IM 9–12 MATH v.1

Unit 2

Polynomials and Rational Functions

ALGEBRA 2

Lesson 25

Summing Up

Illustrative Mathematics®

Kendall Hunt

Learning Goal

Let's figure out a better way to add numbers.

Algebra 2

Illustrative Mathematics®

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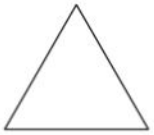
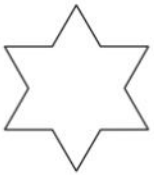
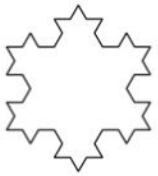
A Snowflake's Return

Warm-up: Notice and Wonder

What do you notice? What do you wonder?

iteration	total number of triangles added since the first
0 	0
1 	3
2 	$3 + 3(4) = 15$
3	$3 + 3(4) + 3(4)^2 = 63$
n	$3 + 3(4) + 3(4)^2 + \ldots + 3(4)^{n-1}$

A Geometric Addition Formula

Earlier, we learned that the nth term of a geometric sequence with an initial value of a and a common ratio of r is $a(r^{n-1})$.

For a Koch Snowflake, it turns out that we can find the number of triangles added on at each iteration by having $a = 3$ and $r = 4$. The sum s of the first n terms in this geometric sequence tell us how many triangles total make up the th iteration of the snowflake

$$s = 3 + 3(4) + 3\left(4^2\right) + \ldots + 3\left(4^{n-1}\right)$$

More generally, the sum of the first terms of any geometric sequence can be expressed as

$$s = a + a(r) + a\left(r^2\right) + \ldots + a\left(r^{n-1}\right)$$

Or

$$s = a\left(1 + r + r^2 + \ldots + r^{n-1}\right)$$

1. What would happen if we multiplied each side of this equation by $(1 - r)$? (hint: $(x - 1)(x^3 + x^2 + x + 1) = x^4 - 1$.)
2. Rewrite the new equation in the form of $s =$.
3. Use this new formula to calculate how many triangles after the original are in the first 5, 10, and 15 iterations of the Koch Snowflake.

The Sum of Antibiotics

Han is prescribed a course of antibiotics for an infection. He is told to take a 150 mg dose of the antibiotic regularly every 12 hours for 15 days. Han is curious about the antibiotic and learns that at the end of the 12 hours, only 5% of the dose is still in his body.

1. How much of the antibiotic is in the body right after the first, second, and third doses?
2. When will the total amount of the antibiotic in Han be highest over the course of the 15 day treatment? Explain your reasoning.

Summing Up

Lesson Synthesis

$$\frac{3}{10}, \frac{3}{100}, \frac{3}{1000}, \frac{3}{10000}, \ldots$$

What would you need to do to figure out the first 30 terms of this sequence?

I understand why the geometric sum formula is true.

Learning Targets

Algebra 2

After the Last Dose

Cool-down

The formula for the sum s of the first n terms in a geometric sequence is given by $s = a\frac{1-r^n}{1-r}$, where a is the initial value and r is the common ratio.

A drug is prescribed for a patient to take 200 mg every 12 hours for 10 days. After 12 hours about 4% of this dose is still in the body. How much of the drug is in the body right after the last dose?

identity

An equation which is true for all values of the variables in it.