IM 9–12 MATH v.1

Unit 2

Polynomials and Rational Functions

ALGEBRA 2

Lesson 18

Graphs of Rational Functions (Part 2)

Illustrative Mathematics®

Kendall Hunt

Learning Goal

Algebra 2

Let's learn about horizontal asymptotes.

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Rewritten Equations

Warm-up

Decide if each of these equations is true or false for x values that do not result in a denominator of 0. Be prepared to explain your reasoning.

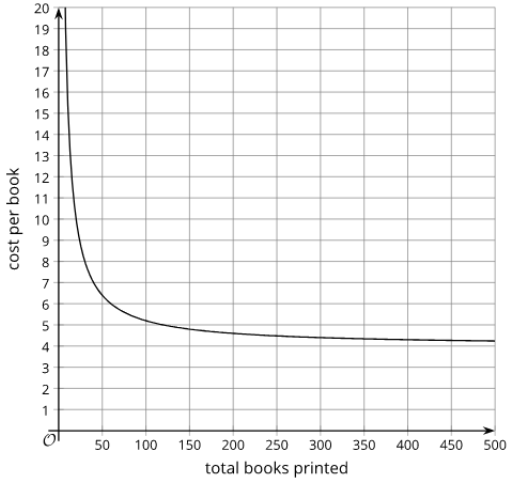
1. $\frac{x+7}{x} = 1 + \frac{7}{x}$

$\frac{x}{x+7} = 1 + \frac{x}{7}$

Publishing a Paperback

Let c be the function that gives the average cost per book $c(x)$, in dollars, when using an online store to print x copies of a self-published paperback book. Here is a graph of

$$c(x) = \frac{120 + 4x}{x}$$

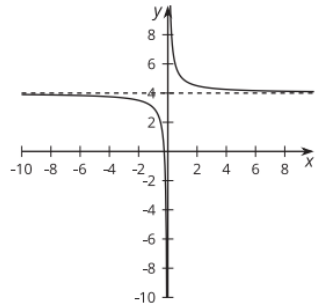


1. What is the approximate cost per book when 50 books are printed? 100 books?
2. The author plans to charge $8 per book. About how many should be printed to make a profit?
3. What is the value of $c(x)$ when $x = ½$? How does this relate to the context?
4. What does the end behavior of the function say about the context?

Horizontal Asymptotes

Here are four graphs of rational functions.

A



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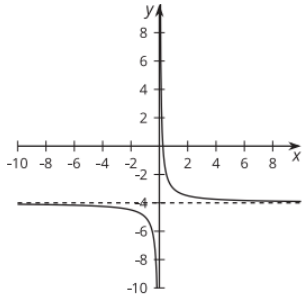

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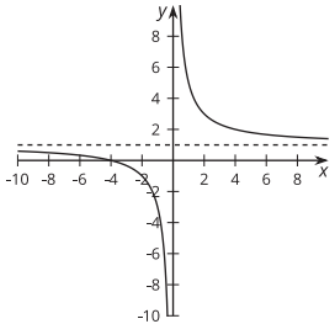

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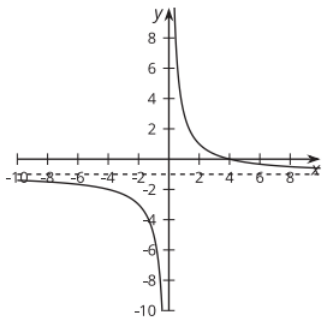


1. Match each function with its graphical representation.

$a.\ a(x) = \frac{4}{x} - 1$

$b.\ b(x) = \frac{1}{x} - 4$

$c.\ c(x) = \frac{1+4x}{x}$

$d.\ d(x) = \frac{x+4}{x}$

$e.\ e(x) = \frac{1-4x}{x}$

$f.\ f(x) = \frac{4-x}{x}$

$g.\ g(x) = 1 + \frac{4}{x}$

$h.\ h(x) = \frac{1}{x} + 4$

1. Where do you see the **horizontal asymptote** of the graph in the expressions for the functions?

Graphs of Rational Functions (Part 2)

Identify the horizontal asymptote of the function.

$$f(x) = \frac{7x + 2}{x}$$

$$g(x) = \frac{7x + 2}{2x}$$

$$h(x) = \frac{7x + 2}{0.5x}$$

- What are two ways you could change a single number in *f(x)* so it has a horizontal asymptote at 35?
- What is the vertical asymptote for each function?
- How could you figure out the horizontal asymptote of something like $k(x) = \frac{7x + 2}{x - 1}$?

I can identify a horizontal asymptote from a graph or an equation of a rational function.

Learning Targets

Algebra 2

Publishing a Paperback, Revisited

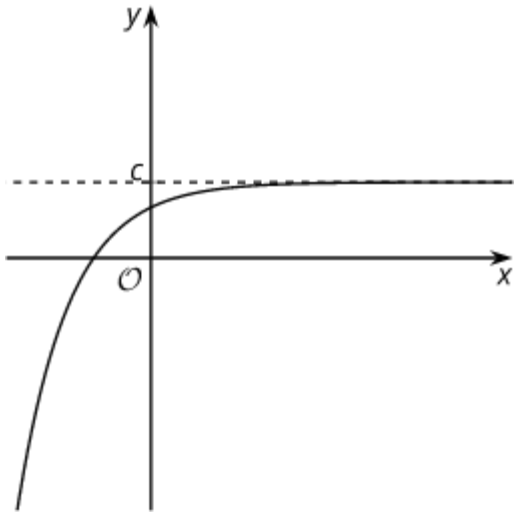
Cool-down

The average cost for printing x copies of a self-published paperback book with Company A is $c(x) = \frac{120 + 4x}{x}$. The average cost for printing copies of a paperback book with Company B is $d(x) = \frac{25 + 10x}{2x}$.

1. Which company would you recommend to an author who wants to print 100 books? Explain your reasoning.
2. Which company would you recommend to an author who thinks their book will be a best seller and needs to print thousands of books? How could you rewrite the equations to make the choice clearer?

horizontal asymptote

The line The line $y = c$ is a horizontal asymptote of a function if the outputs of the function get closer and closer to c as the inputs get larger and larger in either the positive or negative direction. This means the graph gets closer and closer to the line as you move to the right or left along the x-axis.



rational function

A rational function is a function defined by a fraction with polynomials in the numerator and denominator. Rational functions include polynomials because a polynomial can be written as a fraction with denominator 1.

vertical asymptote

The line $x = a$ is a vertical asymptote for a function f if f is undefined at $x = a$ and its outputs get larger and larger in the negative or positive direction when x gets closer and closer to a on each side of the line. This means the graph goes off in the vertical direction on either side of the line.