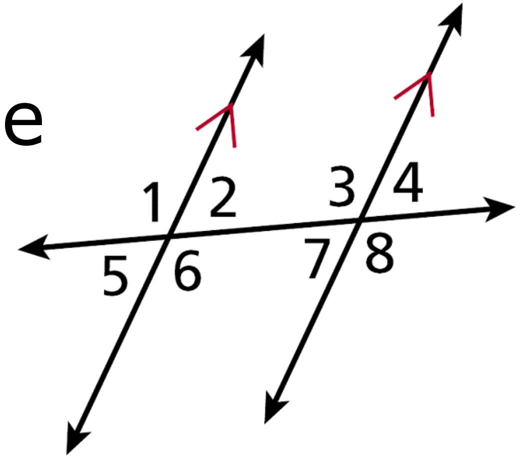
Angle of elevation and depression

Warm Up

1. Identify the pairs of alternate interior angles.



$\angle 2$ and $\angle 7$; $\angle 3$ and $\angle 6$

2. Use your calculator to find tan 30° to the nearest hundredth. 0.58

3. Solve $\tan 54° = \frac{2500}{x}$. Round to the nearest hundredtn.

1816.36

Objective

Solve problems involving angles of elevation and angles of depression.

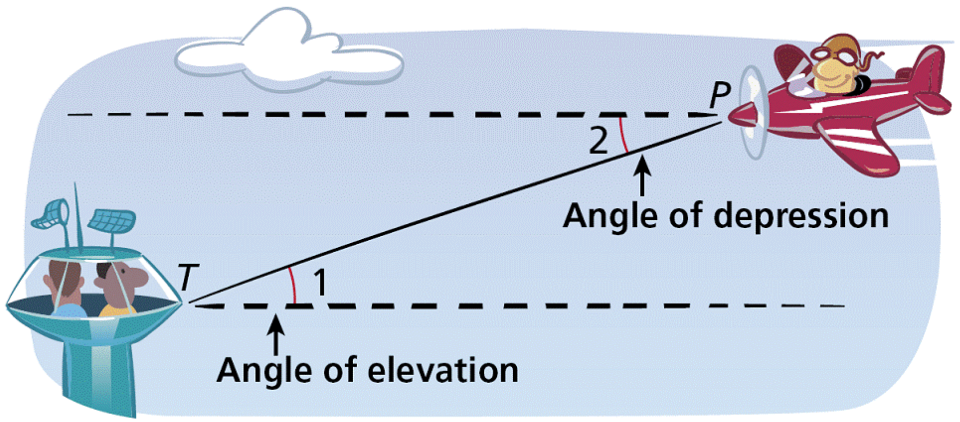
Vocabulary

angle of elevation

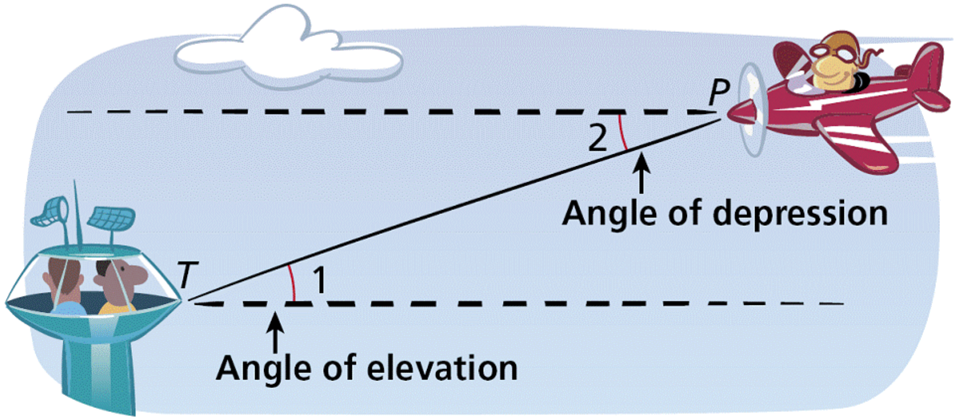
angle of depression

An ***angle of elevation*** is the angle formed by a horizontal line and a line of sight to a point *above* the line. In the diagram, ∠1 is the angle of elevation from the tower *T* to the plane *P*.

An ***angle of depression*** is the angle formed by a horizontal line and a line of sight to a point *below* the line. ∠2 is the angle of depression from the plane to the tower.

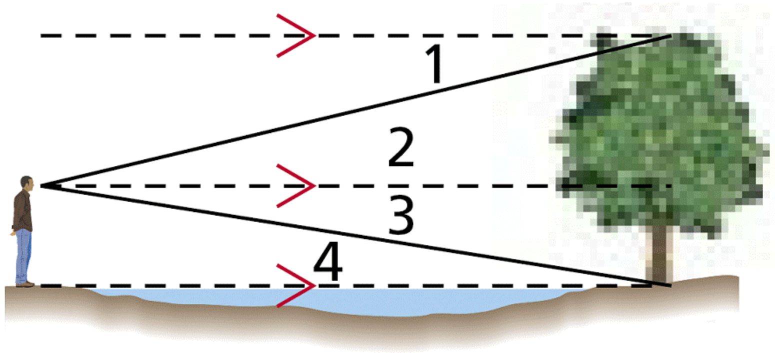


Since horizontal lines are parallel, $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem. Therefore the angle of elevation from one point is congruent to the angle of depression from the other point.



Example 1A: Classifying Angles of Elevation and Depression

Classify each angle as an angle of elevation or an angle of depression.

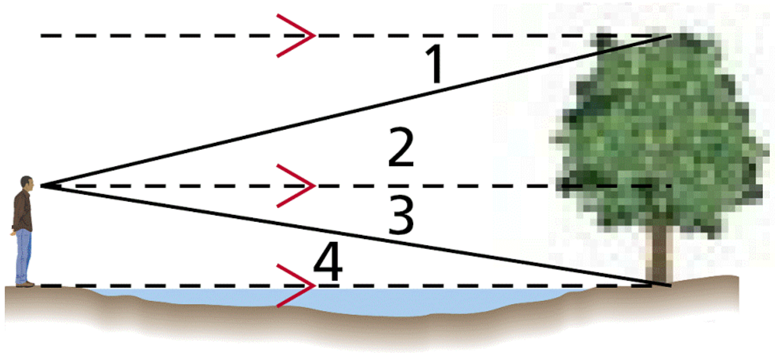


∠1

∠1 is formed by a horizontal line and a line of sight to a point below the line. It is an angle of depression.

Example 1B: Classifying Angles of Elevation and Depression

Classify each angle as an angle of elevation or an angle of depression.

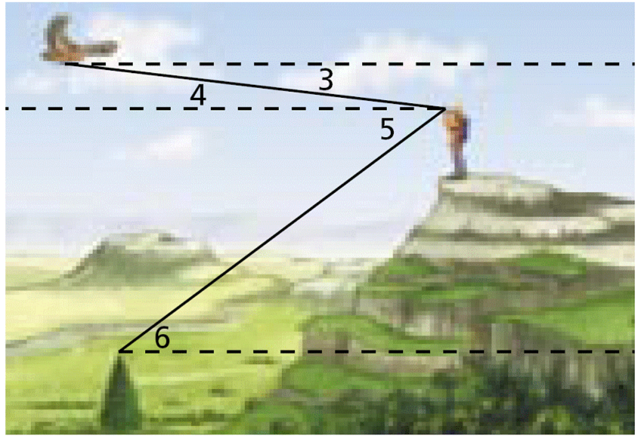


∠4

∠4 is formed by a horizontal line and a line of sight to a point above the line. It is an angle of elevation.

Check It Out! Example 1

Use the diagram above to classify each angle as an angle of elevation or angle of depression.



1a. ∠5

∠5 is formed by a horizontal line and a line of sight to a point below the line. It is an angle of depression.

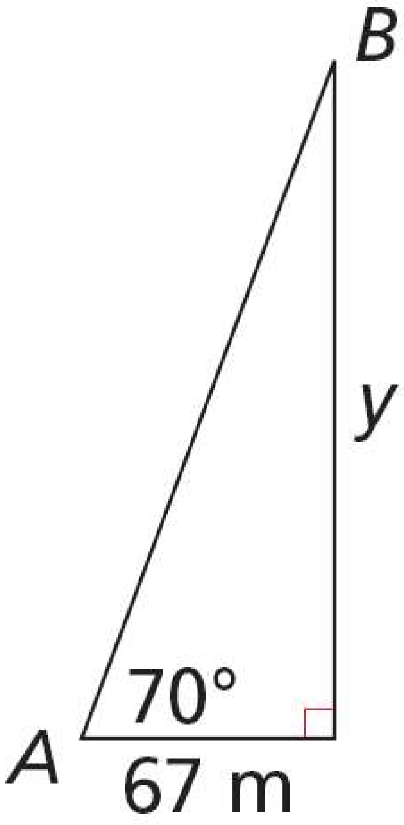
1b. ∠6

∠6 is formed by a horizontal line and a line of sight to a point above the line. It is an angle of elevation.

Example 2: Finding Distance by Using Angle of Elevation

The Seattle Space Needle casts a 67-meter shadow. If the angle of elevation from the tip of the shadow to the top of the Space Needle is 70º, how tall is the Space Needle? Round to the nearest meter.

Draw a sketch to represent the given information. Let *A* represent the tip of the shadow, and let *B* represent the top of the Space Needle. Let *y* be the height of the Space Needle.

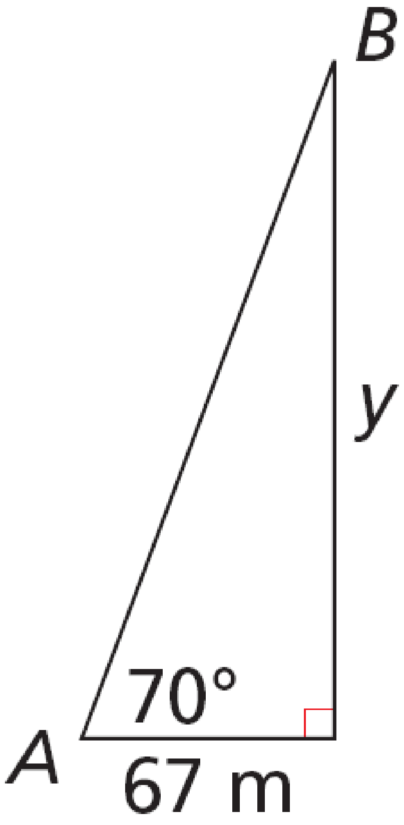


Example 2 Continued

$\tan 70° = \frac{y}{67}$ *You are given the side adjacent to ∠A, and y is the side opposite ∠A. So write a tangent ratio.*

$y = 67 \tan 70°$ *Multiply both sides by 67.*

$y \approx 184$ m *Simplify the expression.*



Check It Out! Example 2

What if...? Suppose the plane is at an altitude of 3500 ft and the angle of elevation from the airport to the plane is 29°. What is the horizontal distance between the plane and the airport? Round to the nearest foot.

$$\tan 29^\circ = \frac{3500}{x}$$

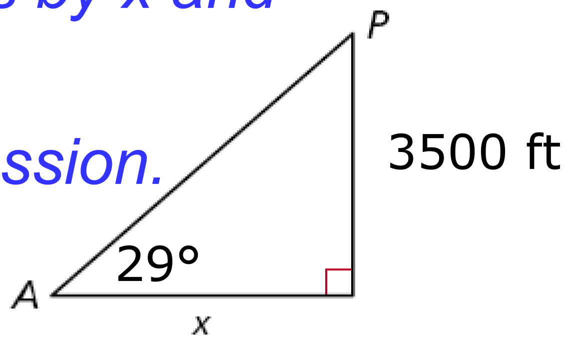
You are given the side opposite ∠A, and x is the side adjacent to ∠A. So write a tangent ratio.

$$x = \frac{3500}{\tan 29^\circ}$$

Multiply both sides by x and divide by tan 29°.

$$x \approx 6314 \text{ ft}$$

Simplify the expression.

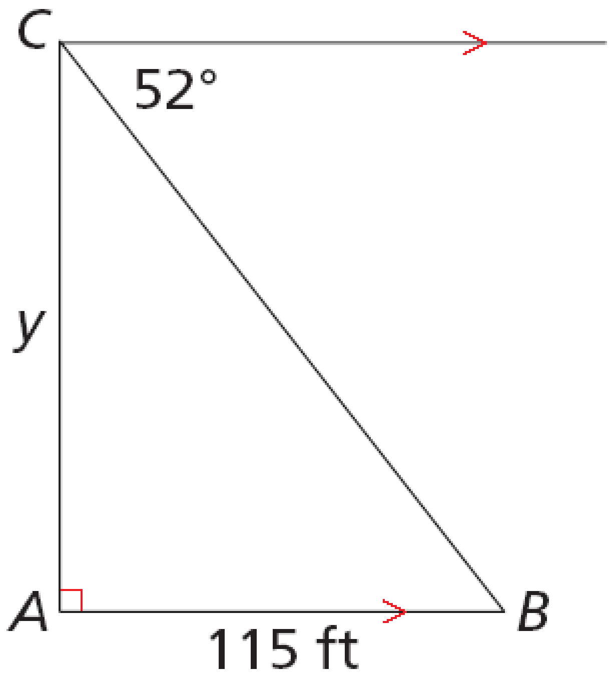


Example 3: Finding Distance by Using Angle of Depression

An ice climber stands at the edge of a crevasse that is 115 ft wide. The angle of depression from the edge where she stands to the bottom of the opposite side is 52º. How deep is the crevasse at this point? Round to the nearest foot.

Example 3 Continued

Draw a sketch to represent the given information. Let *C* represent the ice climber and let *B* represent the bottom of the opposite side of the crevasse. Let *y* be the depth of the crevasse.



Example 3 Continued

By the Alternate Interior Angles Theorem, $m\angle B = 52°$.

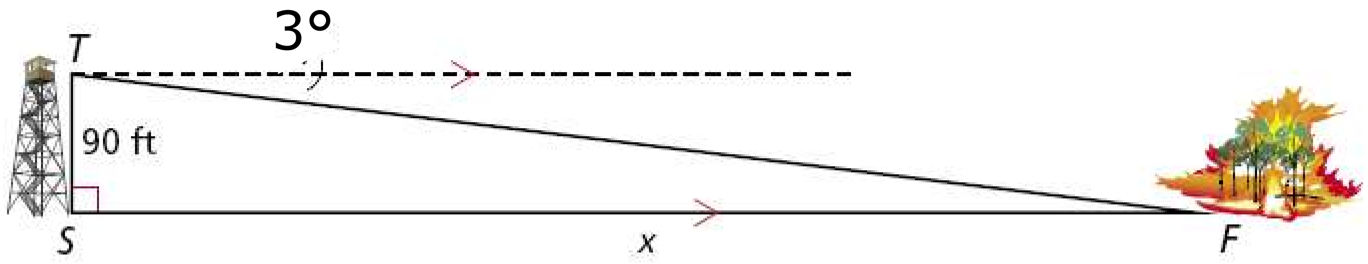
$$\tan 52° = \frac{y}{115}$$ *Write a tangent ratio.*

$y = 115 \tan 52°$ *Multiply both sides by 115.*

$y \approx 147$ ft *Simplify the expression.*

Check It Out! Example 3

What if…? Suppose the ranger sees another fire and the angle of depression to the fire is 3°. What is the horizontal distance to this fire? Round to the nearest foot.



By the Alternate Interior Angles Theorem, $m\angle F = 3°$.

$$\tan 3° = \frac{90}{x}$$ *Write a tangent ratio.*

$$x = \frac{90}{\tan 3°}$$ *Multiply both sides by x and divide by tan 3°.*

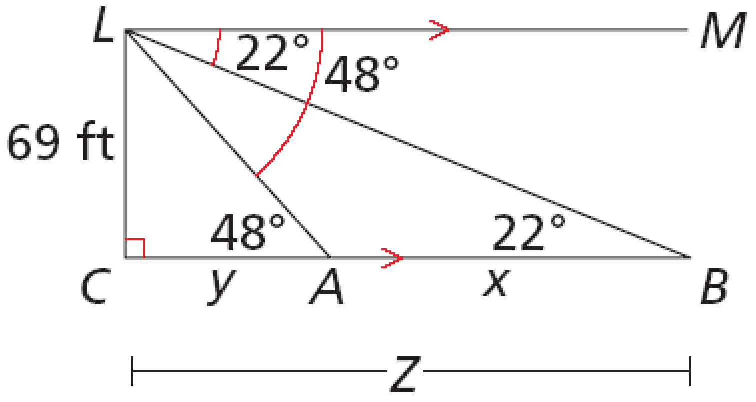
$$x \approx 1717 \text{ ft}$$ *Simplify the expression.*

Example 4: Shipping Application

An observer in a lighthouse is 69 ft above the water. He sights two boats in the water directly in front of him. The angle of depression to the nearest boat is 48º. The angle of depression to the other boat is 22º. What is the distance between the two boats? Round to the nearest foot.

Example 4 Application

Step 1 Draw a sketch. Let *L* represent the observer in the lighthouse and let *A* and *B* represent the two boats. Let *x* be the distance between the two boats.



Example 4 Continued

Step 2 Find *y*.

By the Alternate Interior Angles Theorem, $m\angle CAL = 58°$.

In ΔALC, $\tan 48° = \frac{69}{y}$.

So $y = \frac{69}{\tan 48°} \approx 62.1$ ft.

Example 4 Continued

Step 3 Find z.

By the Alternate Interior Angles Theorem, $m\angle CBL = 22°$.

In ΔBLC, $\tan 22° = \dfrac{69}{z}$.

So $z = \dfrac{69}{\tan 22°} \approx 170.8$ ft.

Example 4 Continued

Step 4 Find *x*.

$x = z - y$

$x \approx 170.8 - 62.1 \approx 109$ ft

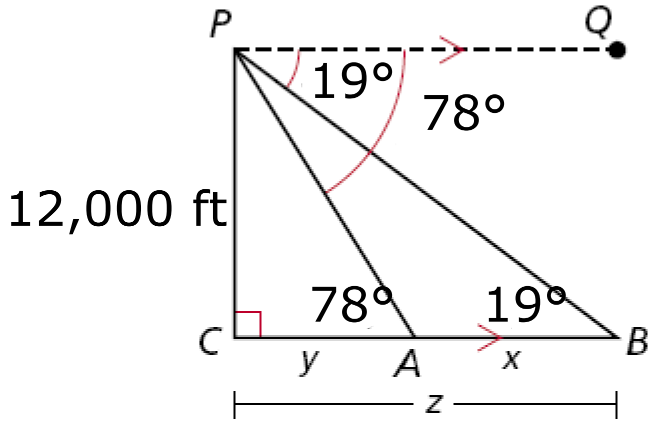
So the two boats are about 109 ft apart.

Check It Out! Example 4

A pilot flying at an altitude of 12,000 ft sights two airports directly in front of him. The angle of depression to one airport is 78°, and the angle of depression to the second airport is 19°. What is the distance between the two airports? Round to the nearest foot.

Check It Out! Example 4 Continued

Step 1 Draw a sketch. Let *P* represent the pilot and let *A* and *B* represent the two airports. Let *x* be the distance between the two airports.



Check It Out! Example 4 Continued

Step 2 Find *y*.

By the Alternate Interior Angles Theorem, $m\angle CAP = 78°$.

In ΔAPC, $\tan 78° = \frac{12{,}000}{y}$.

So $y = \frac{12{,}000}{\tan 78°} \approx 2551 \text{ ft}$.

Check It Out! Example 4 Continued

Step 3 Find *z*.

By the Alternate Interior Angles Theorem, $m\angle CBP = 19°$.

In ΔBPC, $\tan 19° = \frac{12{,}000}{z}$.

So $z = \frac{12{,}000}{\tan 19°} \approx 34{,}851$ ft.

Check It Out! Example 4 Continued

Step 4 Find x.

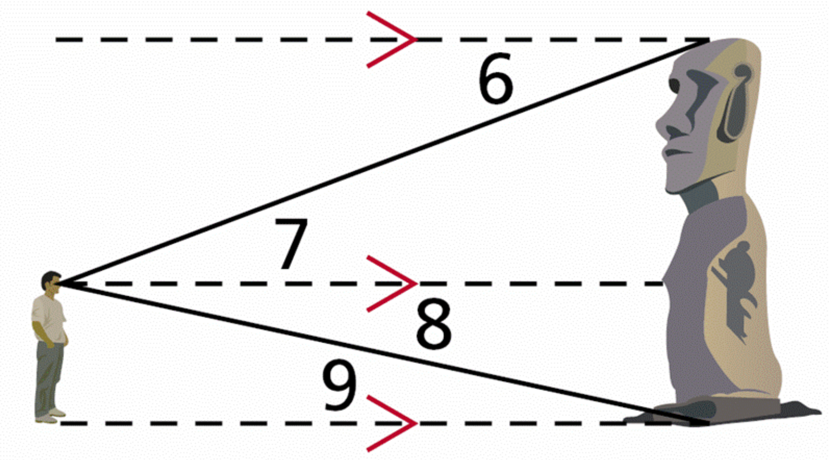
$x = z - y$

$x \approx 34{,}851 - 2551 \approx 32{,}300$ ft

So the two airports are about 32,300 ft apart.

Lesson Quiz: Part I

Classify each angle as an angle of elevation or angle of depression.



1. ∠6 angle of depression

2. ∠9 angle of elevation

Lesson Quiz: Part II

3. A plane is flying at an altitude of 14,500 ft. The angle of depression from the plane to a control tower is 15°. What is the horizontal distance from the plane to the tower? Round to the nearest foot. 54,115 ft

4. A woman is standing 12 ft from a sculpture. The angle of elevation from her eye to the top of the sculpture is 30°, and the angle of depression to its base is 22°. How tall is the sculpture to the nearest foot?

12 ft