IM 9–12 MATH v.1

Unit 2

Polynomials and Rational Functions

ALGEBRA 2

Lesson 15

The Remainder Theorem

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Learning Goal

Let's learn about the Remainder Theorem.

Algebra 2

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Division Leftovers

Warm-up: Notice and Wonder

What do you notice? What do you wonder?

$$\begin{array}{r} 33 \\ 10\overline{)330} \\ \underline{300} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

$$\begin{array}{r} 82 \\ 4\overline{)330} \\ \underline{320} \\ 10 \\ \underline{8} \\ 2 \end{array}$$

$$\begin{array}{r} 66 \\ 5\overline{)330} \\ \underline{300} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

A. $330 = 33\,(10) + 0$

B. $330 = 4\,(82) + 2$

C. $330 = 5\,(66) + 0$

The Unknown Coefficient

Consider the polynomial function $f(x) = x^4 - ux^3 + 24x^2 - 32x + 16$ where u is an unknown real number. If $x - 2$ is a factor, what is the value of u? Explain how you know.

A Study of Remainders

1. Which of these polynomials could have $(x - 2)$ as a factor?
   a. $A(x) = 6x^2 - 7x - 5$
   b. $B(x) = 3x^2 + 15x - 42$
   c. $C(x) = 2x^3 + 13x^2 + 16x + 5$
   d. $D(x) = 3x^3 - 2x^2 - 15x + 14$
   e. $E(x) = 8x^4 - 41x^3 - 18x^2 + 101x + 70$
   f. $F(x) = x^4 + 5x^3 - 27x^2 - 101x - 70$
2. Select one of the polynomials that you said doesn't have (x - 2) as a factor.
   a. Explain how you know (x - 2)is not a factor.
   b. If you have not already done so, divide the polynomial by . What is the remainder?
3. List the remainders for each of the polynomials when divided by $(x - 2)$. How do these values compare to the value of the functions at $x = 2$?

The Remainder Theorem

Lesson Synthesis

The answer is $f(x) = (2x - 1)(x + 2)(x - 8)$. What was the question?

I understand the remainder theorem and why it's true.

Learning Targets

Algebra 2

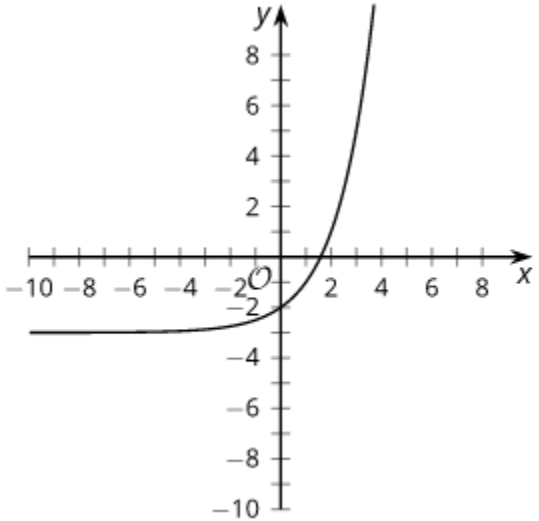
Using Remainder Knowledge

Cool-down

Let p be a polynomial function defined by $p(x) = 2x^3 + 5x^2 - 6x - 9$. Use the fact that $p(-4) = -33$, $p(-3) = 0$, $p(-2) = 7$, and p(-1) = 0 to rewrite the expression for $p(x)$ as the product of linear factors.

end behavior

How the outputs of a function change as we look at input values further and further from 0.



This function shows different end behavior in the positive and negative directions. In the positive direction the values get larger and larger. In the negative direction the values get closer and closer to -3.

multiplicity

The power to which a factor occurs in the factored form of a polynomial. For example, in the polynomial $(x - 1)^2(x + 3)$, the factor $x - 1$ has multiplicity 2 and the factor $x + 3$ has multiplicity 1.