Introduction

Think of all the different kinds of triangles you can create. What are the similarities among the triangles? What are the differences? Are there properties that hold true for all triangles and others that only hold true for certain types of triangles? This lesson will explore angle relationships of triangles. We will examine the relationships of interior angles of triangles as well as the exterior angles of triangles, and how these relationships can be used to find unknown angle measures.

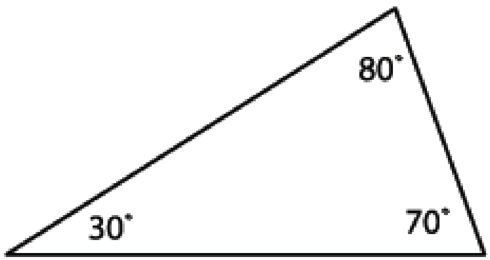
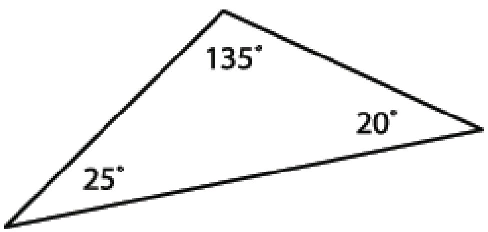
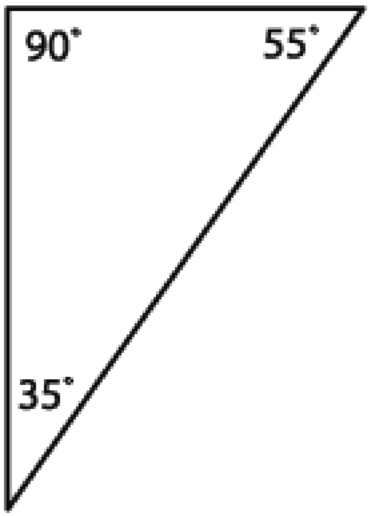
Key Concepts

- There is more to a triangle than just three sides and three angles.
- Triangles can be classified by their angle measures or by their side lengths.
- Triangles classified by their angle measures can be acute, obtuse, or right triangles.

Key Concepts, *continued*

- All of the angles of an **acute triangle** are acute, or less than 90˚.
- One angle of an **obtuse triangle** is obtuse, or greater than 90˚.
- **A right triangle** has one angle that measures 90˚.

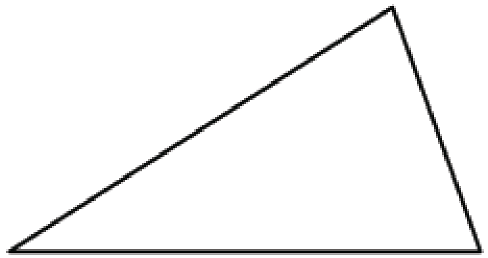
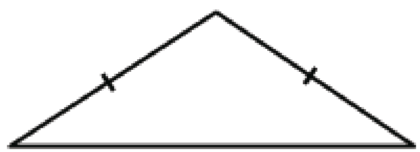
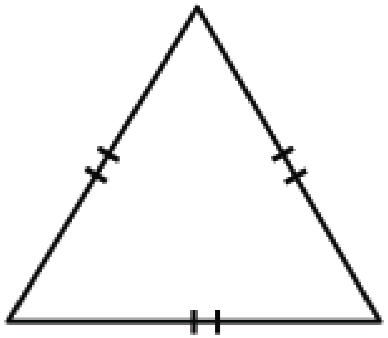
Key Concepts, *continued*

Acute triangle	Obtuse triangle	Right triangle
	 is greater than 90˚.	

Key Concepts, *continued*

- Triangles classified by the number of congruent sides can be scalene, isosceles, or equilateral.
- A **scalene triangle** has no congruent sides.
- An **isosceles triangle** has at least two congruent sides.
- An equilateral triangle has three congruent sides.

Key Concepts, *continued*

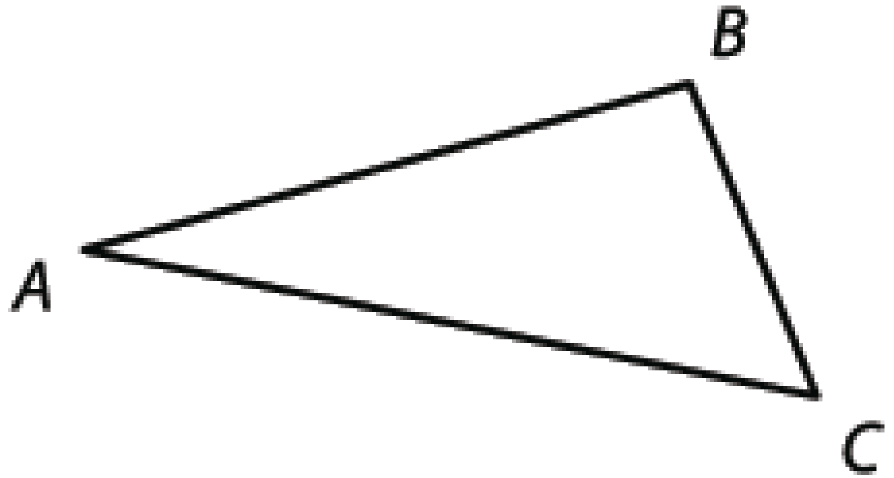
Scalene triangle	Isosceles triangle	Equilateral triangle
	 congruent sides	

Key Concepts, *continued*

- It is possible to create many different triangles, but the sum of the angle measures of every triangle is 180˚. This is known as the Triangle Sum Theorem.

Key Concepts, *continued*

Theorem
Triangle Sum Theorem The sum of the angle measures of a triangle is 180˚.



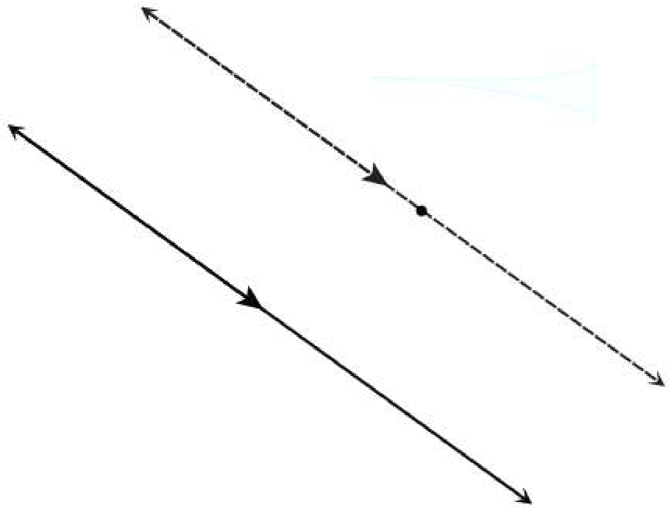
$m\angle A + m\angle B + m\angle C = 180$

Key Concepts, *continued*

- The Triangle Sum Theorem can be proven using the Parallel Postulate.
- The Parallel Postulate states that if a line can be created through a point not on a given line, then that line will be parallel to the given line.
- This postulate allows us to create a line parallel to one side of a triangle to prove angle relationships.

Key Concepts, *continued*

Postulate
Parallel Postulate Given a line and a point not on it, there exists one and only one straight line that passes through that point and never intersects the first line.



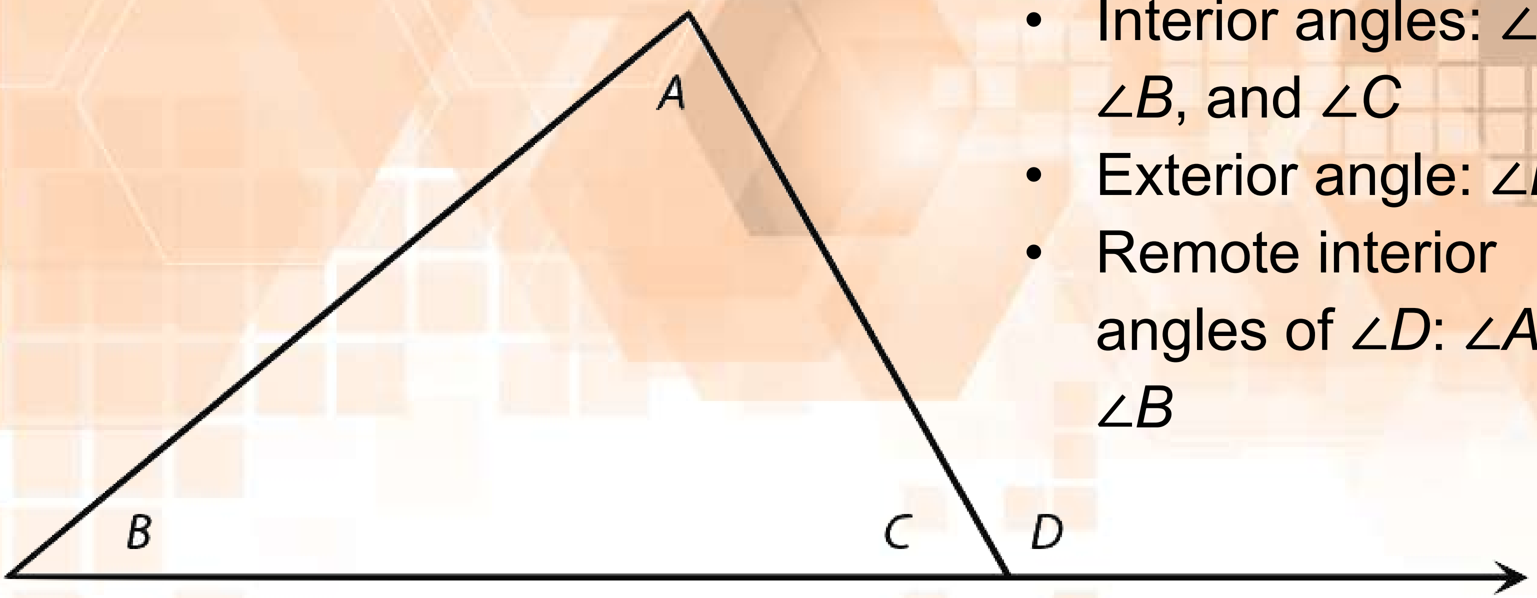
Key Concepts, *continued*

- This theorem can be used to determine a missing angle measure by subtracting the known measures from 180˚.
- Most often, triangles are described by what is known as the **interior angles** of triangles (the angles formed by two sides of the triangle), but exterior angles also exist.

Key Concepts, *continued*

- In other words, interior angles are the angles inside the triangle.
- **Exterior angles** are angles formed by one side of the triangle and the extension of another side.
- The interior angles that are not adjacent to the exterior angle are called the **remote interior angles** of the exterior angle.
- The illustration on the next slide shows the differences among interior angles, exterior angles, and remote interior angles.

Key Concepts, *continued*



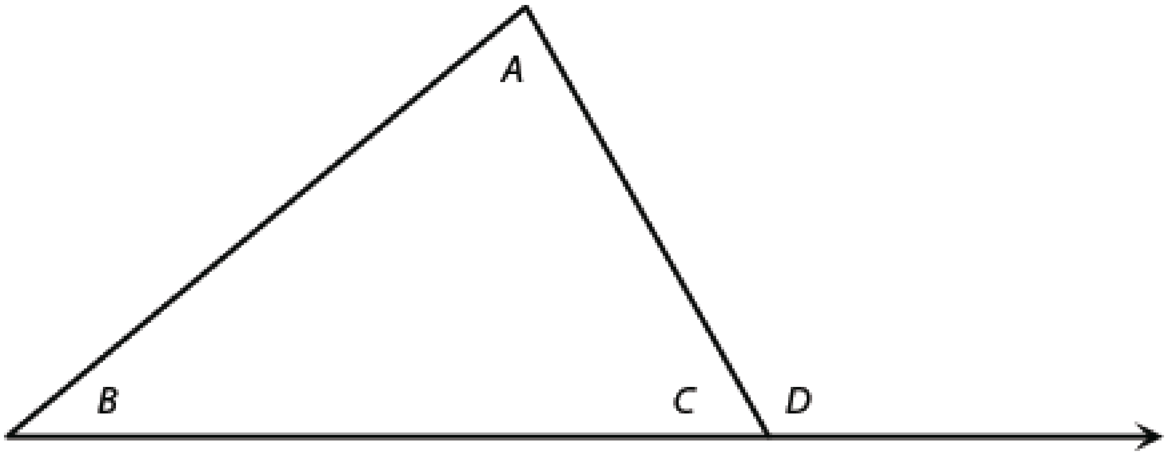
- Interior angles: ∠*A*, ∠*B*, and ∠*C*
- Exterior angle: ∠*D*
- Remote interior angles of ∠*D*: ∠*A* and ∠*B*

- Notice that ∠*C* and ∠*D* are supplementary; that is, together they create a line and sum to 180˚.

Key Concepts, *continued*

- The measure of an exterior angle is equal to the sum of the measure of its remote interior angles. This is known as the Exterior Angle Theorem.

Key Concepts, *continued*

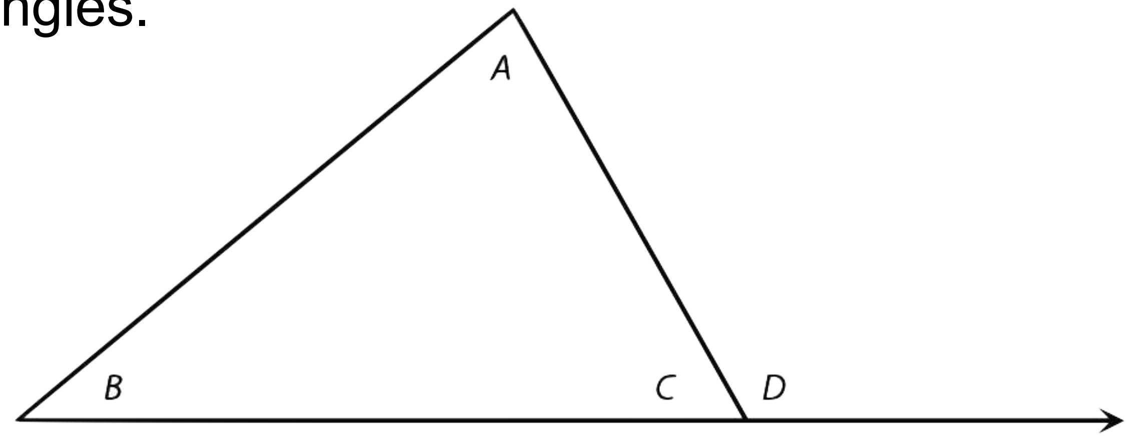
Theorem
Exterior Angle Theorem The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.  $m\angle D = m\angle A + m\angle B$

Key Concepts, *continued*

- This theorem can also be used to determine a missing angle measure of a triangle.
- The measure of an exterior angle will always be greater than either of the remote interior angles. This is known as the Exterior Angle Inequality Theorem.

Key Concepts, *continued*

Theorem
Exterior Angle Inequality Theorem If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.



$m\angle D > m\angle A$

$m\angle D > m\angle B$

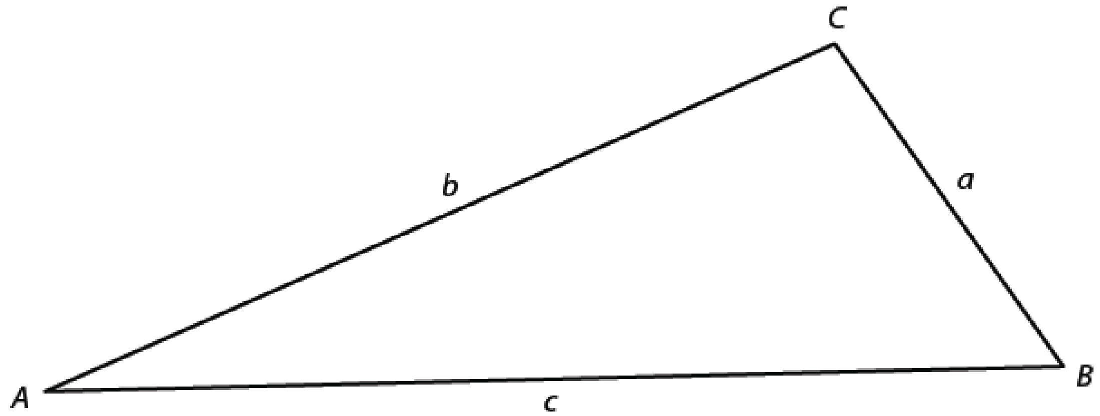
Key Concepts, *continued*

- The following theorems are also helpful when finding the measures of missing angles and side lengths.

Theorem
If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Key Concepts, *continued*

Theorem
If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.



$m\angle A < m\angle B < m\angle C$

$a < b < c$

Key Concepts, *continued*

- The Triangle Sum Theorem and the Exterior Angle Theorem will be proven in this lesson.

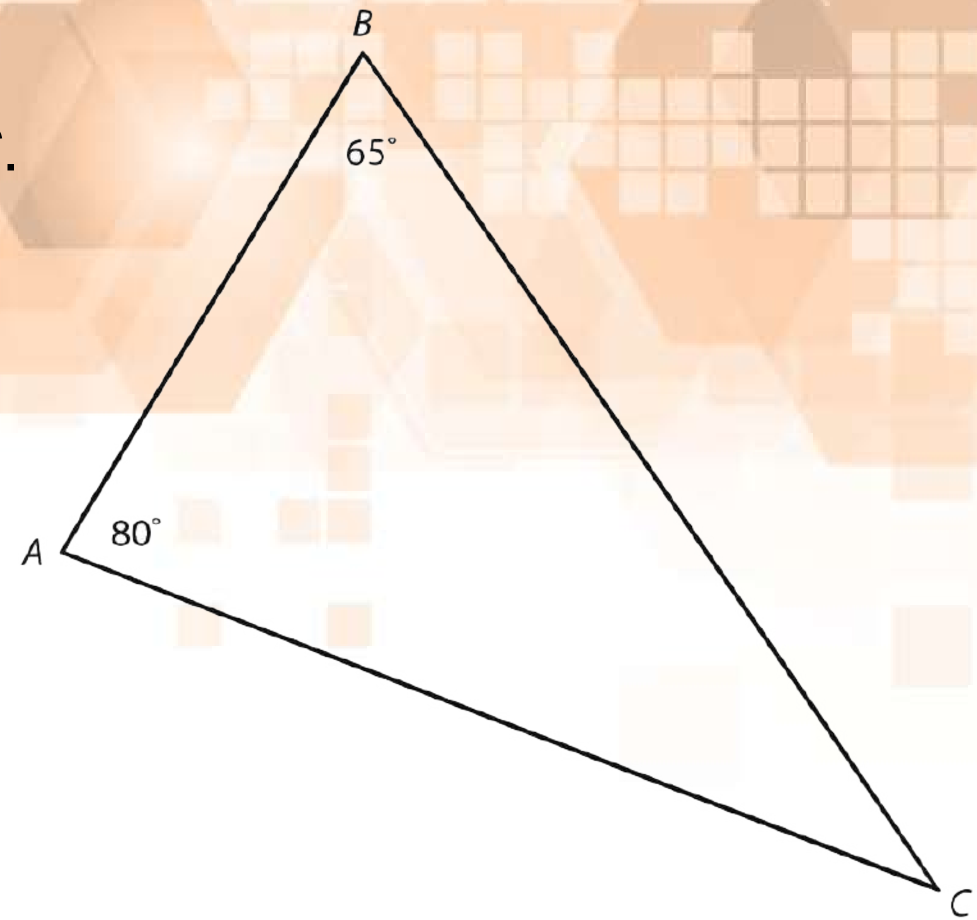
Common Errors/Misconceptions

- incorrectly identifying the remote interior angles associated with an exterior angle
- incorrectly applying theorems to determine missing angle measures
- misidentifying or leaving out theorems, postulates, or definitions when writing proofs

Guided Practice

Example 1

Find the measure of ∠*C*.



Guided Practice: Example 1, *continued*

1. Identify the known information.

Two measures of the three interior angles are given in the problem.

$m\angle A = 80$

$m\angle B = 65$

The measure of $\angle C$ is unknown.

Guided Practice: Example 1, *continued*

2. Calculate the measure of ∠*C*.

The sum of the measures of the interior angles of a triangle is 180˚.

Create an equation to solve for the unknown measure of ∠*C*.

Guided Practice: Example 1, *continued*

$m\angle A + m\angle B + m\angle C = 180$	Triangle Sum Theorem
$80 + 65 + m\angle C = 180$	Substitute values for $m\angle A$ and $m\angle B$.
$145 + m\angle C = 180$	Simplify.
$m\angle C = 35$	Solve for $m\angle C$.

Guided Practice: Example 1, *continued*

3. State the answer.

The measure of $\angle C$ is 35˚.

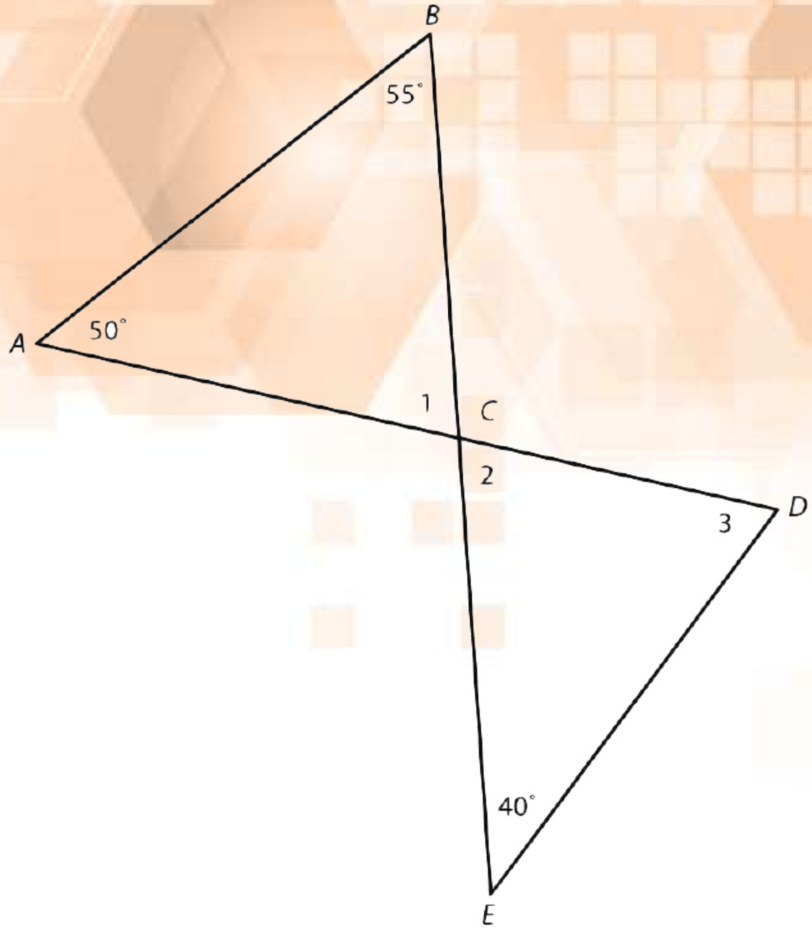
✔

Guided Practice: Example 1, *continued*

Guided Practice

Example 2

Find the missing angle measures.



Guided Practice: Example 2, *continued*

1. Identify the known information.

The figure contains two triangles, $\triangle ABC$ and $\triangle CDE$.

The measures of two of the three interior angles of $\triangle ABC$ are given in the problem.

$m\angle A = 50$

$m\angle B = 55$

The measure of $\angle BCA$ is unknown.

Guided Practice: Example 2, *continued*

The measure of one of the three interior angles of is given in the problem.

$m\angle E = 40$

The measures of $\angle DCE$ and $\angle D$ are unknown.

Guided Practice: Example 2, *continued*

2. Calculate the unknown measures.

The sum of the measures of the interior angles of a triangle is 180˚.

Create an equation to solve for the unknown measure of $\angle BCA$.

Guided Practice: Example 2, *continued*

$m\angle A + m\angle B + m\angle BCA = 180$	Triangle Sum Theorem
$50 + 55 + m\angle BCA = 180$	Substitute values for $m\angle A$ and $m\angle B$.
$105 + m\angle BCA = 180$	Simplify.
$m\angle BCA = 75$	Solve for $m\angle BCA$.

$\angle BCA$ and $\angle DCE$ are vertical angles and are congruent.

$$m\angle DCE = m\angle BCA = 75$$

Guided Practice: Example 2, *continued*

- Create an equation to solve for the unknown measure of $\angle D$.

Guided Practice: Example 2, *continued*

$m\angle DCE + m\angle D + m\angle E = 180$	Triangle Sum Theorem
$75 + m\angle D + 40 = 180$	Substitute values for $m\angle DCE$ and $m\angle E$.
$115 + m\angle D = 180$	Simplify.
$m\angle D = 65$	Solve for $m\angle D$.

Guided Practice: Example 2, *continued*

3. State the answer.

The measure of $\angle BCA$ is 75˚.

The measure of $\angle DCE$ is 75˚.

The measure of $\angle D$ is 65˚.

Guided Practice: Example 2, *continued*