IM 9–12 MATH v.I

Unit 2

Polynomials and Rational Functions

ALGEBRA 2

Lesson 24

Polynomial Identities (Part 2)

Illustrative Mathematics®

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Learning Goal

Let’s explore some other identities.

Algebra 2

Revisiting an Old Theorem

Warm-up

Instructions to make a right triangle:

- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

Follow these instructions to make a few different triangles. Do you think the instructions always produce a right triangle? Be prepared to explain your reasoning.

Theorems and Identities

Here are the instructions to make a right triangle from earlier:

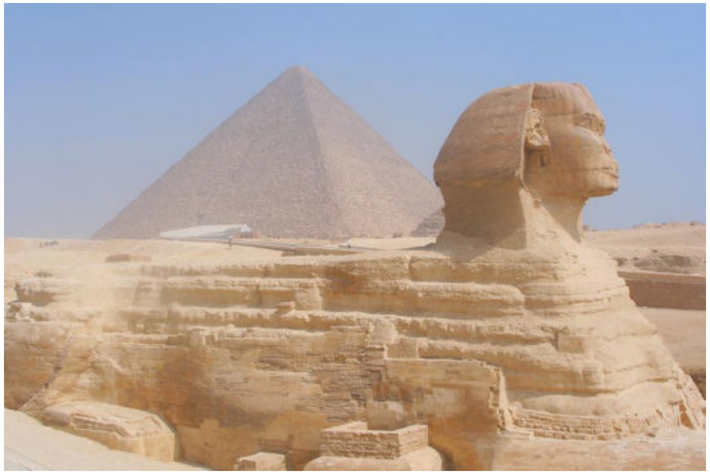
- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.
2. Why do these instructions make a right triangle?

Identifying Identities

Here is a list of equations. Circle all the equations that are identities. Be prepared to explain your reasoning.

1. $a = -a$
2. $a^2 + 2ab + b^2 = (a + b)^2$
3. $a^2 - 2ab + b^2 = (a - b)^2$
4. $a^2 - b^2 = (a - b)(a - b)$
5. $(a + b)(a^2 - ab + b^2) = a^3 - b^3$
6. $(a - b)^3 = a^3 - b^3 - 3ab(a + b)$
7. $a^2 (a - b)^4 - b^2(a - b)^4 = (a - b)^5 (a + b)$



In Ancient Egypt, all non-unit fractions were represented as a sum of distinct unit fractions. For example, $\frac{4}{9}$ would have been written as $\frac{1}{3}+\frac{1}{9}$ (and not as $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ or any other form with the same unit fraction used more than once). Let's look at some different ways we can rewrite $\frac{2}{15}$ as the sum of distinct unit fractions.

1. Use the formula $\frac{2}{d}=\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{6d}$ to rewrite the fraction $\frac{2}{15}$, then show that this formula is an identity.
2. Another way to rewrite fractions of the form $\frac{2}{d}$ is given by the identity $\frac{2}{d}=\frac{1}{d}+\frac{1}{d+1}+\frac{1}{d(d+1)}$ Use it to re-write the fraction $\frac{2}{15}$, then show that it is an identity.

Polynomial Identities (Part 2)

Lesson Synthesis

Look back at the different identities you have investigated so far.

Pick one that you have found interesting and write a few sentences explaining why.

I can justify why identities are true.

Learning Targets

Algebra 2

A Cubic Identification

Cool-down

Is $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ an identity? Explain or show your reasoning.

identity

An equation which is true for all values of the variables in it.