Introduction

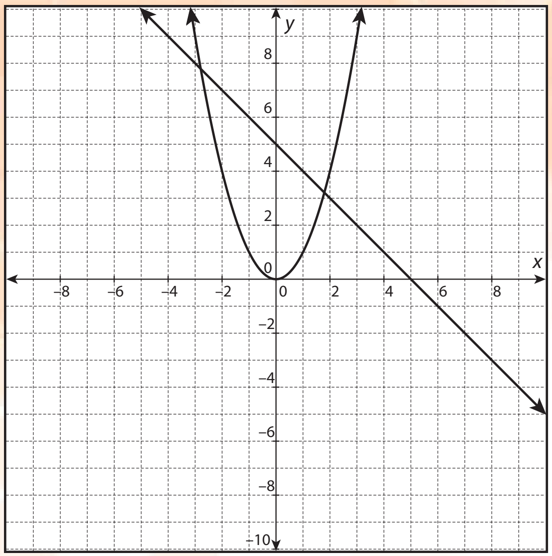
A **system of equations** is a set of equations with the same unknowns. A **quadratic-linear system** is a system of equations in which one equation is quadratic and one is linear. We learned previously how to solve systems of linear equations. In this lesson, we will learn to solve a quadratic-linear system by graphing.

Key Concepts

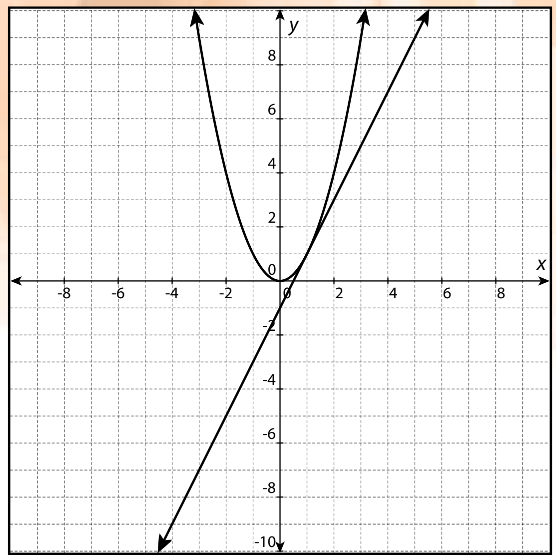
- A quadratic-linear system may have one real solution, two real solutions, or no real solutions.
- The solutions of a quadratic-linear system can be found by graphing.
- The **points of intersection** of the graphed quadratic and linear equation are the ordered pair(s) where graphed functions intersect on a coordinate plane. These are also the solutions to the system.

Key Concepts, *continued*

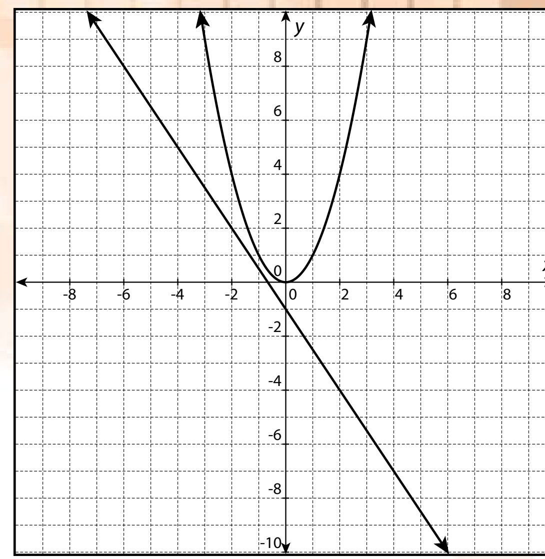
Graphed Quadratic-Linear Systems



If the equations intersect in two places, they have two real solutions.



If the equations intersect in one place, they have one real solution.



If the equations do not intersect, they have no real solutions.

Key Concepts, *continued*

- Real-world problems may appear to have two solutions when graphed, but in fact only have one because of the context of the problem. Traditionally, negative values are ignored. For example, if graphing the distance a car traveled in relation to time, if one algebraic solution had a negative value, it would not be a true solution since distance and time are always positive.

Common Errors/Misconceptions

- not identifying all points of intersection
- incorrectly entering the equations into the calculator

Guided Practice

Example 1

Graph the system of equations below to determine the real solution(s), if any exist.

$$\begin{cases} y = x^2 + 2x - 2 \\ y = 2x - 2 \end{cases}$$

Guided Practice: Example 1, *continued*

1. **Graph the quadratic function,**
   $y = x^2 + 2x - 2$.

If graphing by hand, first plot the y-intercept, (0, –2).

Next, use the equation for the axis of symmetry to determine the *x*-coordinate of the vertex of the parabola.

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$$

Draw the axis of symmetry, $x = -1$.

Guided Practice: Example 1, *continued*

Substitute the *x*-coordinate to determine the corresponding *y*-coordinate of the vertex.

$$y = (-1)^2 + 2(-1) - 2 = 1 - 2 - 2 = -3$$

Plot the vertex, (–1, –3).

Finally, since the *y*-intercept is 1 unit right of the axis of symmetry, an additional point on the parabola is 1 unit left of the axis of symmetry at (–2, –2). Sketch the curve.

If using a graphing calculator, follow the directions appropriate to your model.

Guided Practice: Example 1, *continued*

On a TI-83/84:

Step 1: Press [Y=].

Step 2: At Y1, type [X, T, θ, n][x^2][+][2][X, T, θ, n][–][2].

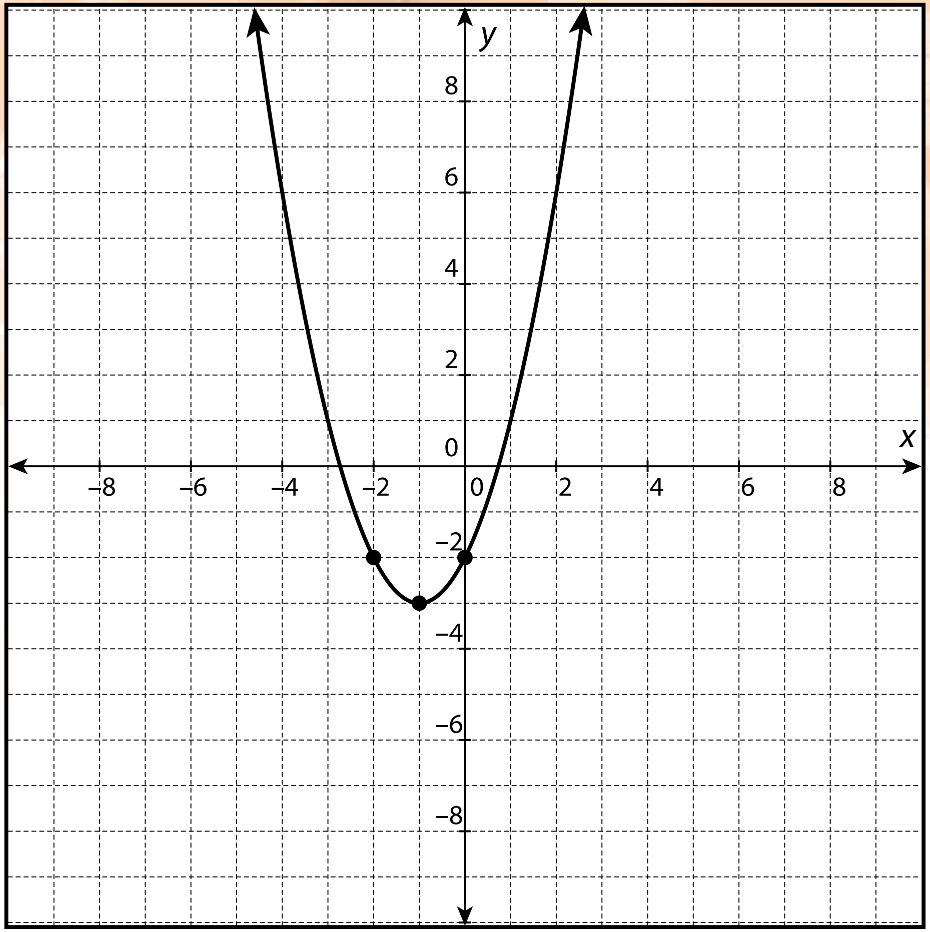
Step 3: Press [GRAPH].

On a TI-Nspire:

Step 1: Press [menu] and select 3: Graph Type, then select 1: Function.

Step 2: At *f1(x)*, enter [*x*], hit the [x^2] key, then type [+][2][*x*][–][2] and press [enter] to graph the quadratic function.

Guided Practice: Example 1, *continued*



Guided Practice: Example 1, *continued*

2. **Graph the linear function, $y = 2x - 2$, on the same grid.**

If graphing by hand, first plot the *y*-intercept, (0, –2).

Note: Since the two functions have the same *y*-intercept, we have already determined one solution.

Next, use the slope of 2 to graph additional points in either direction.

Sketch the line.

Guided Practice: Example 1, *continued*

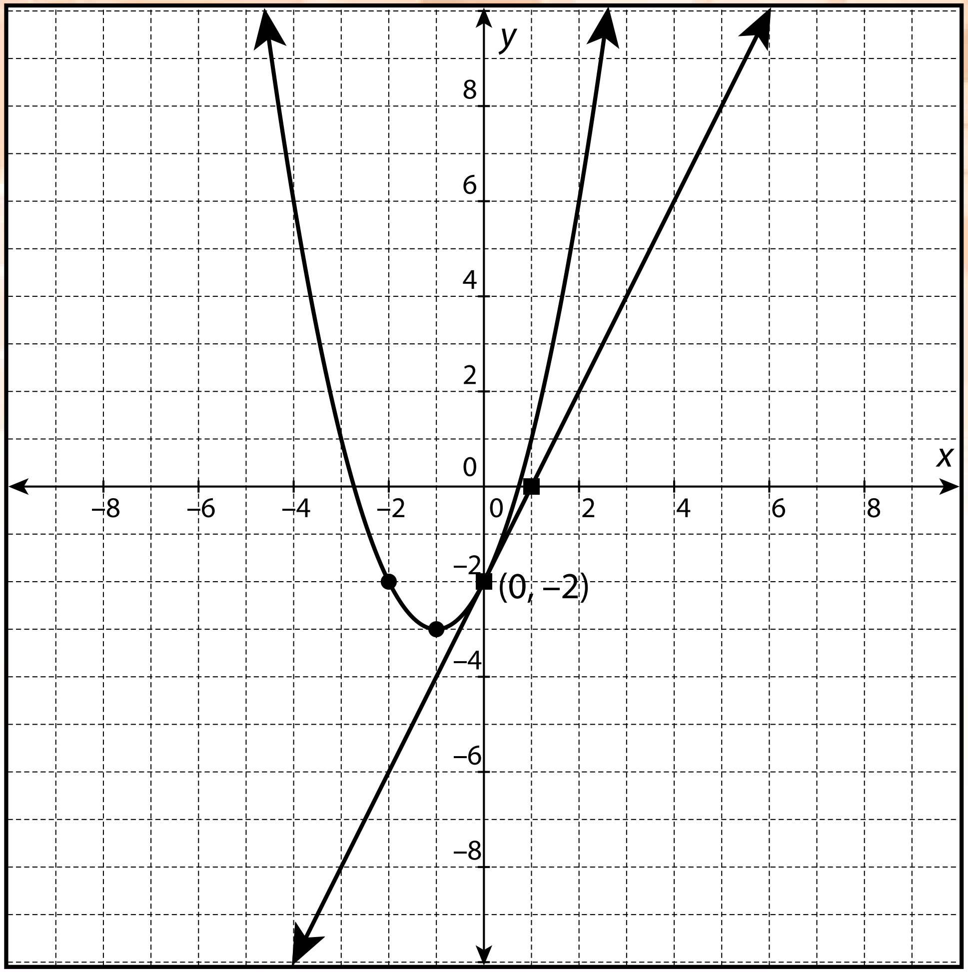
On a TI-83/84:

Step 1: Press [Y=].
Step 2: At Y_2, type [2][X, T, θ, n][–][2].
Step 3: Press [GRAPH].

On a TI-Nspire:

Step 1: Press [menu] and select 3: Graph Type, then select 1: Function.
Step 2: At *f2(x)*, enter [2][*x*][–][2] and press [enter] to graph the linear function.

Guided Practice: Example 1, *continued*



Guided Practice: Example 1, *continued*

3. **Note any intersections that exist as the solution or solutions.**

On a TI-83/84:

Step 1: To approximate the intersections, press [2nd][TRACE] and select intersect.

Step 2: Press [ENTER].

Step 3: At the prompt that reads “First curve?”, press [ENTER].

Step 4: At the prompt that reads “Second curve?”, press [ENTER].

Guided Practice: Example 1, *continued*

Step 5: At the prompt that reads “Guess?”, arrow as close as possible to the first point of intersection and press [ENTER]. Write down the approximate ordered pair.

On a TI-Nspire:

Step 1: Press [menu] and select 7: Points & Lines, then select 3: Intersection Point(s).

Step 2: Select the graphs of both functions.

Guided Practice: Example 1, *continued*

The only point of intersection is $(0,-2)$.

The solution of $\begin{cases} y = x^2 + 2x - 2 \\ y = 2x - 2 \end{cases}$ is $(0,-2)$.

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Guided Practice: Example 1, *continued*

Guided Practice

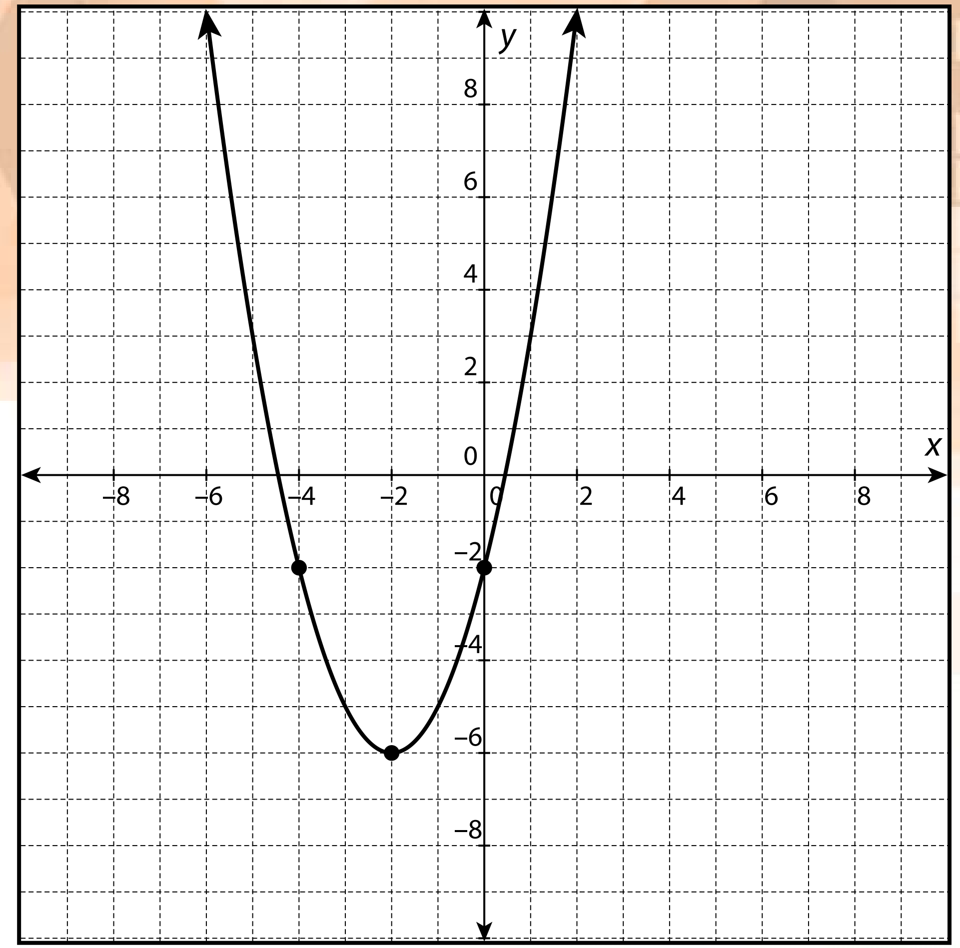
Example 2

Graph the system of equations below to determine the real solution(s), if any exist.

$$\begin{cases} y = 3x \\ y = x^2 + 4x - 2 \end{cases}$$

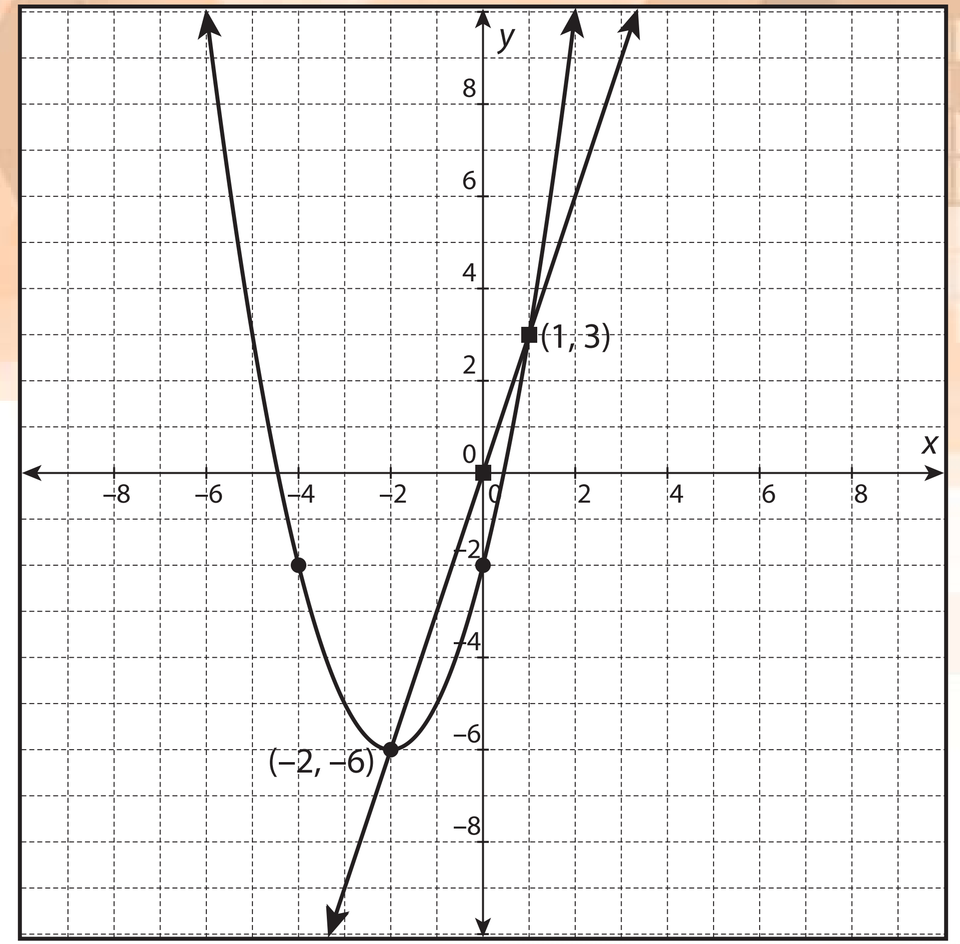
Guided Practice: Example 2, *continued*

1. Graph the quadratic function, $y = x^2 + 4x - 2$, using the methods demonstrated in Example 1.



Guided Practice: Example 2, *continued*

2. Graph the linear function, $y = 3x$, on the same grid.



Guided Practice: Example 2, *continued*

3. Note any intersections that exist as the solution or solutions.

The two points of intersection are $(-2, -6)$ and $(1, 3)$.

The solutions of $\begin{cases} y = 3x \\ y = x^2 + 4x - 2 \end{cases}$ are $(-2, -6)$ and $(1, 3)$.

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Guided Practice: Example 2, *continued*