Introduction

There are many applications for solid geometry in the real world. In this lesson, you will apply the formulas for the volumes of solids to solve real-world problems.

Formulas are derived in many ways, and you will learn to construct arguments for the derivations of the volumes of solids. Specifically, you will use Cavalieri's Principle to construct valid arguments for the volume of a sphere as well as for other solids. You will derive as well as compare the volumes of different solids. You will also apply the formulas for the volumes of prisms, cylinders, cones, pyramids, and spheres to solve real-world problems.

Key Concepts

- A **sphere** is a three-dimensional surface that has all its points the same distance from its center.
- The volume of a sphere can be derived in several ways, including by using Cavalieri's Principle and a limit process.
- The formula for the volume of sphere is $V = \frac{4}{3}\pi r^3$.

Key Concepts, *continued*

- Many application problems can be solved using volume formulas.
- You can make valid comparisons of volume formulas, which can be useful in developing other volume formulas.

Common Errors/Misconceptions

- confusing formulas; for example, confusing volume of a sphere with surface area of a sphere
- performing miscalculations with the number π
- making arithmetic errors when dealing with the formulas, especially with fractions
- using incorrect notation in word problems, such as using cm^2 instead of cm^3 for a volume problem

Guided Practice

Example 2

A teenager buying some chewing gum is comparing packages of gum in order to get the most gum possible. Each package costs the same amount. Package 1 has 20 pieces of gum shaped like spheres. Each piece has a radius of 5 mm. Package 2 has 5 pieces of gum shaped like spheres. Each piece has a radius of 10 mm. Which package should the teenager buy? Round to the nearest millimeter.

Guided Practice: Example 2, *continued*

1. **Find the volume of a piece of gum in package 1 by using the volume formula for a sphere.**

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$V = \frac{4}{3}\pi (5)^3$	Substitute 5 for *r*.
$V = 524 \text{ mm}^3$	

Guided Practice: Example 2, *continued*

2. **Multiply the volume of the single piece of gum in package 1 by 20 to get the total volume of the gum in the package.**

   $(524)(20) = 10{,}480 \text{ mm}^3$

Guided Practice: Example 2, *continued*

3. The radius of each piece of gum in package 2 is 10 mm. Find the volume of a piece of gum in package 2 by using the volume formula for a sphere.

$V=\frac{4}{3}\pi r^3$	Formula for volume of a sphere
$V=\frac{4}{3}\pi(10)^3$	Substitute 10 for *r*.
$V\approx 4189\text{ mm}^3$	

Guided Practice: Example 2, *continued*

4. Multiply the volume of a single piece of gum in package 2 by 5 to get the total volume of gum in the package.

   $(4189)(5) = 20{,}945 \text{ mm}^3$

Guided Practice: Example 2, *continued*

5. **Compare the volumes of the two packages to determine the best purchase.**

   10,480 < 20,945

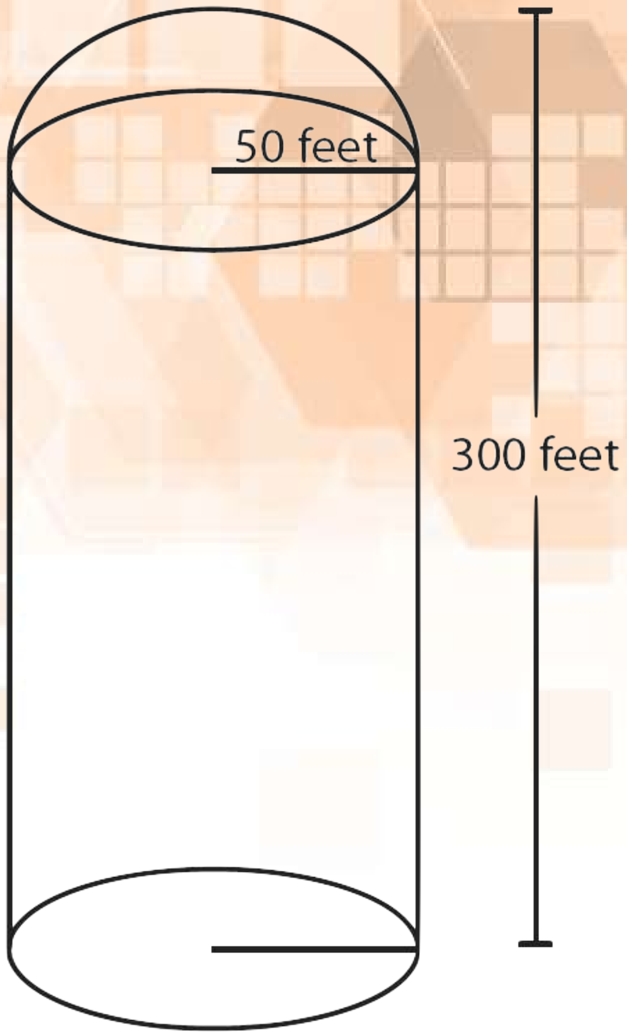
   The teenager should buy package 2.

Guided Practice: Example 2, *continued*

Guided Practice

Example 3

Pictured to the right is a cylindrical grain silo. It can be completely filled to the top of the dome. The dome is in the shape of a hemisphere. The height of the silo is 300 feet to the top of the dome and the radius of the dome is 50 feet. How much grain can fit in the silo? Round to the nearest cubic foot.



Guided Practice: Example 3, *continued*

1. **Find the height of the cylinder by subtracting the radius of the hemisphere (which is also the same as the height of the hemisphere) from the total height.**

   300 – 50 = 250 feet

Guided Practice: Example 3, *continued*

2. Calculate the volume of the cylinder using the formula $V = \pi r^2 h$. Substitute 50 for *r* and 250 for *h*.

$$V = \pi(50)^2 \bullet (250) = 1{,}962{,}500 \text{ ft}^3$$

Guided Practice: Example 3, *continued*

3. **Calculate the volume of the hemisphere, which is half the volume of a sphere.**

$V=\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Formula for volume of a hemisphere
$V=\frac{2}{3}\pi r^3$	Simplify.
$V=\frac{2}{3}\pi(50)^3$	Substitute 50 for *r*.
$V=261{,}667\text{ ft}^3$	

Guided Practice: Example 3, *continued*

4. **Add the volume of the cylinder to the volume of the hemisphere to find the total volume of the grain silo.**

$$1{,}962{,}500 + 261{,}667 = 2{,}224{,}167$$

The total volume of the grain silo is 2,224,167 ft^3.

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Guided Practice: Example 3, *continued*