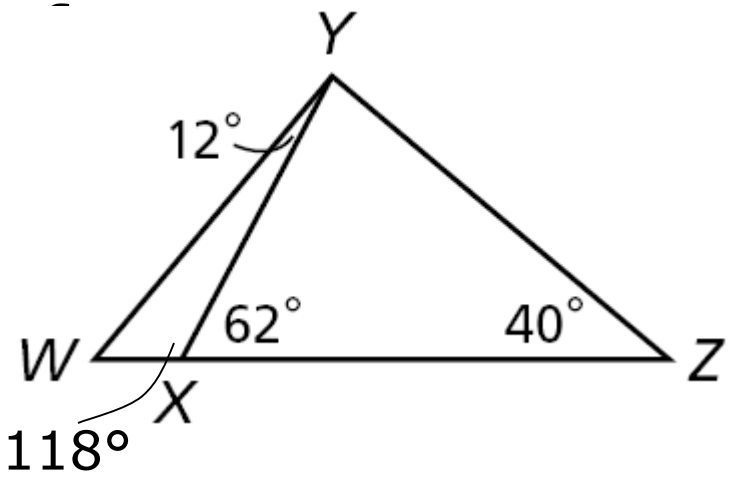
m∠*YXZ* + m∠*WXY* = 180° *Lin. Pair Thm. and ∠ Add. Post.*

62 + m∠*WXY* = 180 *Substitute 62 for m∠YXZ.*

m∠*WXY* = 118° *Subtract 62 from both sides.*

Example 1B: Application Continued

After an accident, the positions cars are measured by law enforcement to investigate the collision. Use the diagram draw from the information collected to find m∠*YWZ*.



Step 2 Find m∠*YWZ*.

m∠*YWX* + m∠*WXY* + m∠*XYW* = 180° △ *Sum. Thm*

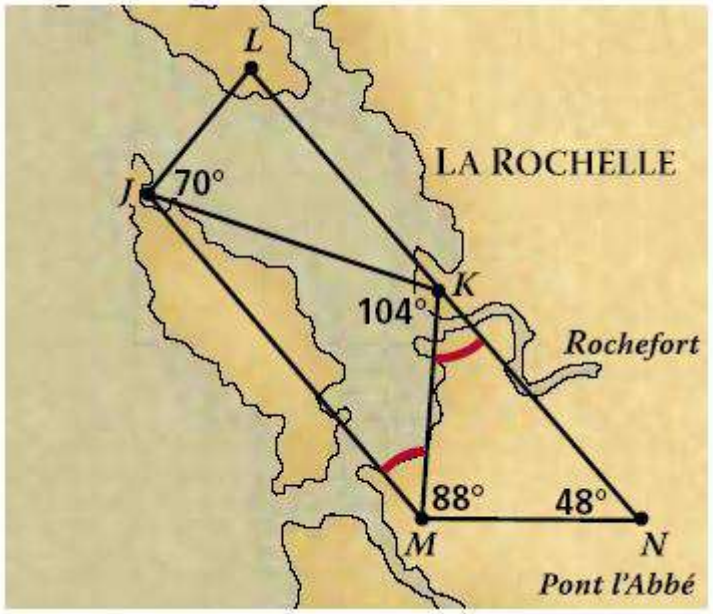
m∠*YWX* + 118 + 12 = 180 *Substitute 118 for m∠WXY and 12 for m∠XYW.*

m∠*YWX* + 130 = 180 *Simplify.*

m∠*YWX* = 50° *Subtract 130 from both sides.*

Check It Out! Example 1

Use the diagram to find m∠*MJK*.



$m\angle MJK + m\angle JKM + m\angle KMJ = 180°$ △ *Sum. Thm*

$m\angle MJK + 104 + 44 = 180$ *Substitute 104 for m∠JKM and 44 for m∠KMJ.*

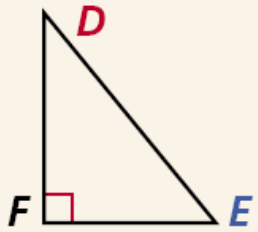
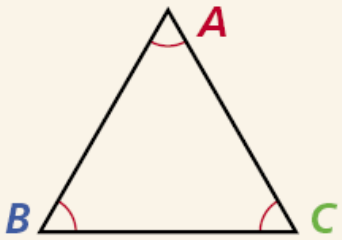
$m\angle MJK + 148 = 180$ *Simplify.*

$m\angle MJK = 32°$ *Subtract 148 from both sides.*

4-2 Angle Relationships in Triangles

A **corollary** is a theorem whose proof follows directly from another theorem. Here are two corollaries to the Triangle Sum Theorem.

Corollaries

COROLLARY		HYPOTHESIS	CONCLUSION
4-2-2	The acute angles of a right triangle are complementary.		$\angle D$ and $\angle E$ are complementary. $m\angle D + m\angle E = 90°$
4-2-3	The measure of each angle of an equiangular triangle is 60°.		$m\angle A = m\angle B = m\angle C = 60°$

Example 2: Finding Angle Measures in Right Triangles

One of the acute angles in a right triangle measures 2x°. What is the measure of the other acute angle?

Let the acute angles be $\angle A$ and $\angle B$, with $m\angle A = 2x°$.

$m\angle A + m\angle B = 90°$ *Acute ∠s of rt. △ are comp.*

$2x + m\angle B = 90$ *Substitute 2x for m∠A.*

$m\angle B = (90 - 2x)°$ *Subtract 2x from both sides.*

Check It Out! Example 2a

The measure of one of the acute angles in a right triangle is 63.7°. What is the measure of the other acute angle?

Let the acute angles be $\angle A$ and $\angle B$, with $m\angle A = 63.7°$.

$m\angle A + m\angle B = 90°$ *Acute ∠s of rt. △ are comp.*

$63.7 + m\angle B = 90$ *Substitute 63.7 for $m\angle A$.*

$m\angle B = 26.3°$ *Subtract 63.7 from both sides.*

Check It Out! Example 2b

The measure of one of the acute angles in a right triangle is $x°$. What is the measure of the other acute angle?

Let the acute angles be $\angle A$ and $\angle B$, with $m\angle A = x°$.

$m\angle A + m\angle B = 90°$	*Acute ∠s of rt. △ are comp.*
$x + m\angle B = 90$	*Substitute x for m∠A.*
$m\angle B = (90 - x)°$	*Subtract x from both sides.*

Check It Out! Example 2c

The measure of one of the acute angles in a right triangle is $48\frac{2}{5}°$. What is the measure of the other acute angle?

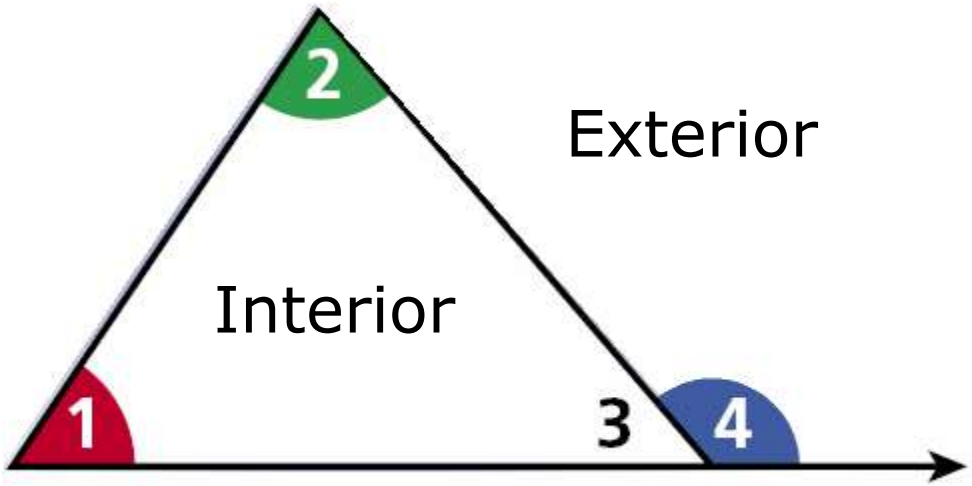
Let the acute angles be $\angle A$ and $\angle B$, with $m\angle A = 48\frac{2}{5}°$.

$m\angle A + m\angle B = 90°$ *Acute ∠s of rt. △ are comp.*

$48\frac{2}{5} + m\angle B = 90$ *Substitute $48\frac{2}{5}$ for $m\angle A$.*

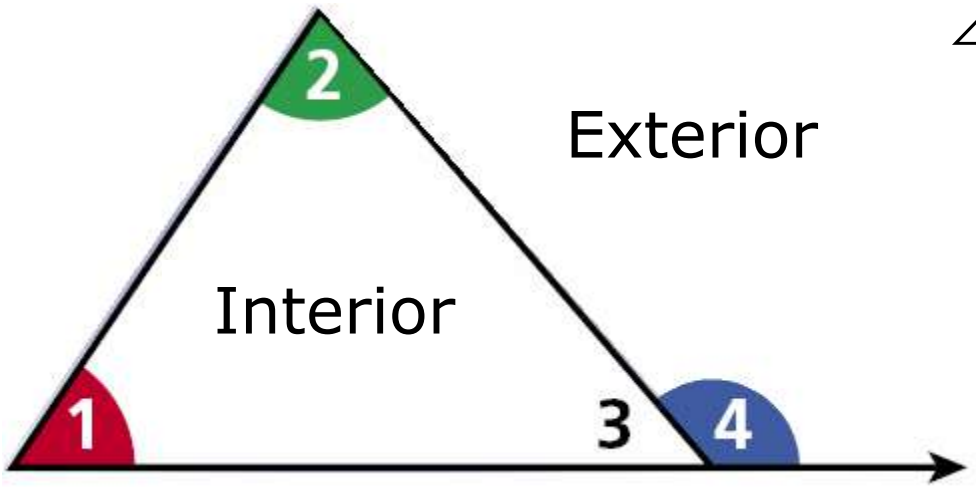
$m\angle B = 41\frac{3}{5}°$ *Subtract $48\frac{2}{5}$ from both sides.*

The **interior** is the set of all points inside the figure. The **exterior** is the set of all points outside the figure.



4-2 Angle Relationships in Triangles

An **interior angle** is formed by two sides of a triangle. An **exterior angle** is formed by one side of the triangle and extension of an adjacent side.

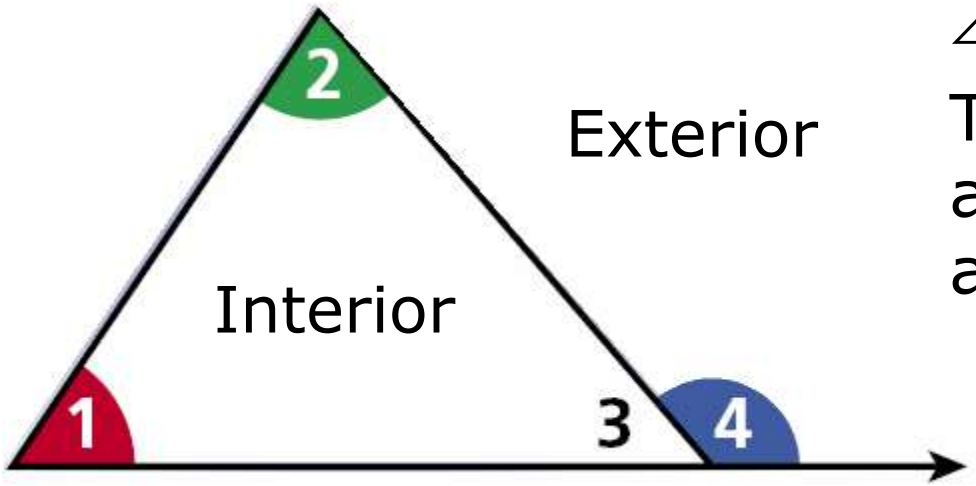


∠4 is an exterior angle.

∠3 is an interior angle.

4-2 Angle Relationships in Triangles

Each exterior angle has two remote interior angles. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.



$\angle 4$ is an exterior angle.

The remote interior angles of $\angle 4$ are $\angle 1$ and $\angle 2$.

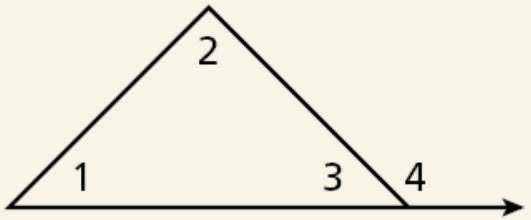
$\angle 3$ is an interior angle.

4-2 Angle Relationships in Triangles

Theorem 4-2-4 **Exterior Angle Theorem**

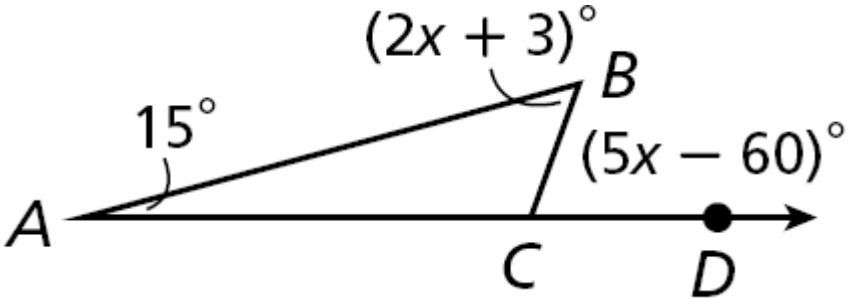
The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

$m\angle 4 = m\angle 1 + m\angle 2$



Example 3: Applying the Exterior Angle Theorem

Find m∠*B*.



$m\angle A + m\angle B = m\angle BCD$ — *Ext. ∠ Thm.*

$15 + 2x + 3 = 5x - 60$ — *Substitute 15 for m∠A, 2x + 3 for m∠B, and 5x – 60 for m∠BCD.*

$2x + 18 = 5x - 60$ — *Simplify.*

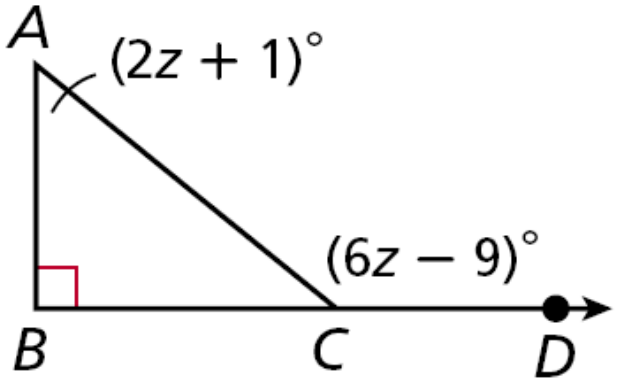
$78 = 3x$ — *Subtract 2x and add 60 to both sides.*

$26 = x$ — *Divide by 3.*

$m\angle B = 2x + 3 = 2(26) + 3 = 55°$

Check It Out! Example 3

Find m∠*ACD*.



$m\angle ACD = m\angle A + m\angle B$ *Ext. ∠ Thm.*

$6z - 9 = 2z + 1 + 90$ *Substitute 6z – 9 for m∠ACD, 2z + 1 for m∠A, and 90 for m∠B.*

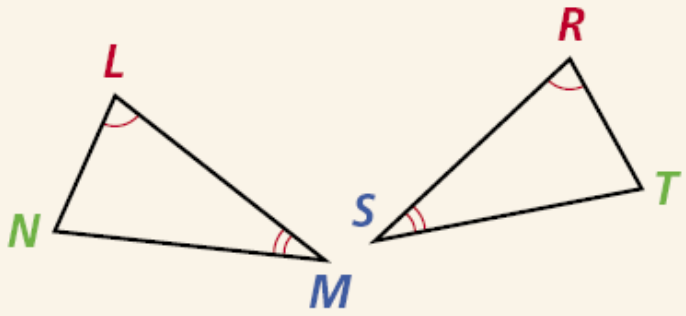
$6z - 9 = 2z + 91$ *Simplify.*

$4z = 100$ *Subtract 2z and add 9 to both sides.*

$z = 25$ *Divide by 4.*

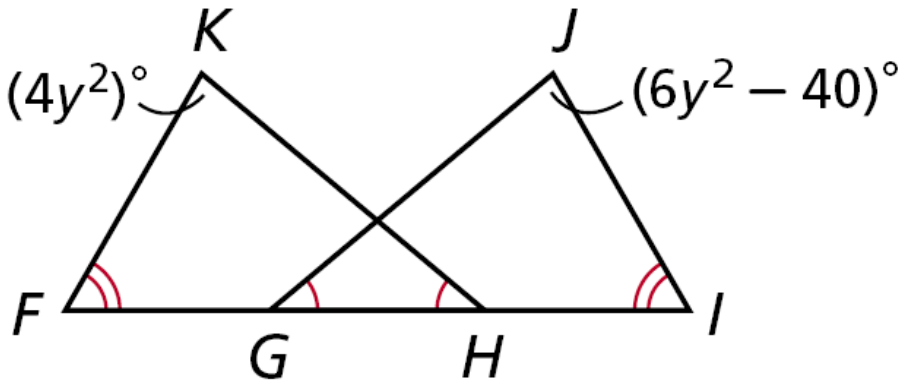
$m\angle ACD = 6z - 9 = 6(25) - 9 = 141°$

Theorem 4-2-5 Third Angles Theorem

THEOREM	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.		$\angle N \cong \angle T$

Example 4: Applying the Third Angles Theorem

Find m∠*K* and m∠*J*.



$\angle K \cong \angle J$ *Third ∠s Thm.*

$m\angle K = m\angle J$ *Def. of ≅ ∠s.*

$4y^2 = 6y^2 - 40$ *Substitute $4y^2$ for m∠K and $6y^2 - 40$ for m∠J.*

$-2y^2 = -40$ *Subtract $6y^2$ from both sides.*

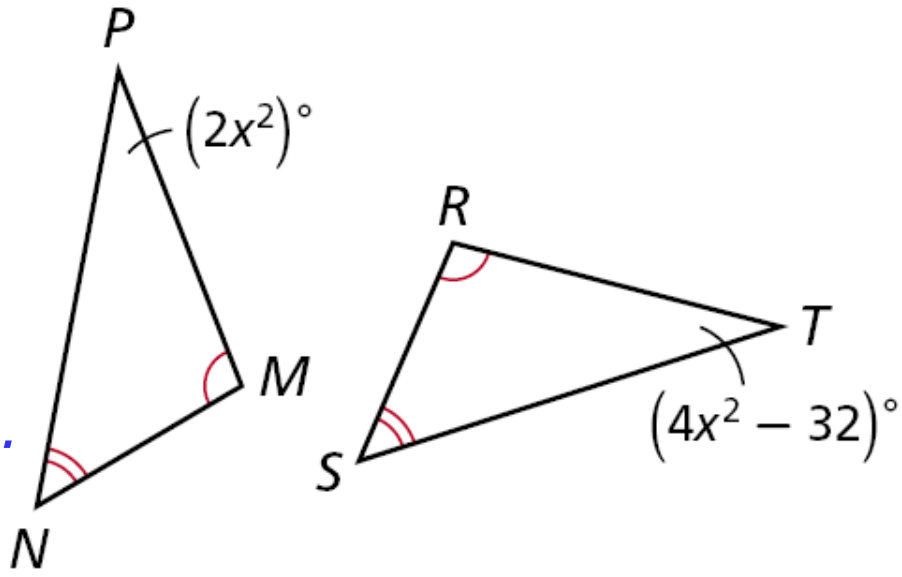
$y^2 = 20$ *Divide both sides by -2.*

So $m\angle K = 4y^2 = 4(20) = 80°$.

Since $m\angle J = m\angle K$, $m\angle J = 80°$.

Check It Out! Example 4

Find m∠*P* and m∠*T*.



$\angle P \cong \angle T$ *Third ∠s Thm.*

$m\angle P = m\angle T$ *Def. of ≅ ∠s.*

$2x^2 = 4x^2 - 32$ *Substitute $2x^2$ for m∠P and $4x^2 - 32$ for m∠T.*

$-2x^2 = -32$ *Subtract $4x^2$ from both sides.*

$x^2 = 16$ *Divide both sides by -2.*

So $m\angle P = 2x^2 = 2(16) = 32°$.

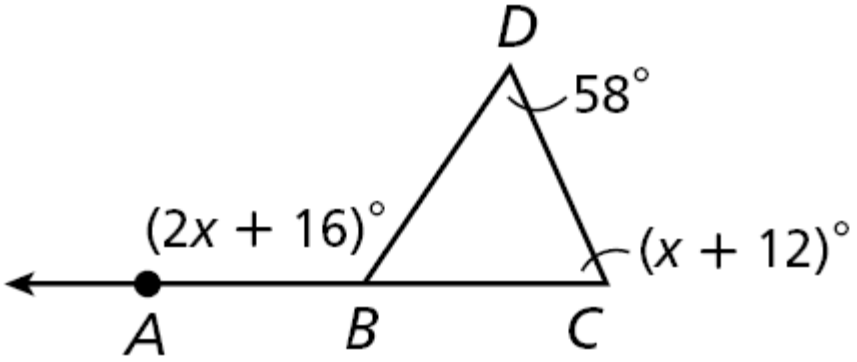
Since $m\angle P = m\angle T$, $m\angle T = 32°$.

Lesson Quiz: Part I

1. The measure of one of the acute angles in a right triangle is $56\frac{2}{3}°$. What is the measure of the other acute angle?

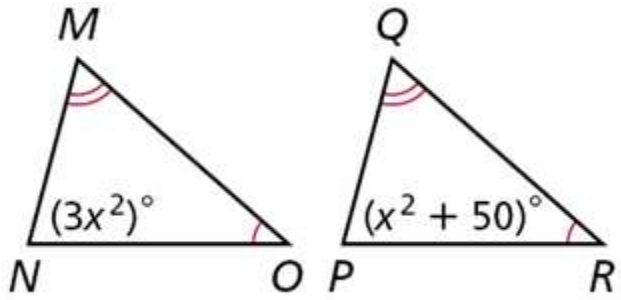
$33\frac{1}{3}°$

2. Find m∠*ABD*.



124°

3. Find m∠*N* and *m*∠P.



75°; 75°

Lesson Quiz: Part II

4. The diagram is a map showing John's house, Kay's house, and the grocery store. What is the angle the two houses make with the store?

30°