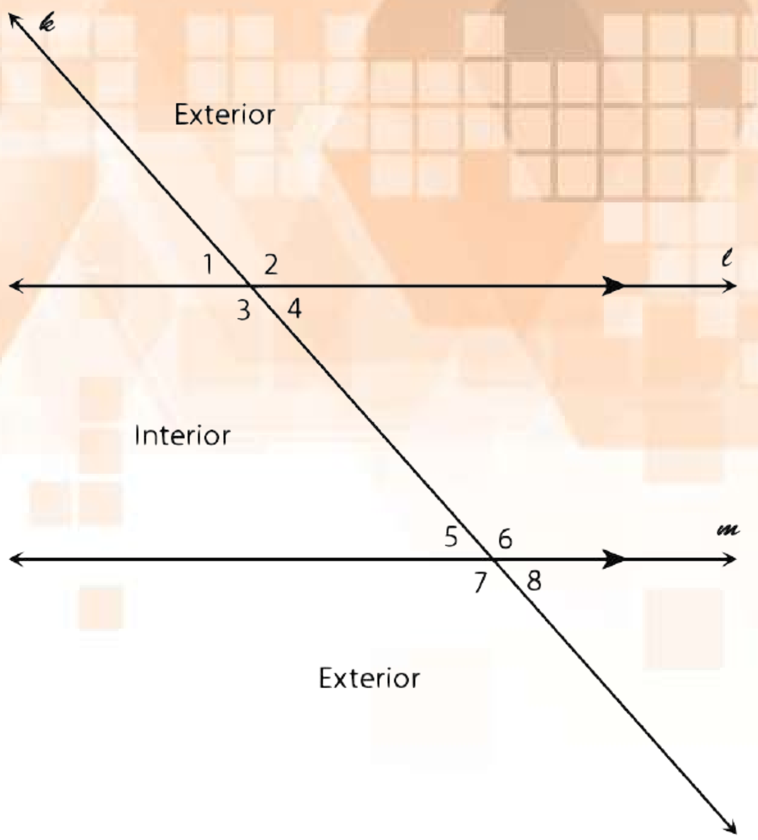
Introduction

Think about all the angles formed by parallel lines intersected by a transversal. What are the relationships among those angles? In this lesson, we will prove those angle relationships. First, look at the diagram on the next slide of a pair of parallel lines and notice the interior angles versus the exterior angles.

Introduction, *continued*

The **interior angles** lie between the parallel lines and the **exterior angles** lie outside the pair of parallel lines. In the following diagram, line k is the transversal. A **transversal** is a line that intersects a system of two or more lines. Lines l and m are parallel. The exterior angles are ∠1, ∠2, ∠7, and ∠8. The interior angles are ∠3, ∠4, ∠5, and ∠6.

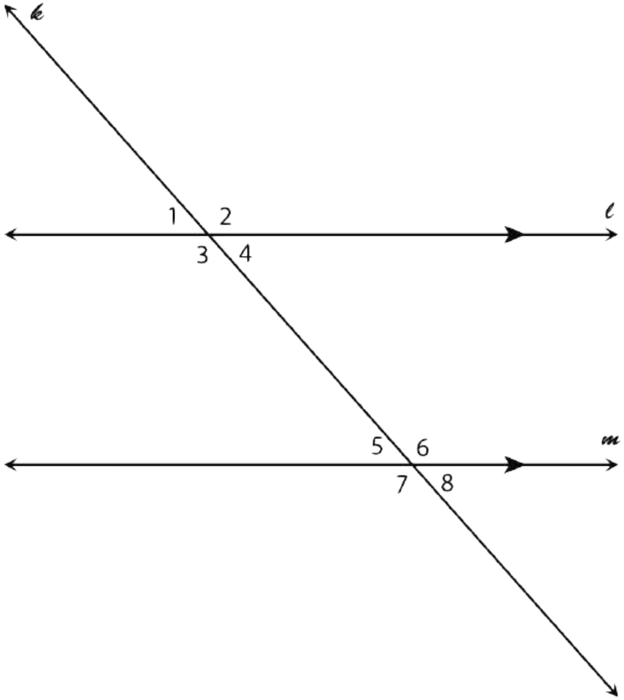


Key Concepts

- A straight line has a constant slope and parallel lines have the same slope.
- If a line crosses a set of parallel lines, then the angles in the same relative position have the same measures.
- Angles in the same relative position with respect to the transversal and the intersecting lines are corresponding angles.
- If the lines that the transversal intersects are parallel, then corresponding angles are congruent.

Key Concepts, *continued*

Postulate
Corresponding Angles Postulate If two parallel lines are cut by a transversal, then corresponding angles are congruent. Corresponding angles: $\angle 1 \cong \angle 5,\ \angle 2 \cong \angle 6,\ \angle 3 \cong \angle 7,\ \angle 4 \cong \angle 8$ The converse is also true. If corresponding angles of lines that are intersected by a transversal are congruent, then the lines are parallel.

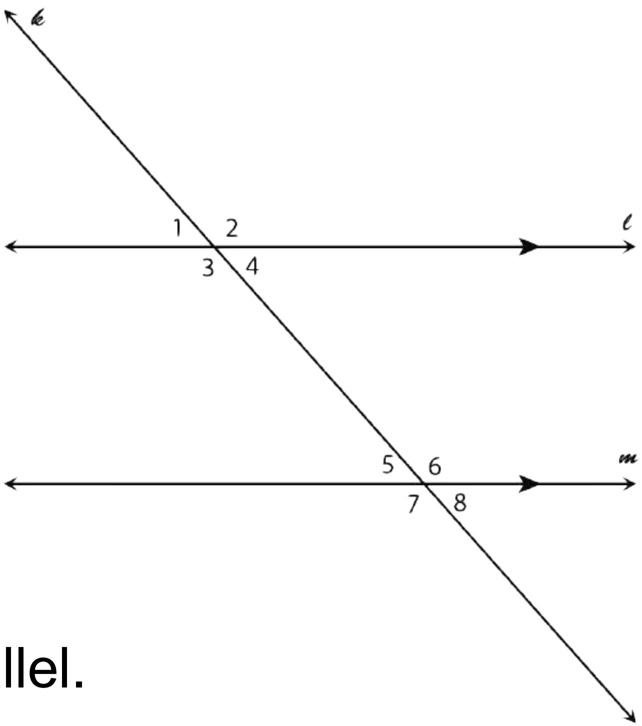


Key Concepts, *continued*

- **Alternate interior angles** are angles that are on opposite sides of the transversal and lie on the interior of the two lines that the transversal intersects.
- If the two lines that the transversal intersects are parallel, then alternate interior angles are congruent.

Key Concepts, *continued*

Theorem
Alternate Interior Angles Theorem If two parallel lines are intersected by a transversal, then alternate interior angles are congruent. Alternate interior angles: $\angle 3 \cong \angle 6, \ \angle 4 \cong \angle 5$ The converse is also true. If alternate interior angles of lines that are intersected by a transversal are congruent, then the lines are parallel.



Key Concepts, *continued*

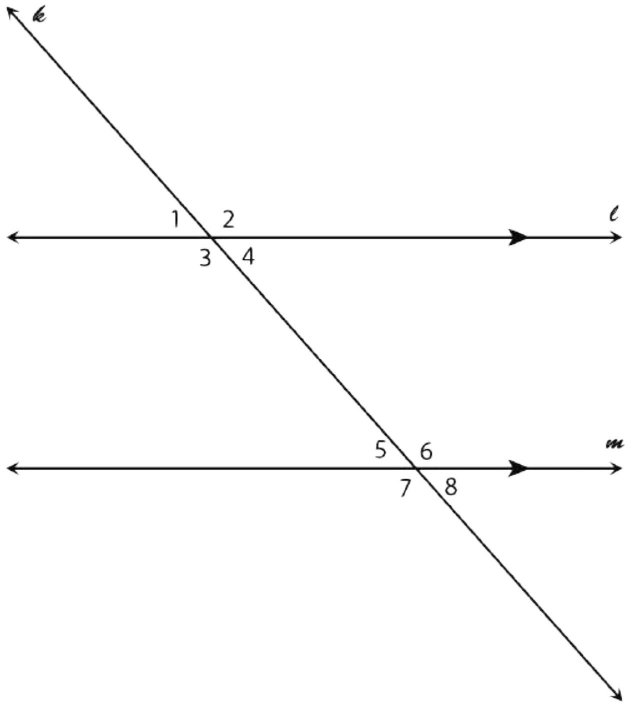
- **Same-side interior angles** are angles that lie on the same side of the transversal and are in between the lines that the transversal intersects.
- If the lines that the transversal intersects are parallel, then same-side interior angles are supplementary.
- Same-side interior angles are sometimes called consecutive interior angles.

Key Concepts, *continued*

Theorem

Same-Side Interior Angles Theorem

If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.



Same-side interior angles:

$m\angle 3 + m\angle 5 = 180$

$m\angle 4 + m\angle 6 = 180$

The converse is also true. If same-side interior angles of lines that are intersected by a transversal are supplementary, then the lines are parallel.

Key Concepts, *continued*

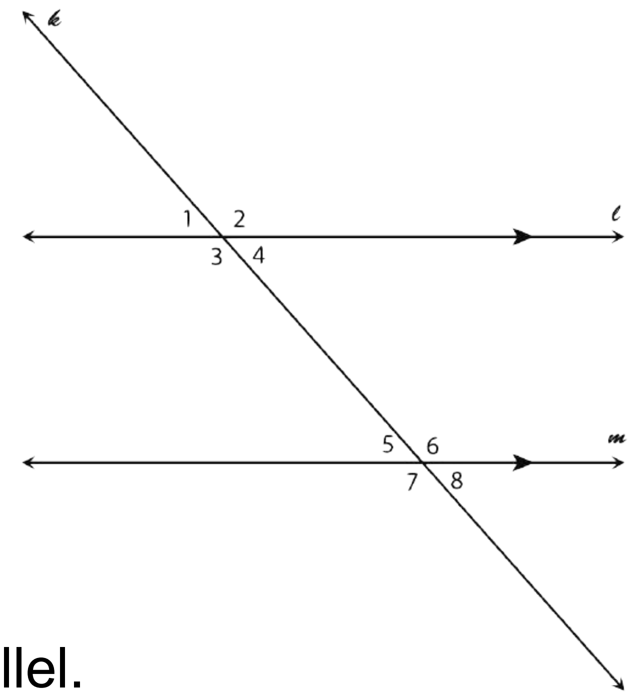
- **Alternate exterior angles** are angles that are on opposite sides of the transversal and lie on the exterior (outside) of the two lines that the transversal intersects.
- If the two lines that the transversal intersects are parallel, then alternate exterior angles are congruent.

Key Concepts, *continued*

Theorem

Alternate Exterior Angles Theorem

If parallel lines are intersected by a transversal, then alternate exterior angles are congruent.



Alternate exterior angles:

$\angle 1 \cong \angle 8,\ \angle 2 \cong \angle 7$

The converse is also true. If alternate exterior angles of lines that are intersected by a transversal are congruent, then the lines are parallel.

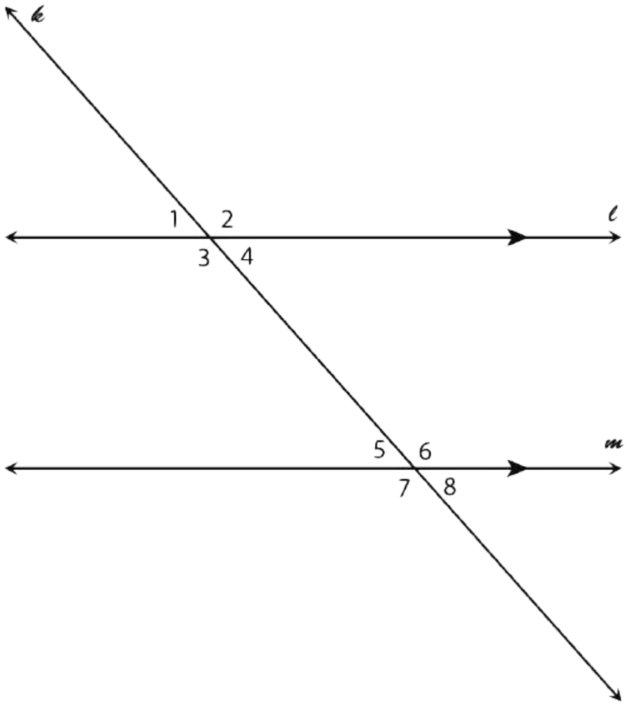
Key Concepts, *continued*

- **Same-side exterior angles** are angles that lie on the same side of the transversal and are outside the lines that the transversal intersects.
- If the lines that the transversal intersects are parallel, then same-side exterior angles are supplementary.
- Same-side exterior angles are sometimes called consecutive exterior angles.

Key Concepts, *continued*

Theorem
Same-Side Exterior Angles Theorem

If two parallel lines are intersected by a transversal, then same-side exterior angles are supplementary.



Same-side exterior angles:

$m\angle 1 + m\angle 7 = 180$

$m\angle 2 + m\angle 8 = 180$

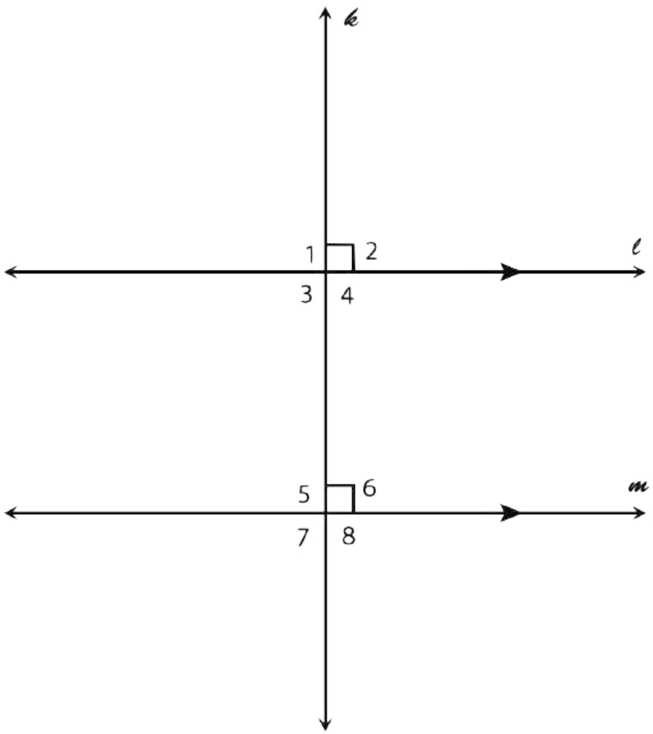
The converse is also true. If same-side exterior angles of lines that are intersected by a transversal are supplementary, then the lines are parallel.

Key Concepts, *continued*

- When the lines that the transversal intersects are parallel and perpendicular to the transversal, then all the interior and exterior angles are congruent right angles.

Key Concepts, *continued*

Theorem
Perpendicular Transversal Theorem If a line is perpendicular to one line that is parallel to another, then the line is perpendicular to the second parallel line. The converse is also true. If a line intersects two lines and is perpendicular to both lines, then the two lines are parallel.



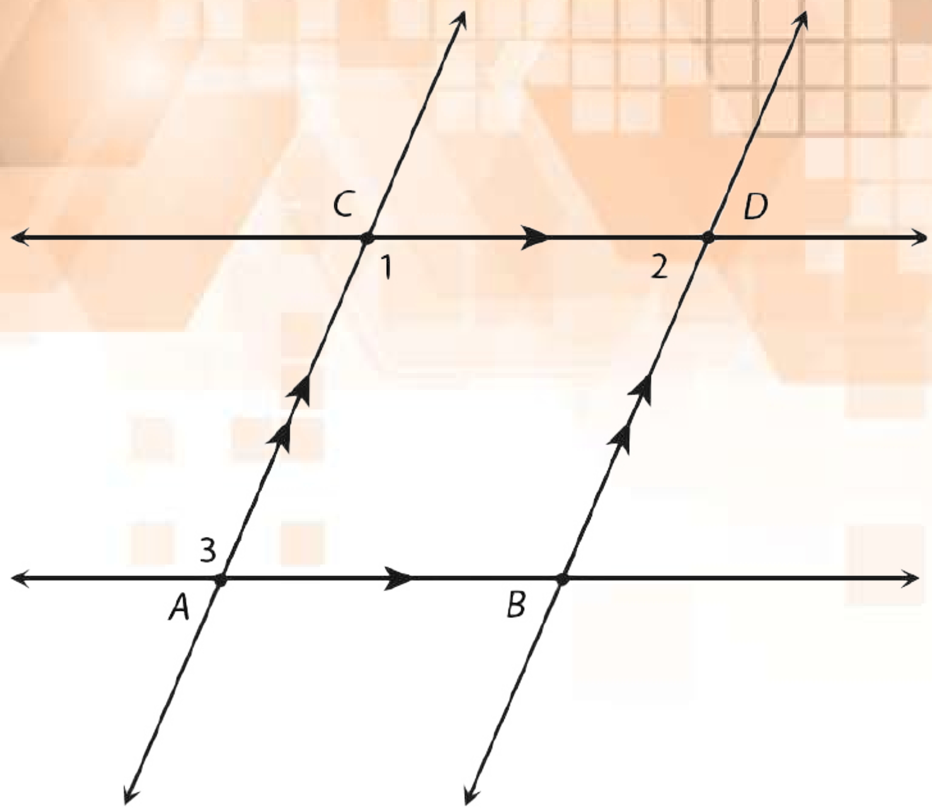
Common Errors/Misconceptions

- setting expressions equal to each other instead of setting up expressions as a supplemental relationship and vice versa
- not being able to recognize the relative positions of the angles in a set of parallel lines intersected by a transversal
- misidentifying or not being able to identify the theorem or postulate to apply
- leaving out definitions or other steps in proofs
- assuming information not given in a diagram or problem statement that cannot be assumed
- assuming drawings are to scale

Guided Practice

Example 3

In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$. If $m\angle 1 = 3(x + 15)$, $m\angle 2 = 2x + 55$, and $m\angle 3 = 4y + 9$, find the measures of the unknown angles and the values of x and y.



Guided Practice: Example 3, *continued*

1. Find the relationship between two angles that have the same variable.

∠1 and ∠2 are same-side interior angles and are both expressed in terms of *x*.

Guided Practice: Example 3, *continued*

2. Use the Same-Side Interior Angles Theorem.

Same-side interior angles are supplementary. Therefore, $m\angle 1 + m\angle 2 = 180$.

Guided Practice: Example 3, *continued*

3. Use substitution and solve for x.

$m\angle 1 = 3(x + 15)$ and $m\angle 2 = 2x + 55$	Given
$m\angle 1 + m\angle 2 = 180$	Same-Side Interior Angles Theorem
$[3(x + 15)] + (2x + 55) = 180$	Substitute $3(x + 15)$ for $m\angle 1$ and $2x + 55$ for $m\angle 2$.
$(3x + 45) + (2x + 55) = 180$	Distribute.
$5x + 100 = 180$	Combine like terms.
$5x = 80$	Subtract 100 from both sides of the equation.
$x = 16$	Divide both sides by 5.

Guided Practice: Example 3, *continued*

4. Find $m\angle 1$ and $m\angle 2$ using substitution.

$m\angle 1 = 3(x + 15)$; $x = 16$

$m\angle 1 = [3(16) + 15)]$

$m\angle 1 = 3(31)$

$m\angle 1 = 93$

$m\angle 2 = 2x + 55$; $x = 16$

$m\angle 2 = 2(16) + 55$

$m\angle 2 = 32 + 55$

$m\angle 2 = 87$

Guided Practice: Example 3, *continued*

After finding $m\angle 1$, to find $m\angle 2$ you could alternately use the Same-Side Interior Angles Theorem, which says that same-side interior angles are supplementary.

$$m\angle 1 + m\angle 2 = 180$$

$$(93) + m\angle 2 = 180$$

$$m\angle 2 = 180 - 93$$

$$m\angle 2 = 87$$

Guided Practice: Example 3, *continued*

5. **Find the relationship between one of the known angles and the last unknown angle, ∠3.**

   ∠1 and ∠3 lie on the opposite side of the transversal on the interior of the parallel lines. This means they are alternate interior angles.

Guided Practice: Example 3, *continued*

6. Use the Alternate Interior Angles Theorem.

The Alternate Interior Angles Theorem states that alternate interior angles are congruent if the transversal intersects a set of parallel lines. Therefore, $\angle 1 \cong \angle 3$.

Guided Practice: Example 3, *continued*

7. **Use the definition of congruence and substitution to find $m\angle 3$.**

$\angle 1 \cong \angle 3$, so $m\angle 1 = m\angle 3$.

$m\angle 1 = 93$

Using substitution, $93 = m\angle 3$.

Guided Practice: Example 3, *continued*

8. Use substitution to solve for *y*.

$m\angle 3 = 4y + 9$	Given
$93 = 4y + 9$	Substitute 93 for $m\angle 3$.
$84 = 4y$	Subtract 9 from both sides of the equation.
$y = 21$	Simplify.

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Guided Practice: Example 3, *continued*

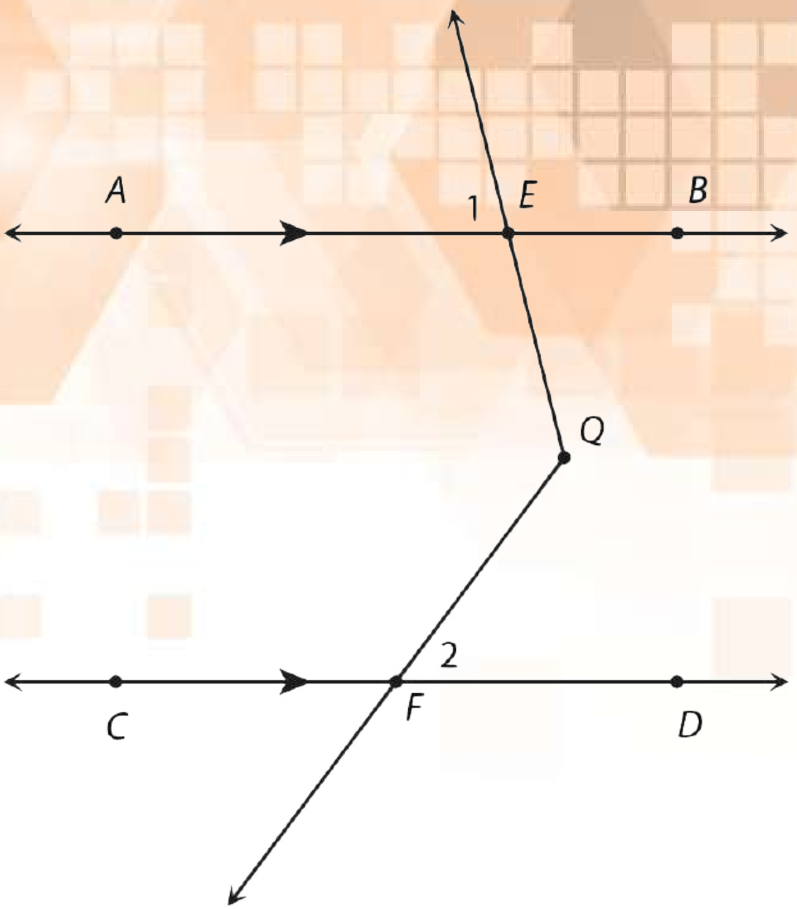
Guided Practice

Example 4

In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

If $m\angle 1 = 35$ and

$m\angle 2 = 65$, find $m\angle EQF$.

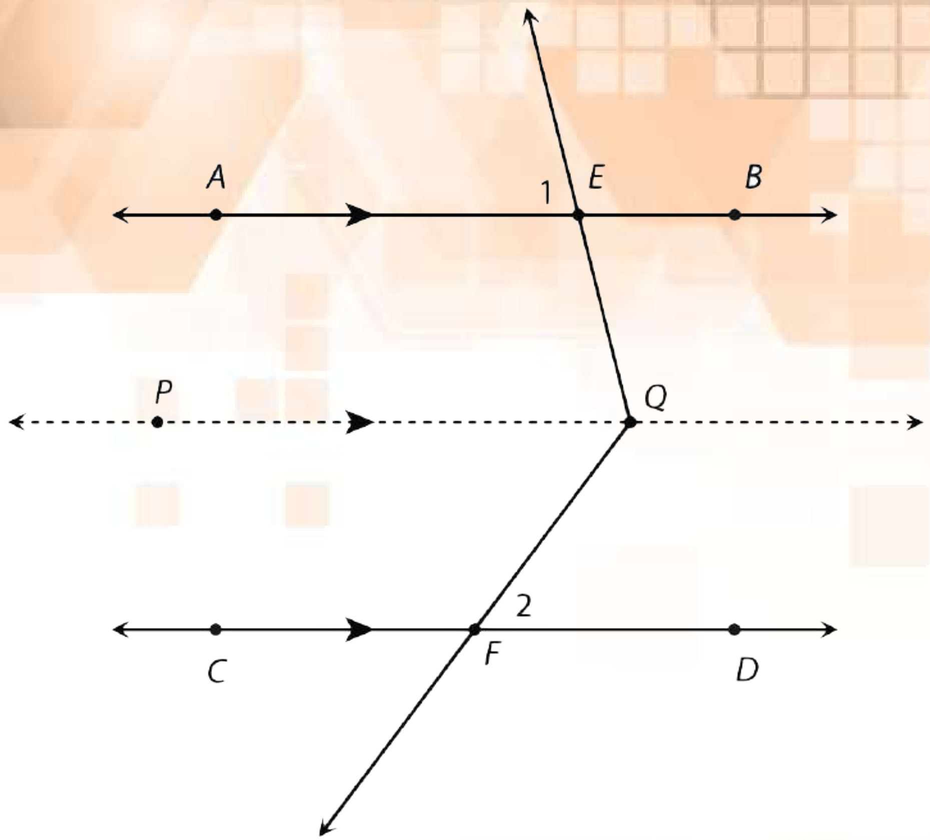


Guided Practice: Example 4, *continued*

1. Draw a third parallel line that passes through point *Q*.

Label a second point on the line as *P*.

$$\overleftrightarrow{PQ} \parallel \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$$



Guided Practice: Example 4, *continued*

2. Use $\overleftrightarrow{QE}$ as a transversal to $\overleftrightarrow{AB}$ and $\overleftrightarrow{PQ}$ and identify angle relationships.

$\angle 1 \cong \angle BEQ$ because they are vertical angles.

$\angle BEQ \cong \angle EQP$ because they are alternate interior angles.

$\angle 1 \cong \angle EQP$ by the Transitive Property.

It was given that $m\angle 1 = 35$.

By substitution, $m\angle EQP = 35$.

Guided Practice: Example 4, *continued*

3. **Use $\overleftrightarrow{QF}$ as a transversal to $\overleftrightarrow{PQ}$ and $\overleftrightarrow{CD}$ and identify angle relationships.**

   $\angle 2 \cong \angle FQP$ because they are alternate interior angles.

   It was given that $m\angle 2 = 65$.

   By substitution, $m\angle FQP = 65$.

Guided Practice: Example 4, *continued*

4. Use angle addition.

Notice that the angle measure we are looking for is made up of two smaller angle measures that we just found.

$$m\angle EQF = m\angle EQP + m\angle FQP$$

$$m\angle EQF = 35 + 65$$

$$m\angle EQF = 100$$

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Guided Practice: Example 4, *continued*