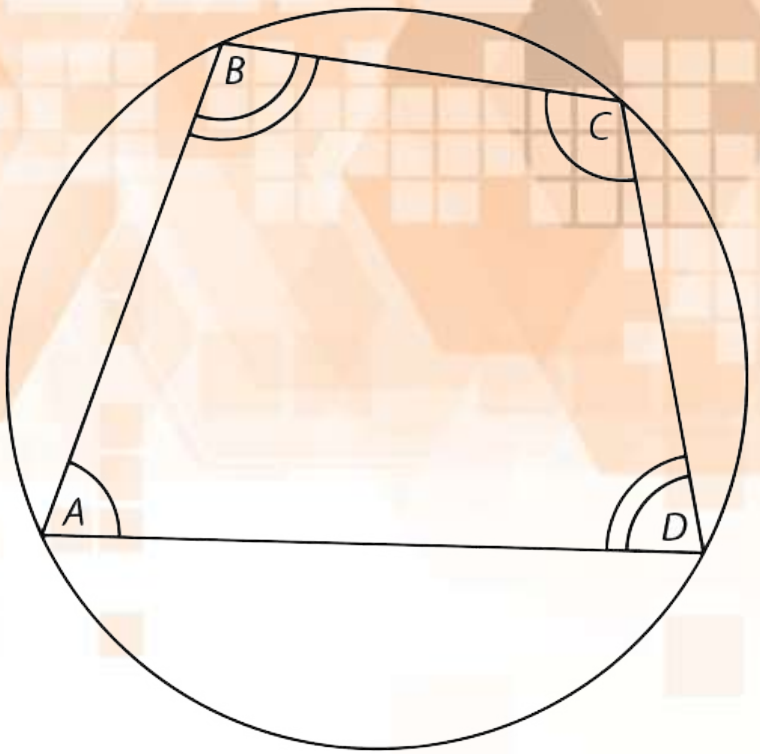
Introduction

One of the most famous drawings of all time is Leonardo da Vinci's Vitruvian Man. Da Vinci's sketch was of a man enclosed by a circle that touched the man's feet and hands. In this lesson, we will investigate the properties of quadrilaterals inscribed in a circle.

Key Concepts

- An **inscribed quadrilateral** is a quadrilateral whose vertices are on a circle.
- The opposite angles of an inscribed quadrilateral are supplementary.
  - $m\angle A + m\angle C = 180$
  - $m\angle B + m\angle D = 180$



Key Concepts

- Remember that the measure of an inscribed angle is half the measure of the intercepted arc.
- Rectangles and squares can always be inscribed within a circle.

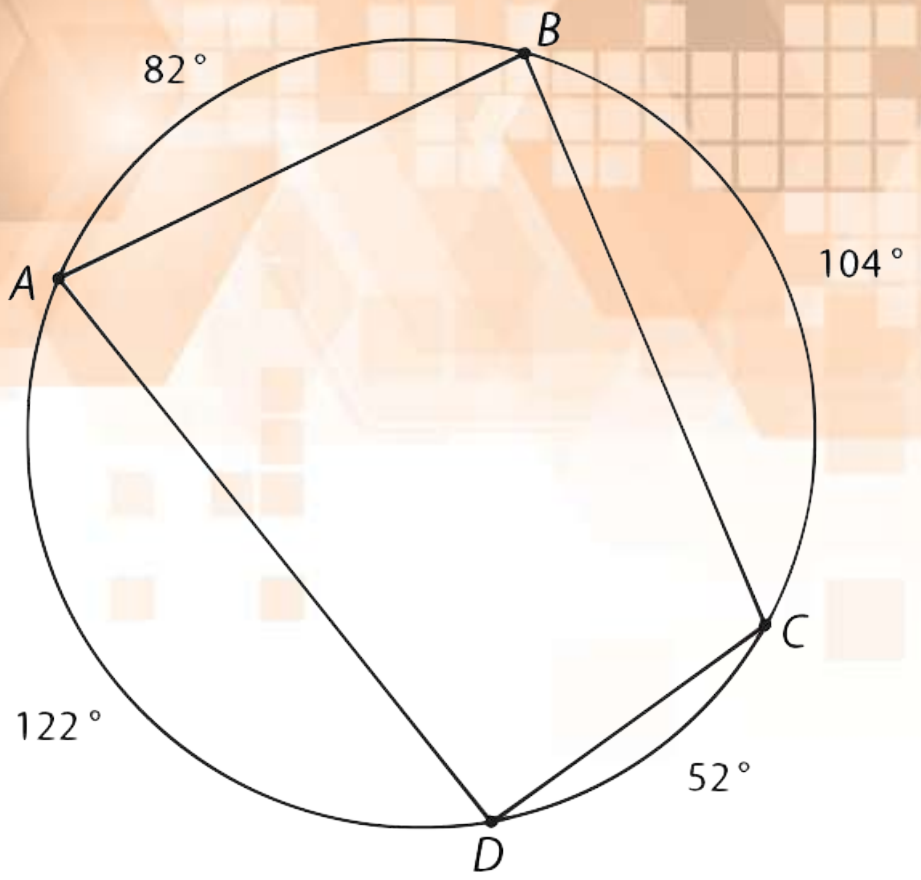
Common Errors/Misconceptions

- mistakenly thinking that the diagonal of any inscribed quadrilateral is a diameter of the circle

Guided Practice

Example 1

Consider the inscribed quadrilateral in the diagram at right. What are the relationships between the measures of the angles of an inscribed quadrilateral?



Guided Practice: Example 1, *continued*

1. Find the measure of $\angle B$.

$\angle B$ is an inscribed angle. Therefore, its measure will be equal to half the measure of the intercepted arc.

The intercepted arc $\overset{\frown}{ADC}$ has a measure of 122 + 52, or 174°.

The measure of $\angle B$ is $\frac{1}{2}$ of 174, or 87°.

Guided Practice: Example 1, *continued*

2. Find the measure of $\angle D$.

The intercepted arc $\overset{\frown}{ABC}$ has a measure of 82 + 104, or 186°.

The measure of $\angle D$ is $\frac{1}{2}$ of 186, or 93°.

Guided Practice: Example 1, *continued*

3. What is the relationship between $\angle B$ and $\angle D$?

Since the sum of the measures of $\angle B$ and $\angle D$ equals 180°, $\angle B$ and $\angle D$ are supplementary angles.

Guided Practice: Example 1, *continued*

4. Does this same relationship exist between $\angle A$ and $\angle C$?

The intercepted arc $\overset{\frown}{BCD}$ has a measure of 104 + 52, or 156°.
The measure of $\angle A$ is $\frac{1}{2}$ of 156, or 78°.

The intercepted arc $\overset{\frown}{BAD}$ has a measure of 82 + 122, or 204°.
The measure of $\angle C$ is $\frac{1}{2}$ of 204, or 102°.

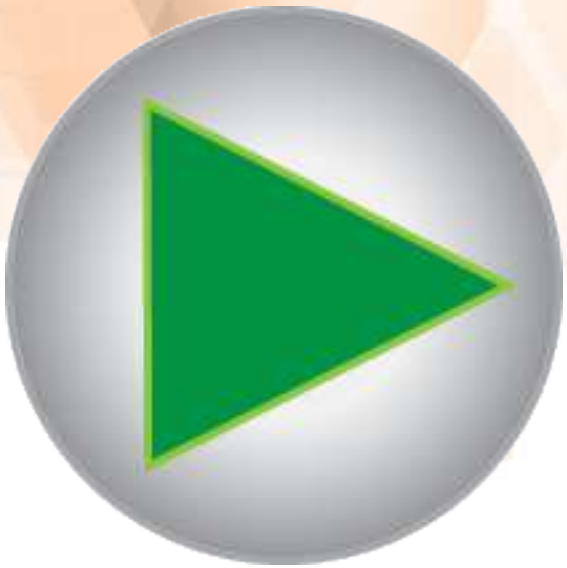
The sum of the measures of $\angle A$ and $\angle C$ also equals 180°; therefore, $\angle A$ and $\angle C$ are supplementary.

Guided Practice: Example 1, *continued*

5. State your conclusion.

The opposite angles of an inscribed quadrilateral are supplementary.

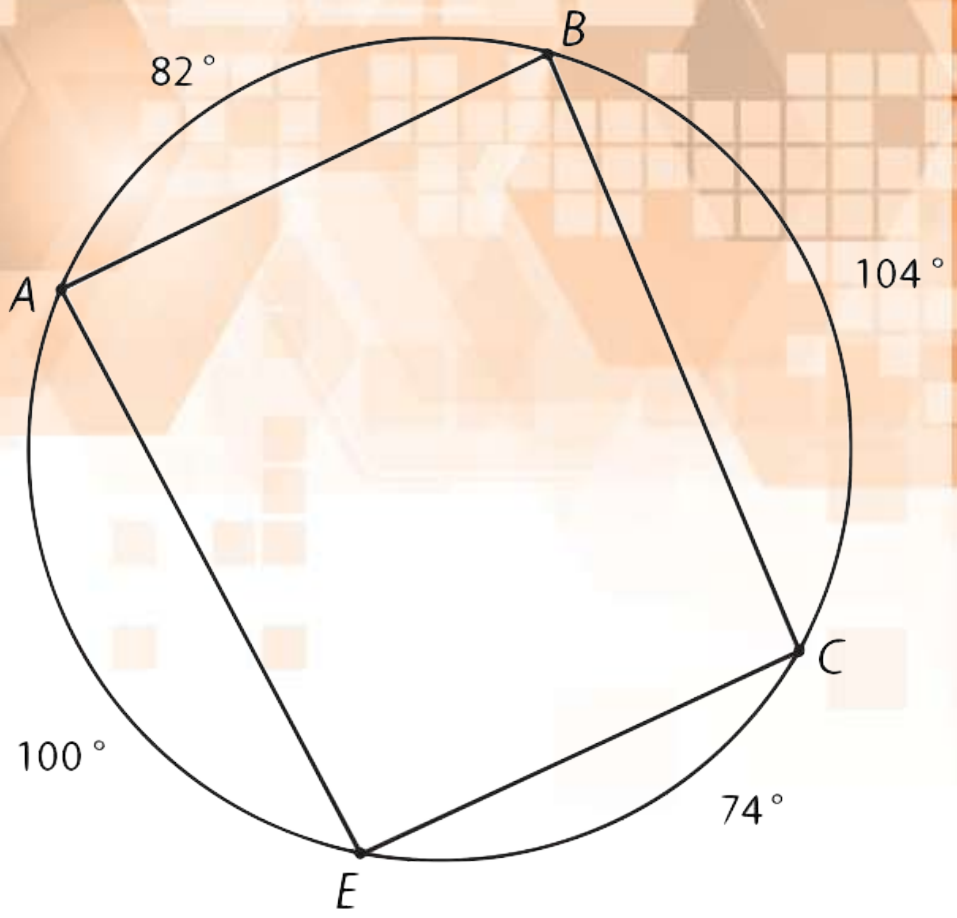
Guided Practice: Example 1, *continued*



Guided Practice

Example 2

Consider the inscribed quadrilateral to the right. Do the relationships discovered between the angles in Example 1 still hold for the angles in this quadrilateral?



Guided Practice: Example 2, *continued*

1. Calculate the measures of all four angles of quadrilateral *ABCE*.

$\angle A$ intercepts $\overset{\frown}{BCE}$ so the measure of $\angle A$ is half the measure of $\overset{\frown}{BCE}$

$$m\angle A = \frac{1}{2}(104 + 74) = 89$$

Guided Practice: Example 2, *continued*

$\angle B$ intercepts $\overset{\frown}{AEC}$ so the measure of $\angle B$ is half the measure of $\overset{\frown}{AEC}$

$$m\angle B = \frac{1}{2}(100 + 74) = 87$$

$\angle C$ intercepts $\overset{\frown}{BAE}$ so the measure of $\angle C$ is half the measure of $\overset{\frown}{BAE}$

$$m\angle C = \frac{1}{2}(100 + 82) = 91$$

Guided Practice: Example 2, *continued*

$\angle E$ intercepts $\overset{\frown}{ABC}$ so the measure of $\angle E$ is half the measure of $\overset{\frown}{ABC}$.

$$m\angle E = \frac{1}{2}(104 + 82) = 93$$

Guided Practice: Example 2, *continued*

2. Find the sum of the measures of $\angle A$ and $\angle C$.

The sum of the measures of $\angle A$ and $\angle C$ is equal to 89 + 91 = 180.

Guided Practice: Example 2, *continued*

3. State your conclusion.

The measures of $\angle A$ and $\angle C$ sum to 180°, as do the measures of $\angle B$ and $\angle E$; therefore, it is still true that opposite angles of an inscribed quadrilateral are supplementary.

Guided Practice: Example 2, *continued*