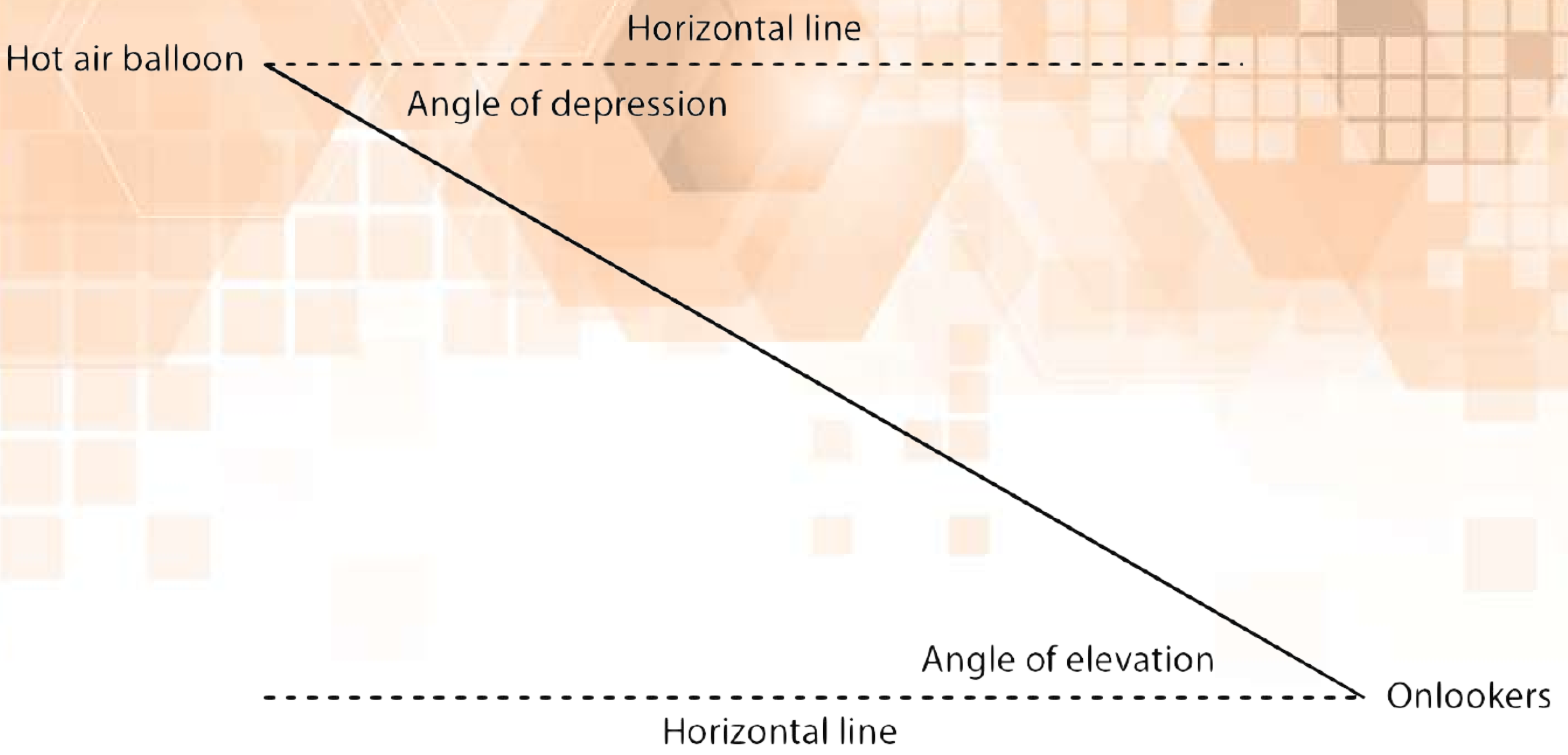
Introduction

Imagine it is a beautiful afternoon in northern Georgia. Your class has raised enough money to take a ride in a hot air balloon that overlooks Lake Lanier, Stone Mountain, Kennesaw Mountain, and the Atlanta skyline, among other sights. As you fly high in the sky, you observe the geography below, and onlookers below marvel at the beauty of the hot air balloon. The angles of observation from both parties, downward toward the sights and upward toward the balloon, have specific names.

Introduction, *continued*

As you look down toward the onlookers, you view the landscape at an **angle of depression**, which is the angle created by a horizontal line and a downward line of sight. The onlookers view the hot air balloon at an **angle of elevation**, which is the angle created by a horizontal line and an upward line of sight. In the following diagram, notice the labeled angles given the horizontals.

Introduction, *continued*



Introduction, *continued*

These two angles are very important to understand and will be the basis for our study and practice during this lesson. Specifically, you will study and practice calculating angles of depression and elevation and use these angles to calculate distances that are not easily measured by common devices.

Introduction, *continued*

It is important to note that, in this example, the horizontal lines are parallel to each other, and therefore the line of sight behaves as a transversal to the parallel lines. As such, the angle of elevation and angle of depression are alternate interior angles and, therefore, their angle measures are congruent.

Key Concepts

- The angle of depression is the angle created by a horizontal line and a downward line of sight to an object below the observer.
- The angle of elevation is the angle created by a horizontal line and an upward line of sight to an object above the observer.

Key Concepts

- Remember:
  - Given the angle θ, $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$.
  - Given the angle θ, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.
  - Given the angle θ, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.
  - Given right triangle *ABC*, $a^2 + b^2 = c^2$, where *a* and *b* are the legs and *c* is the hypotenuse.

Common Errors/Misconceptions

- marking an angle of depression or elevation in reference to the vertical instead of the horizontal
- not recognizing the pair of alternate interior angles formed in angles of elevation and depression

Guided Practice

Example 2

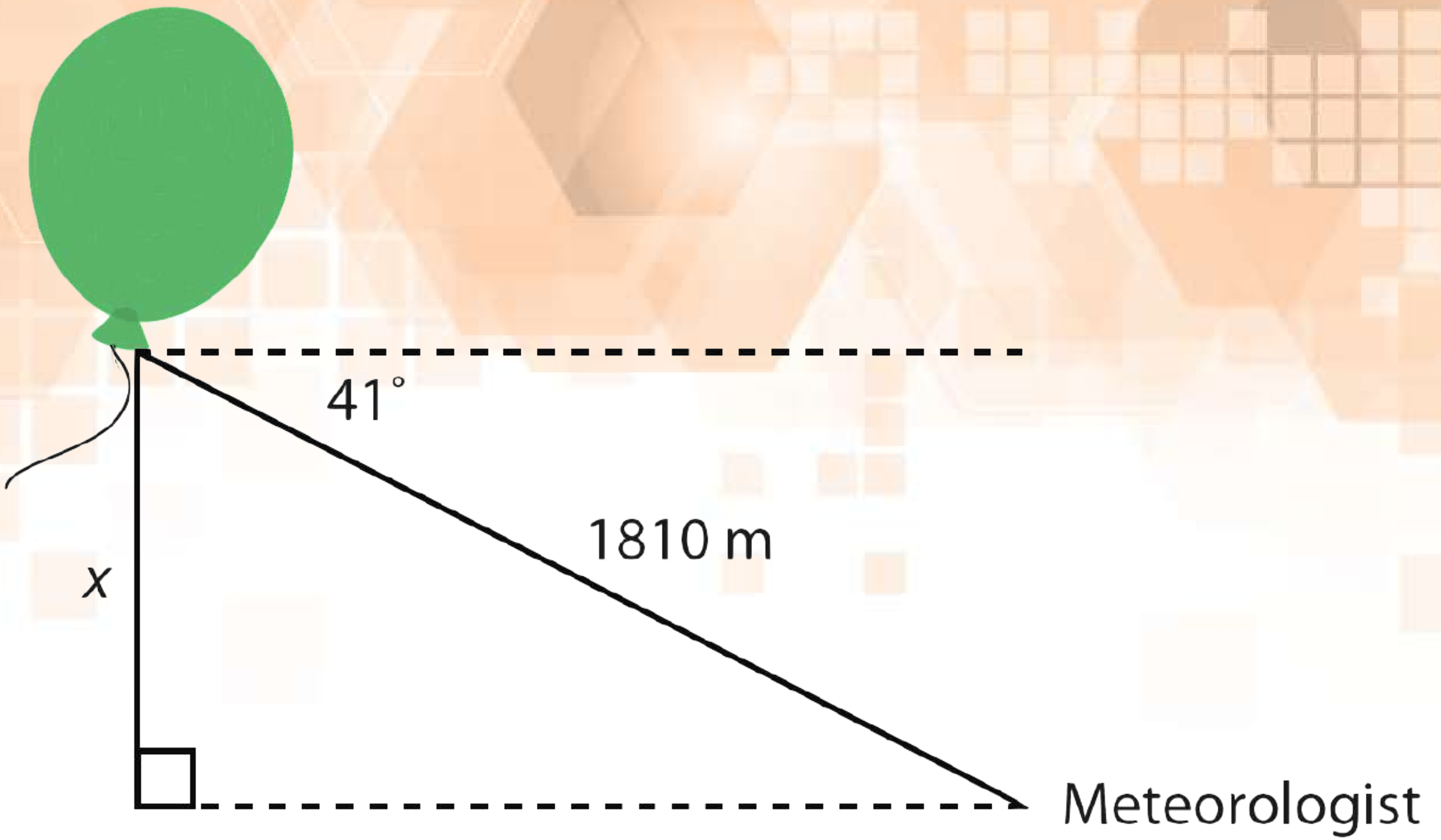
A meteorologist reads radio signals to get information from a weather balloon. The last alert indicated that the angle of depression of the weather balloon to the meteorologist was 41º and the balloon was 1,810 meters away from his location on the diagonal. To the nearest meter, how high above the ground was the balloon?

Guided Practice: Example 2, *continued*

1. Make a drawing of the scenario.

Remember, the angle of depression is above the diagonal line of sight and below the horizontal.

Guided Practice: Example 2, *continued*

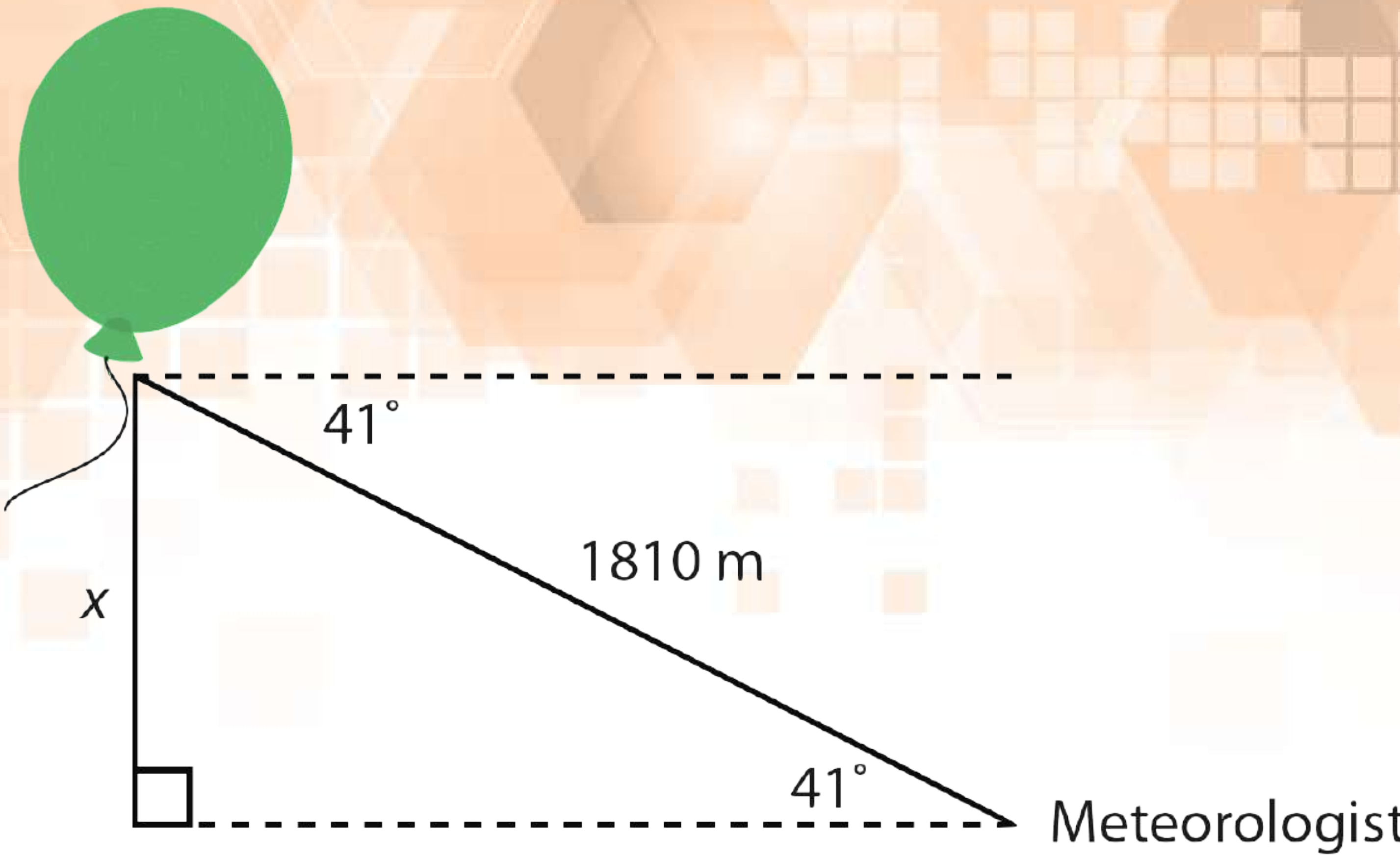


Guided Practice: Example 2, *continued*

2. Identify the correct trigonometric function to use given the angle of depression.

Recall that the angles of depression and elevation are congruent, so we can use this to determine the trigonometric function.

Guided Practice: Example 2, *continued*



Guided Practice: Example 2, *continued*

Since *x* is opposite the angle of elevation (which yields the same information as the angle of depression), and the distance from the weather balloon to the meteorologist is represented by the hypotenuse of the right triangle, use sine to find the vertical height.

Guided Practice: Example 2, *continued*

3. **Set up the function and solve for the height.**

$$\sin 41° = \frac{x}{1810}$$

$$1810 \sin 41° = x$$

Guided Practice: Example 2, *continued*

On a TI-83/84:

Step 1: Press [1810][SIN][41].

Step 2: Press [ENTER].

$x \approx 1187.467$ m

On a TI-Nspire:

Step 1: In the calculate window from the home screen, press [1810][sin][41].

Step 2: Press [enter].

$x \approx 1187.467$ m

Guided Practice: Example 2, *continued*

The weather balloon was, vertically, about 1,188 meters from the meteorologist's location.

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Guided Practice: Example 2, *continued*

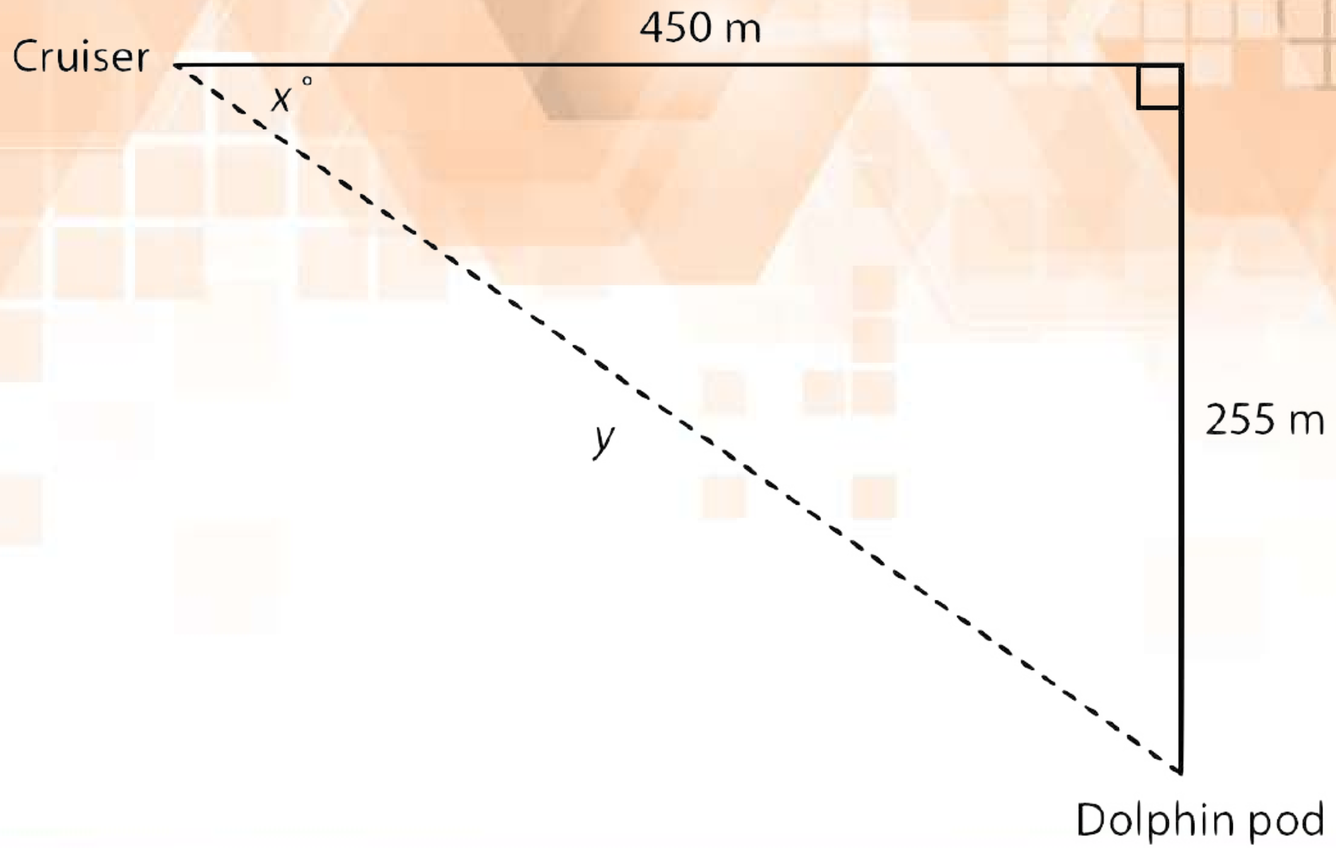
Guided Practice

Example 3

A sonar operator on an anchored cruiser detects a pod of dolphins feeding at a depth of about 255 meters directly below. If the cruiser travels 450 meters west and the dolphins remain at the same depth to feed, what is the angle of depression, *x*, from the cruiser to the pod? What is the distance, *y*, between the cruiser and the pod? Round your answers to the nearest whole number.

Guided Practice: Example 3, *continued*

1. Make a drawing of the scenario.



Guided Practice: Example 3, *continued*

2. **Find the angle of depression. Identify the given information to determine which trigonometric function to use.**

   Since you are calculating an angle, use the inverse of the trigonometric function. Notice that because of the orientation of the triangle and the horizontal side, the angle of depression lies above the diagonal.

Guided Practice: Example 3, *continued*

We are given the distances opposite and adjacent to the angle of depression. Therefore, use the tangent function.

$$\tan x = \frac{255}{450}$$

$$x = \tan^{-1}\left(\frac{255}{450}\right)$$

Guided Practice: Example 3, *continued*

On a TI-83/84:

Step 1: Press [2ND][TAN][250][÷][450].

Step 2: Press [ENTER].

$x \approx 29.539$

On a TI-Nspire:

Step 1: In the calculate window from the home screen, press [tan][250][÷][450].

Step 2: Press [enter].

$x \approx 29.539$

The angle of depression from the cruiser is about 30º.

Guided Practice: Example 3, *continued*

3. Determine the distance, *y*.

Since two side lengths were given, to determine the distance between the cruiser and the pod, there is the option of using either a trigonometric ratio or the Pythagorean Theorem. However, since the value of the angle of depression was not given and had to be approximated, using the Pythagorean Theorem given the two distances will yield a more precise answer.

Guided Practice: Example 3, *continued*

$$450^2 + 255^2 = y^2$$
$$202{,}500 + 65{,}025 = y^2$$
$$267{,}525 = y^2$$
$$\pm\sqrt{267{,}525} = y$$
$$y \approx 517$$

The distance from the cruiser to the dolphin pod after travelling 450 meters west of the vertical is about 517 meters.

Guided Practice: Example 3, *continued*