Lines That Intersect Circles

Warm Up

Lesson Presentation

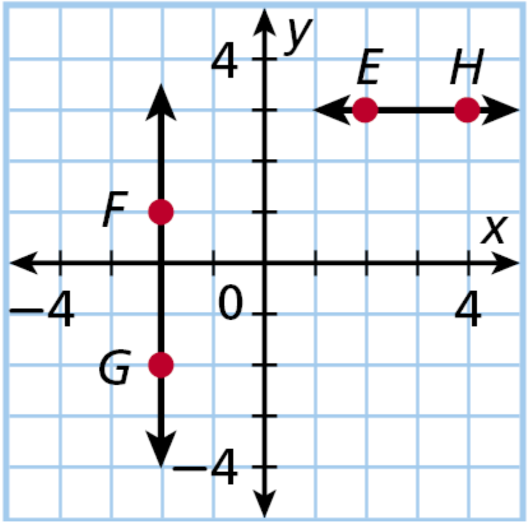
Lesson Quiz

Warm Up

Write the equation of each item.

1. $\overleftrightarrow{FG}$ $x = -2$

2. $\overleftrightarrow{EH}$ $y = 3$



3. $2(25 - x) = x + 2$

$x = 16$

4. $3x + 8 = 4x$

$x = 8$

Objectives

Identify tangents, secants, and chords.

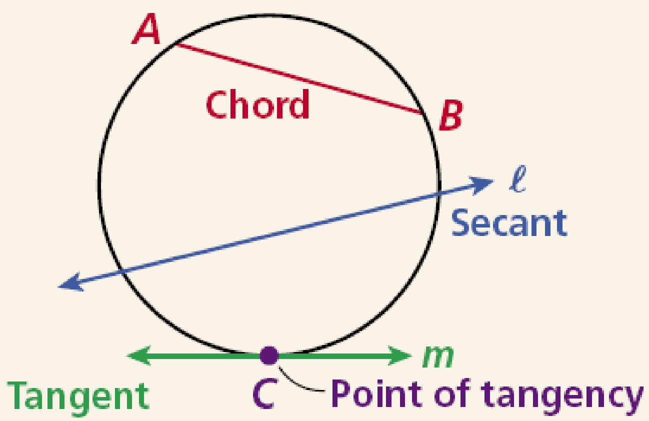
Use properties of tangents to solve problems.

Vocabulary

interior of a circleconcentric circles
exterior of a circletangent circles
chordcommon tangent
secant
tangent of a circle
point of tangency
congruent circles

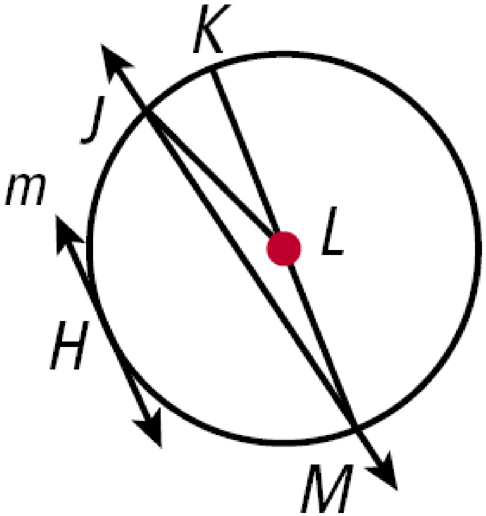
The **interior of a circle** is the set of all points inside the circle. The **exterior of a circle** is the set of all points outside the circle.

Lines and Segments That Intersect Circles

TERM	DIAGRAM
A **chord** is a segment whose endpoints lie on a circle.	
A **secant** is a line that intersects a circle at two points.	
A **tangent** is a line in the same plane as a circle that intersects it at exactly one point.	
The point where the tangent and a circle intersect is called the **point of tangency**.	

Example 1: Identifying Lines and Segments That Intersect Circles

Identify each line or segment that intersects $\odot L$.



chords: $\overline{JM}$ and $\overline{KM}$

secant: $\overleftrightarrow{JM}$

tangent: m

diameter: $\overline{KM}$

radii: $\overline{LK}$, $\overline{LJ}$, and $\overline{LM}$

Check It Out! Example 1

Identify each line or segment that intersects ⊙*P*.

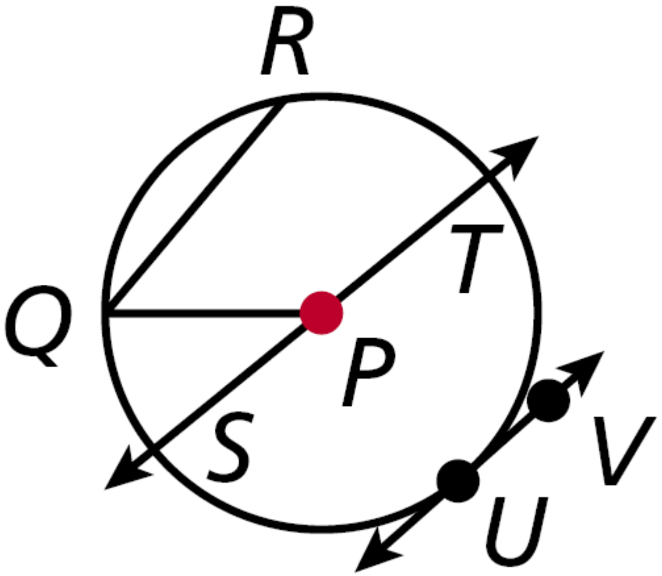
chords: $\overline{QR}$ and $\overline{ST}$

secant: $\overleftrightarrow{ST}$

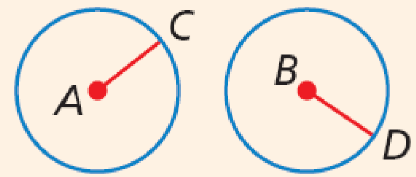
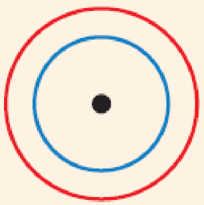
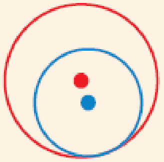
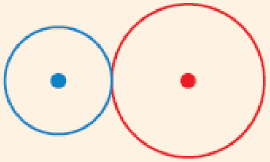
tangent: $\overleftrightarrow{UV}$

diameter: $\overline{ST}$

radii: $\overline{PQ}$, $\overline{PT}$, and $\overline{PS}$



Pairs of Circles

TERM	DIAGRAM
Two circles are **congruent circles** if and only if they have congruent radii.	 $\odot A \cong \odot B$ *if* $\overline{AC} \cong \overline{BD}$. $\overline{AC} \cong \overline{BD}$ *if* $\odot A \cong \odot B$.
Concentric circles are coplanar circles with the same center.	
Two coplanar circles that intersect at exactly one point are called **tangent circles**.	 Internally tangent circles  Externally tangent circles

Example 2: Identifying Tangents of Circles

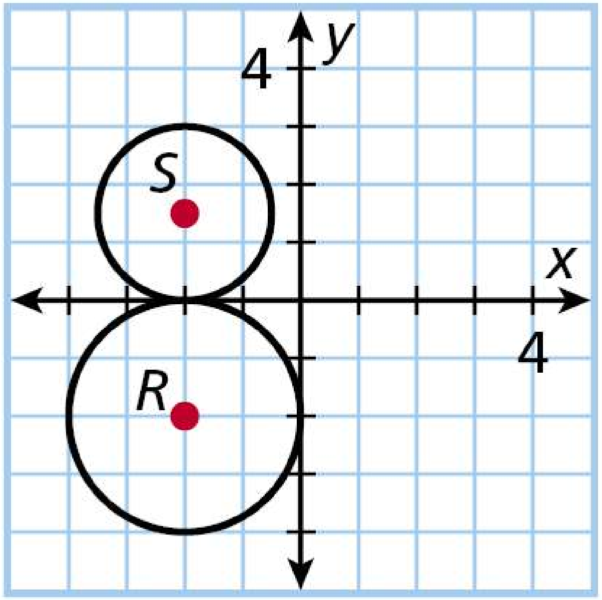
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of ⊙*R*: 2

Center is (–2, –2). Point on ⊙ is (–2,0). Distance between the 2 points is 2.

radius of ⊙*S*: 1.5

Center is (–2, 1.5). Point on ⊙ is (–2,0). Distance between the 2 points is 1.5.



Example 2 Continued

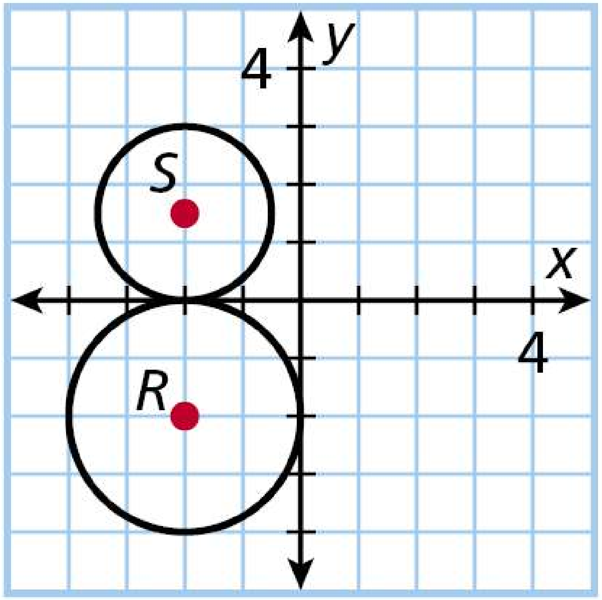
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

point of tangency: (–2, 0)

Point where the ⊙s and tangent line intersect

equation of tangent line: $y = 0$

Horizontal line through (–2,0)



Check It Out! Example 2

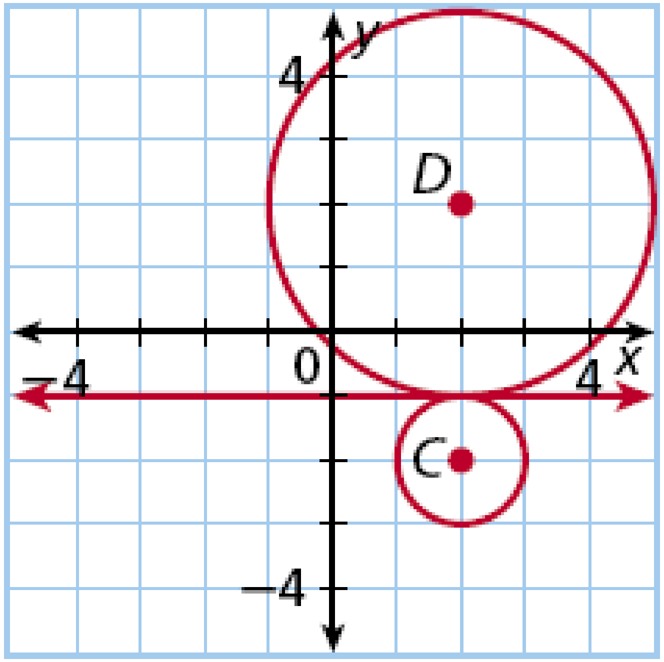
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

radius of $\odot C$: 1

Center is (2, –2). Point on $\odot$ is (2, –1). Distance between the 2 points is 1.

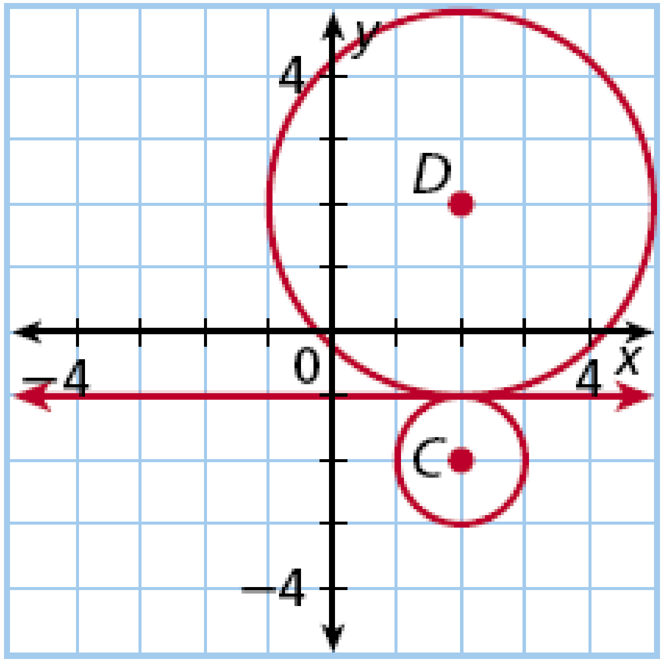
radius of $\odot D$: 3

Center is (2, 2). Point on $\odot$ is (2, –1). Distance between the 2 points is 3.



Check It Out! Example 2 Continued

Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



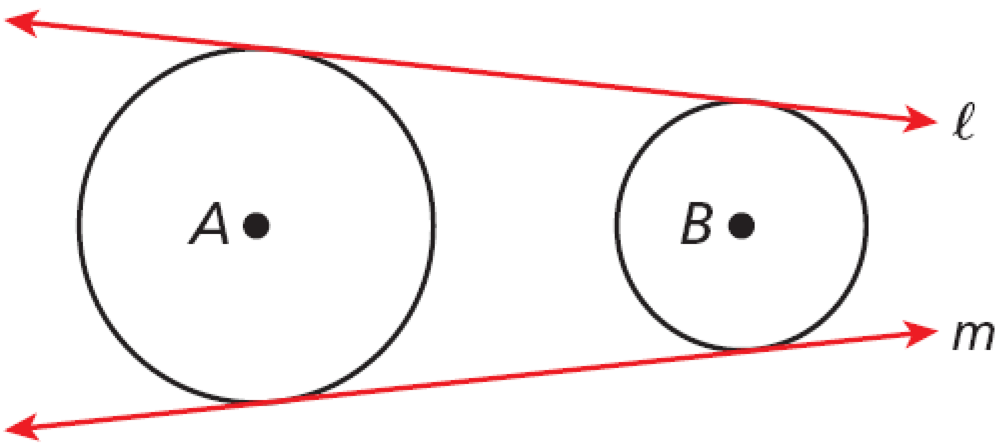
Pt. of tangency: **(**2, –1**)**

Point where the ⊙s and tangent line intersect

eqn. of tangent line: $y = -1$

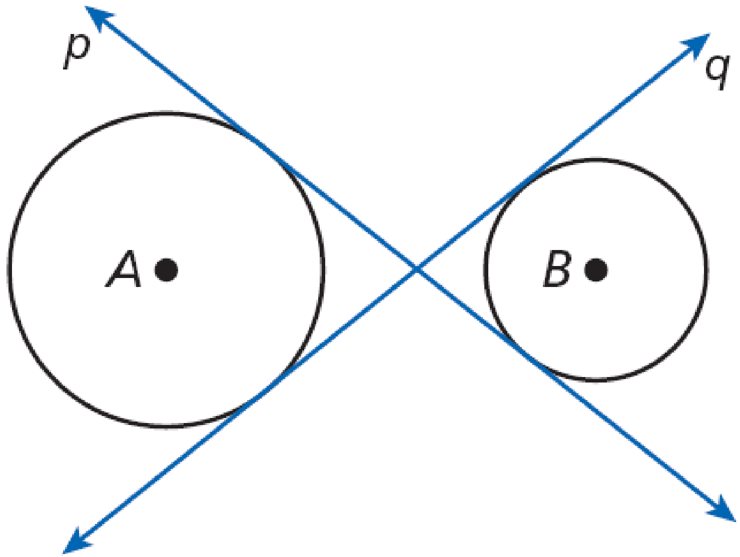
Horizontal line through (2,-1)

A **common tangent** is a line that is tangent to two circles.



Lines ℓ and m are common external tangents to $\odot A$ and $\odot B$.

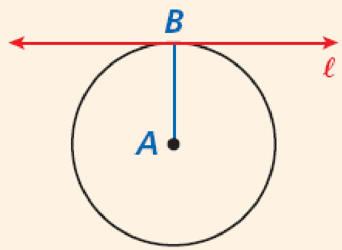
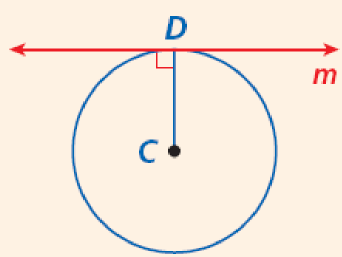
A **common tangent** is a line that is tangent to two circles.



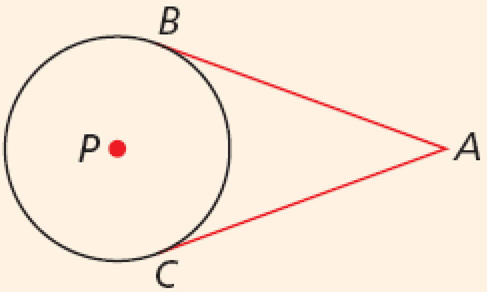
Lines *p* and *q* are common internal tangents to ⊙*A* and ⊙*B*.

Lines That Intersect Circles

Theorems

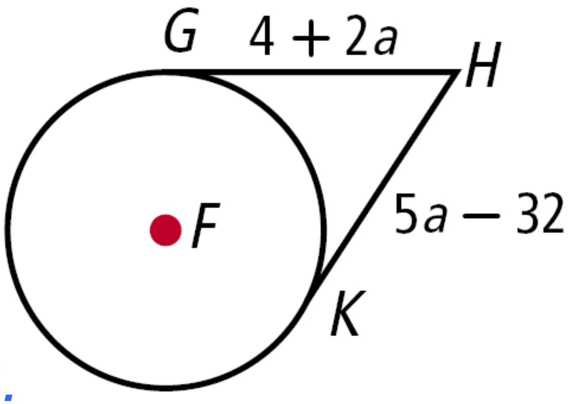
THEOREM	HYPOTHESIS	CONCLUSION
11-1-1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (line tangent to ⊙ → line ⊥ to radius)	 ℓ is tangent to ⊙*A*	$\ell \perp \overline{AB}$
11-1-2 If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle. (line ⊥ to radius → line tangent to ⊙)	 *m* is ⊥ to $\overline{CD}$ at *D*	*m* is tangent to ⊙*C*.

Theorem 11-1-3

THEOREM	HYPOTHESIS	CONCLUSION
If two segments are tangent to a circle from the same external point, then the segments are congruent. (2 segs. tangent to ⊙ from same ext. pt. $\rightarrow$ segs. $\cong$)	 $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot P$.	$\overline{AB} \cong \overline{AC}$

Example 4: Using Properties of Tangents

$\overline{HK}$ and $\overline{HG}$ are tangent to $\odot F$. Find HG.



$HK = HG$ — 2 segments tangent to ⊙ from same ext. point → segments ≅.

$5a - 32 = 4 + 2a$ — Substitute 5a – 32 for HK and 4 + 2a for HG.

$3a - 32 = 4$ — Subtract 2a from both sides.

$3a = 36$ — Add 32 to both sides.

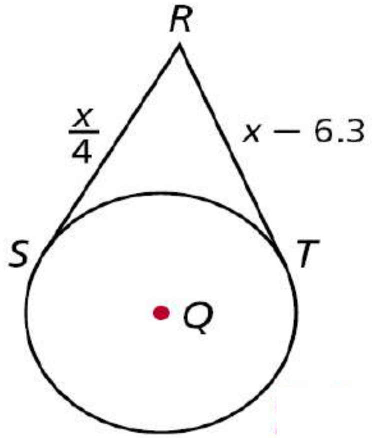
$a = 12$ — Divide both sides by 3.

$HG = 4 + 2(12)$ — Substitute 12 for a.

$= 28$ — Simplify.

Check It Out! Example 4a

$\overline{RS}$ and $\overline{RT}$ are tangent to $\odot Q$. Find RS.



$RS = RT$ — *2 segments tangent to ⊙ from same ext. point → segments ≅.*

$\frac{x}{4} = x - 6.3$ — *Substitute $\frac{x}{4}$ for RS and x – 6.3 for RT.*

$x = 4x - 25.2$ — *Multiply both sides by 4.*

$-3x = -25.2$ — *Subtract 4x from both sides.*

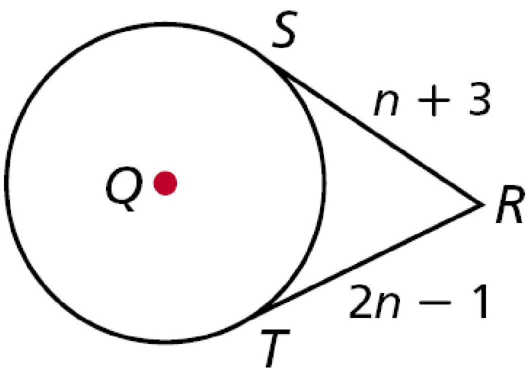
$x = 8.4$ — *Divide both sides by –3.*

$RS = \frac{8.4}{4}$ — *Substitute 8.4 for x.*

$= 2.1$ — *Simplify.*

Check It Out! Example 4b

$\overline{RS}$ and $\overline{RT}$ are tangent to ⊙*Q*. Find *RS*.



$RS = RT$	*2 segments tangent to ⊙ from same ext. point → segments ≅.*
$n + 3 = 2n - 1$	*Substitute n + 3 for RS and 2n – 1 for RT.*
$4 = n$	*Simplify.*
$RS = 4 + 3$	*Substitute 4 for n.*
$= 7$	*Simplify.*

Lesson Quiz: Part I

1. Identify each line or segment that intersects ⊙Q.

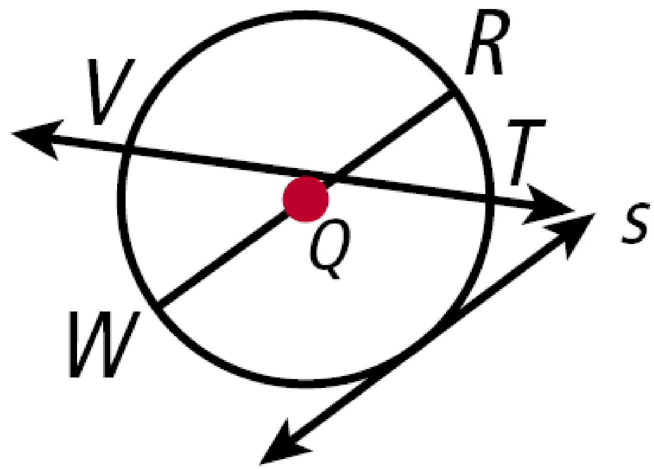
chords $\overline{VT}$ and $\overline{WR}$

secant: $\overleftrightarrow{VT}$

tangent: s

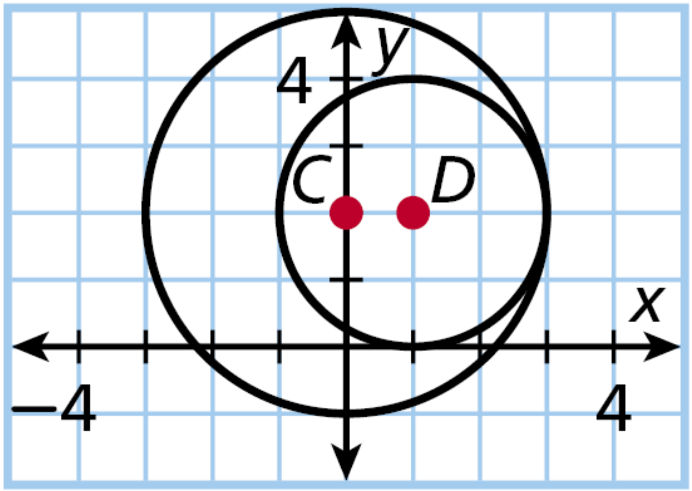
diam.: $\overline{WR}$

radii: $\overline{QW}$ and $\overline{QR}$



Lesson Quiz: Part II

2. Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.



radius of ⊙*C*: 3

radius of ⊙*D*: 2

pt. of tangency: **(**3, 2**)**

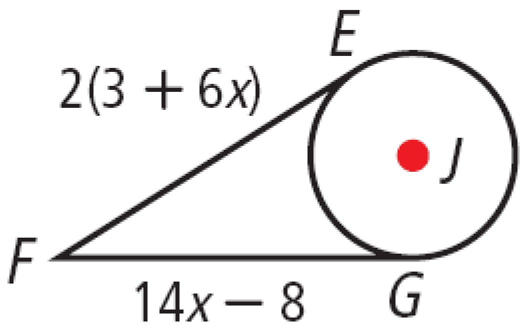
eqn. of tangent line: $x = 3$

Lesson Quiz: Part III

3. Mount Mitchell peaks at 6,684 feet. What is the distance from this peak to the horizon, rounded to the nearest mile?

≈ 101 mi

4. $\overline{FE}$ and $\overline{FG}$ are tangent to ⊙*F*. Find *FG*.



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