IM 9–12 MATH v.1

Unit 2

Polynomials and Rational Functions

ALGEBRA 2

Lesson 19

End Behavior of Rational Functions

Illustrative Mathematics®

Kendall Hunt

Learning Goal

Let's explore the end behavior of rational functions.

Algebra 2

Illustrative Mathematics®

Kendall Hunt

Different Divisions, Revisited

Warm-up

Complete all three representations of the polynomial division following the forms of the integer division.

$$\begin{array}{r} 252 \\ 11\overline{)2775} \\ \underline{2200} \\ 575 \\ \underline{550} \\ 25 \\ \underline{22} \\ 3 \end{array}$$

$$\begin{array}{r} 2x^2 \\ x+1\overline{)2x^3+7x^2+7x+5} \end{array}$$

$$2775 = 11(252) + 3$$

$$2x^3 + 7x^2 + 7x + 5 =$$

$$\frac{2775}{11} = 252 + \frac{3}{11}$$

$$\frac{2x^3 + 7x^2 + 7x + 5}{x+1} =$$

Combined Fuel Economy

In 2000, the Environmental Protection Agency (EPA) reported a combined fuel efficiency for cars that assumes 55% city driving and 45% highway driving. The expression for the combined fuel efficiency of a car that gets x mpg in the city and h mpg on the highway can be written as $\frac{100xh}{55x + 45h}$.

1. Several conventional cars have a fuel economy for highway driving is that is about 10 mpg higher than for city driving. That is, $h = x + 10$. Write a function f that represents the combined fuel efficiency for cars like these in terms of x.
2. Rewrite f in the form $q(x) + \frac{r(x)}{b(x)}$ where $q(x)$, $r(x)$, and $b(x)$ are polynomials.

Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $q(x) + \frac{r(x)}{b(x)}$	end behavior
$g(x) = -\frac{5}{x+2}$				
$h(x) = \frac{7x-5}{x+2}$				
$j(x) = \frac{3x^2+7x-5}{x+2}$				
$k(x) = \frac{2x^3+3x^2+7x-5}{x+2}$				
$m(x) = \frac{x+2}{2x^3+3x^2+7x-5}$				

1. Complete the table to explore the end behavior for rational functions.
2. What do you notice about the end behavior of different types of rational functions?

End Behavior of Rational Functions

Lesson Synthesis

There are many cylinders with volume 452 cm^3. Let r represent the radius of these cylinders, h represent the height, and S represent the surface area.

- The height of these cylinders is defined by the function $h(r) = \frac{452}{\pi r^2}$. What is the end behavior of this function, and what does it tell us about the situation?
- The surface area of these cylinders is defined by the function $S(r) = \frac{2\pi r^2 + 904}{r}$. What is the end behavior of this function, and what does it tell us about the situation?

I can find the end behavior of a rational function by rewriting it as $f(x) = q(x) + \frac{r(x)}{b(x)}$.

Learning Targets

Algebra 2

Finding End Behavior

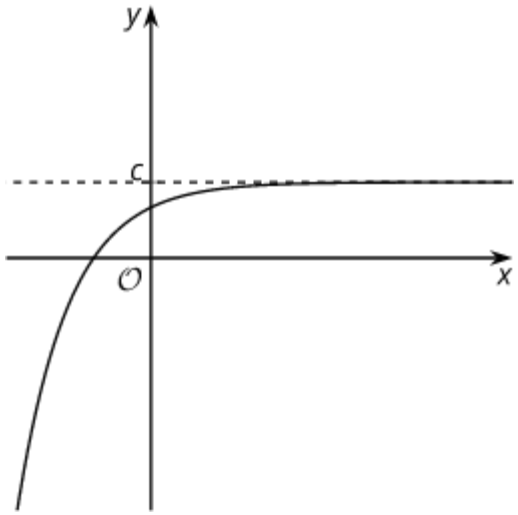
Cool-down

1. Rewrite $c(x) = \frac{x^2 + 5x + 2}{x - 3}$ in the form $y = q(x) + \frac{r(x)}{b(x)}$ where r has degree less than b.

1. What is the end behavior of ?$y = c(x)$

horizontal asymptote

The line The line $y = c$ is a horizontal asymptote of a function if the outputs of the function get closer and closer to c as the inputs get larger and larger in either the positive or negative direction. This means the graph gets closer and closer to the line as you move to the right or left along the x-axis.



rational function

A rational function is a function defined by a fraction with polynomials in the numerator and denominator. Rational functions include polynomials because a polynomial can be written as a fraction with denominator 1.

vertical asymptote

The line $x = a$ is a vertical asymptote for a function f if f is undefined at $x = a$ and its outputs get larger and larger in the negative or positive direction when x gets closer and closer to a on each side of the line. This means the graph goes off in the vertical direction on either side of the line.