Introduction

Geometry includes many definitions and statements. Once a statement has been shown to be true, it is called a **theorem**. Theorems, like definitions, can be used to show other statements are true. One of the most well known theorems of geometry is the Pythagorean Theorem, which relates the length of the hypotenuse of a right triangle to the lengths of its legs.

Introduction, *continued*

The theorem states that the sum of the squares of the lengths of the legs (a and b) of a right triangle is equal to the square of the length of the hypotenuse (c). This can be written algebraically as $a^2 + b^2 = c^2$. The Pythagorean Theorem has many applications and can be very helpful when solving real-world problems. There are several ways to prove the Pythagorean Theorem; one way is by using similar triangles and similarity statements.

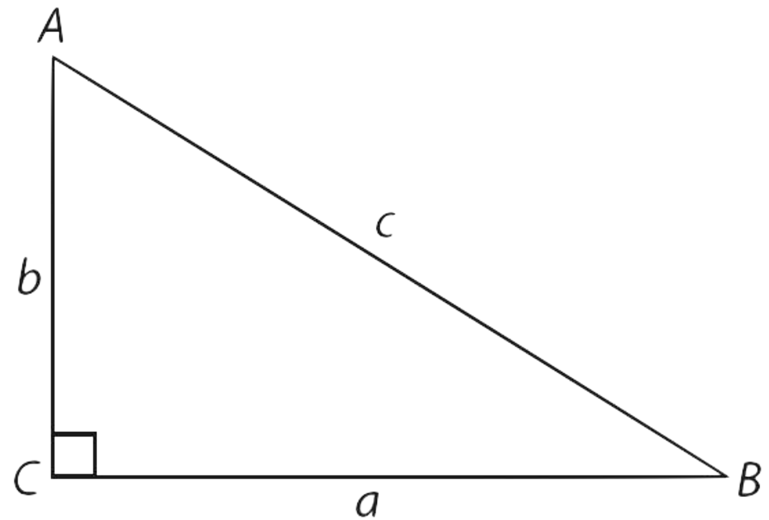
Key Concepts

The Pythagorean Theorem

- The Pythagorean Theorem is often used to find the lengths of the sides of a right triangle, a triangle that includes one 90˚ angle.

Key Concepts, *continued*

Theorem
Pythagorean Theorem The sum of the squares of the lengths of the legs (a and b) of a right triangle is equal to the square of the length of the hypotenuse (c). $a^2 + b^2 = c^2$



Key Concepts, *continued*

- In the triangle on the previous slide, angle C is 90˚, as shown by the square.
- The longest side of the right triangle, c, is called the hypotenuse and is always located across from the right angle.
- The legs of the right triangle, a and b, are the two shorter sides.

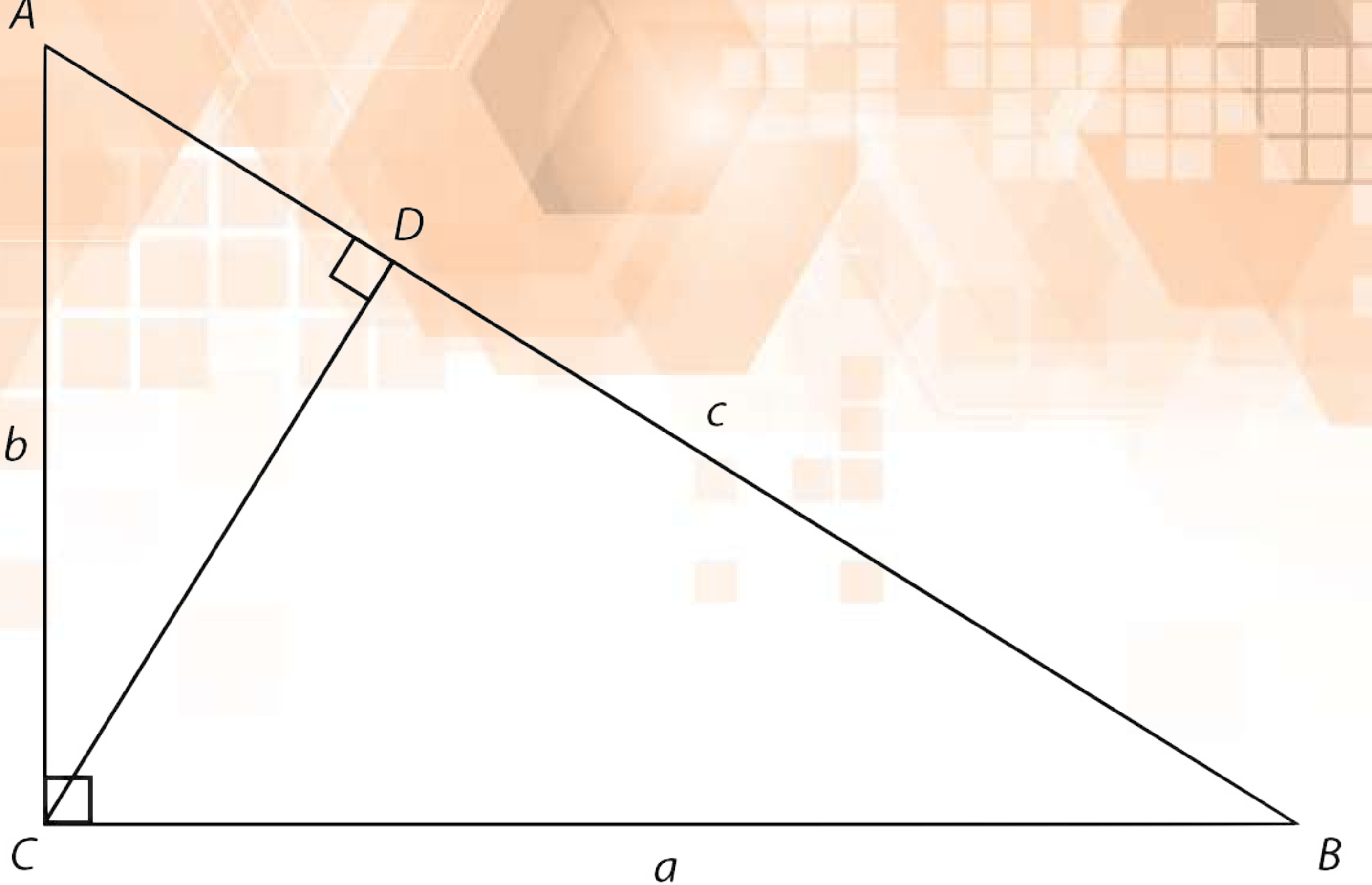
Key Concepts, *continued*

- It is also true that if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.
- This is known as the **converse of the Pythagorean Theorem**.
- To prove the Pythagorean Theorem using similar triangles, you must first identify the similar triangles.

Key Concepts, *continued*

- In this example, there is only one triangle given.
- Begin by drawing the altitude, the segment from angle *C* that is perpendicular to the line containing the opposite side, *c*.
- See the illustration on the following slide.

Key Concepts, *continued*



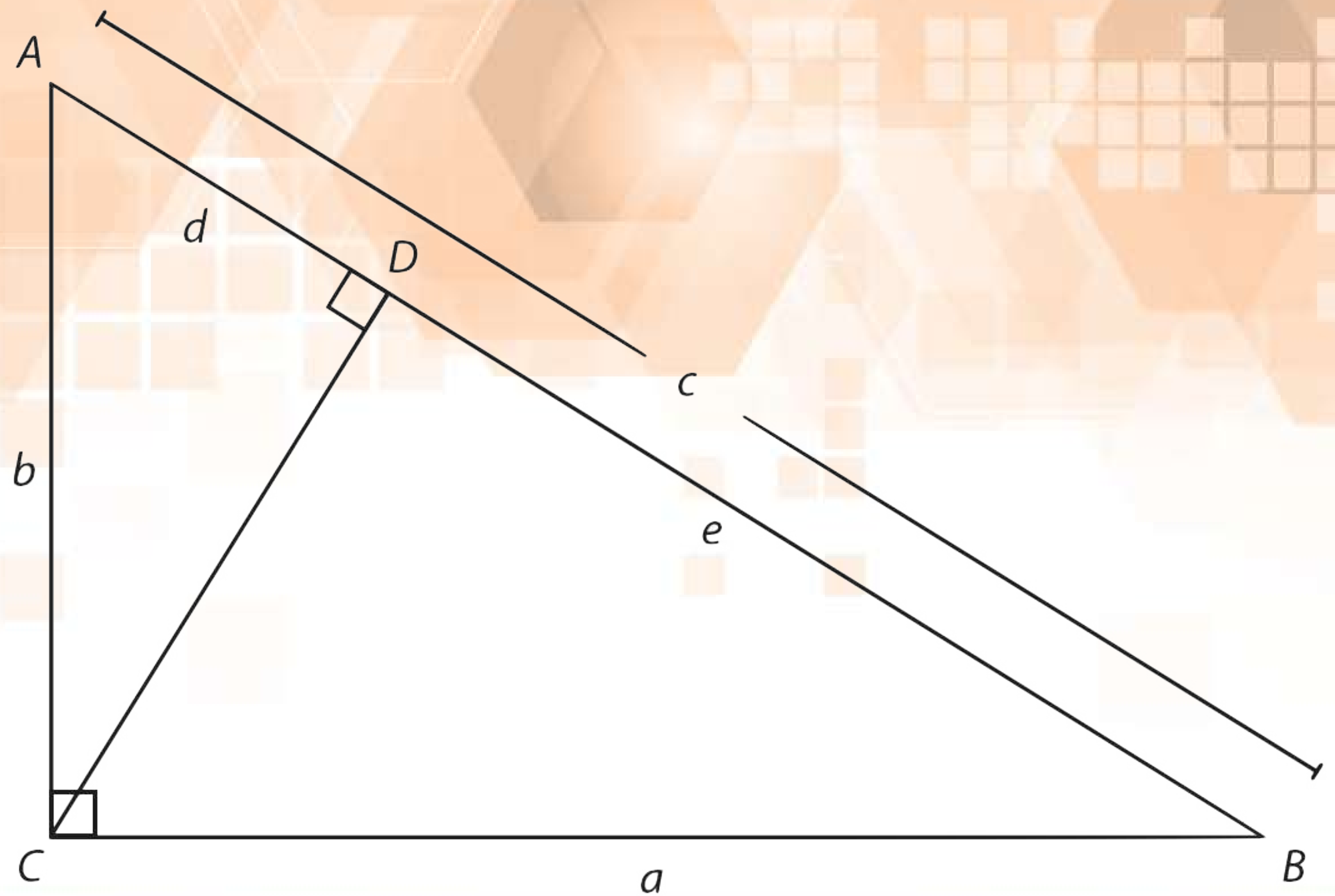
Key Concepts, *continued*

- Notice that by creating the altitude $\overline{CD}$, we have created two smaller right triangles, $\triangle ADC$ and $\triangle BDC$, within the larger given right triangle, $\triangle ACB$.
- $\angle ACB$ and $\angle ADC$ are 90° and are therefore congruent.
- $\angle A$ of $\triangle ADC$ is congruent to $\angle A$ of $\triangle ACB$ because of the Reflexive Property of Congruence.
- According to the Angle-Angle (AA) Similarity Statement, if two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar; therefore, $\triangle ADC \sim \triangle ACB$.

Key Concepts, *continued*

- $\angle ACB$ and $\angle BDC$ are 90° and are therefore congruent.
- $\angle B$ of $\triangle BDC$ is congruent to $\angle B$ of $\triangle ACB$ because of the Reflexive Property of Congruence.
- Two angles in $\triangle BDC$ are congruent to two angles in $\triangle ACB$; therefore, $\triangle BDC \sim \triangle ACB$.
- Similarity is transitive. Since $\triangle ADC \sim \triangle ACB$ and $\triangle BDC \sim \triangle ACB$, then $\triangle ADC \sim \triangle BDC$.

Key Concepts, *continued*



Key Concepts, *continued*

- Corresponding sides of similar triangles are proportional; therefore, $\frac{c}{b}=\frac{b}{d}$ and $\frac{c}{a}=\frac{a}{e}$.
- Determining the cross products of each proportion leads to the Pythagorean Theorem.

Key Concepts, *continued*

$$\frac{c}{b} = \frac{b}{d}$$

$$cd = b^2$$

$$\frac{c}{a} = \frac{a}{e}$$

$$ce = a^2$$

$cd + ce = a^2 + b^2$ Add both equations.

$c(e + d) = a^2 + b^2$ Factor.

$c^2 = a^2 + b^2$ $(e + d)$ is equal to c because of segment addition.

Key Concepts, *continued*

- The converse of the Pythagorean Theorem can be useful when proving right triangles using similar triangles.

Key Concepts, *continued*

Types of Proofs

- Paragraph proofs are statements written out in complete sentences in a logical order to show an argument.
- Flow proofs are a graphical method of presenting the logical steps used to show an argument.
- In a flow proof, the logical statements are written in boxes and the reason for each statement is written below the box.

Key Concepts, *continued*

- Another accepted form of proof is a two-column proof.
- Two-column proofs include numbered statements and corresponding reasons that show the argument in a logical order.
- Two-column proofs appear in the Guided Practice examples that follow.

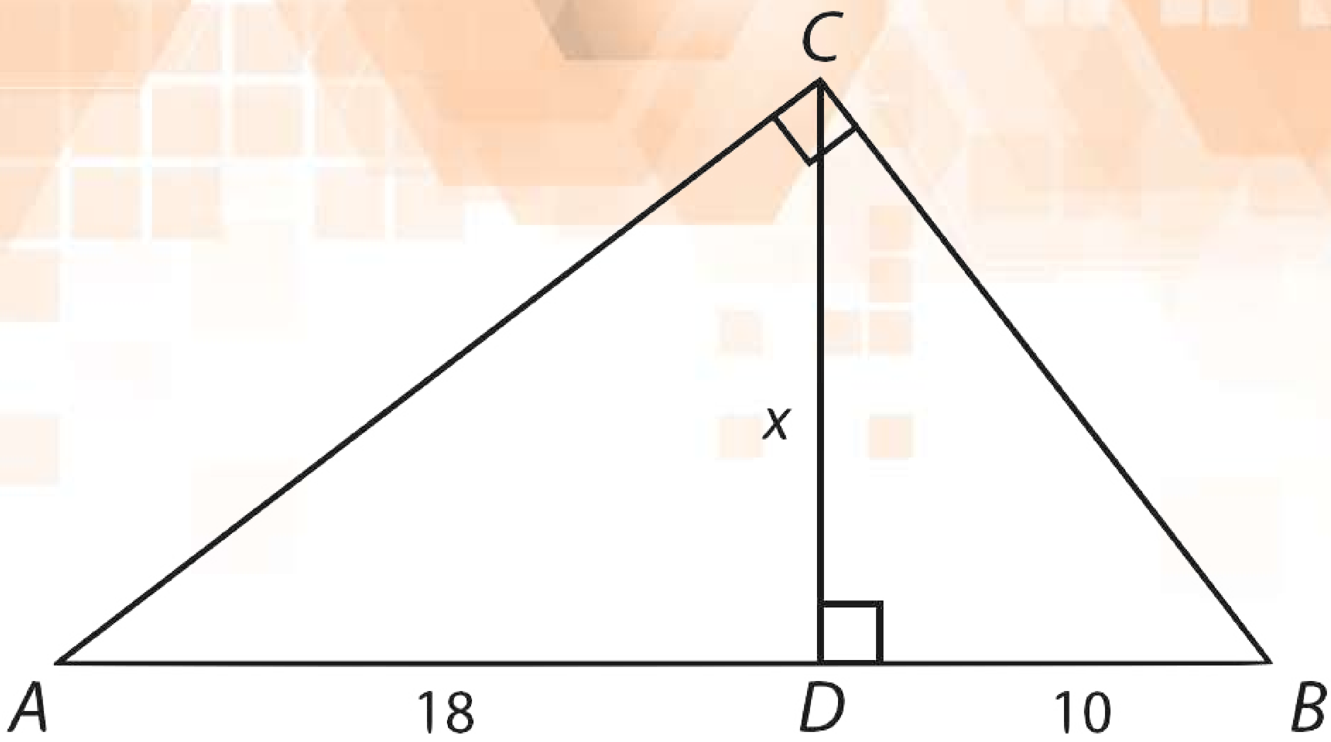
Common Errors/Misconceptions

- misidentifying the altitudes of triangles
- incorrectly simplifying expressions with square roots

Guided Practice

Example 2

Find the length of the altitude, x, of $\triangle ABC$.



Guided Practice: Example 2, *continued*

1. Identify the similar triangles.

$\triangle ABC$ is a right triangle.

The altitude of $\triangle ABC$ is drawn from right $\angle ACB$ to the opposite side, creating two smaller similar triangles.

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

Guided Practice: Example 2, *continued*

2. **Use corresponding sides to write a proportion containing *x*.**

$$\frac{\text{shorter leg of } \triangle ACD}{\text{shorter leg of } \triangle CBD} = \frac{\text{longer leg of } \triangle ACD}{\text{longer leg of } \triangle CBD}$$

$\frac{x}{10} = \frac{18}{x}$	Substitute values for each side.
$(x)(x) = (10)(18)$	Find the cross products.
$x^2 = 180$	Simplify.
$x = 6\sqrt{5} \approx 13.4$	Take the positive square root of each side.

Guided Practice: Example 2, *continued*

3. Summarize your findings.

The length of the altitude, x, of $\triangle ABC$ is $6\sqrt{5}$ units, or approximately 13.4 units.

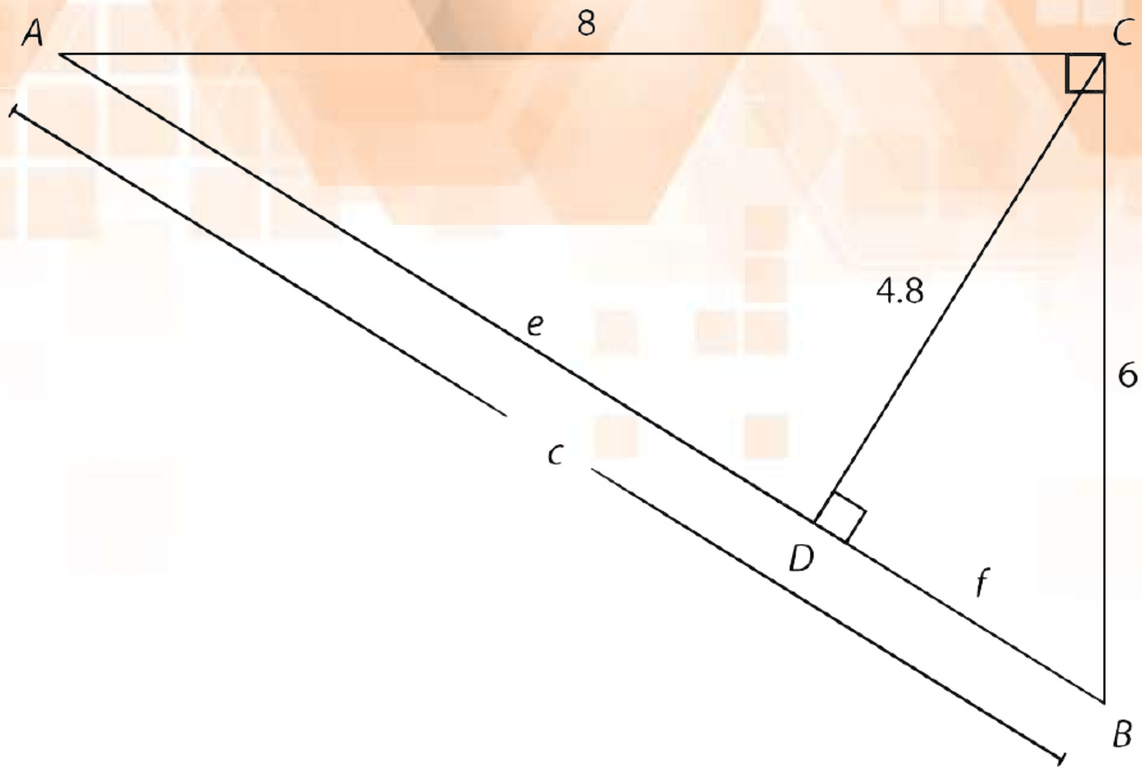
✔

Guided Practice: Example 2, *continued*

Guided Practice

Example 3

Find the unknown values in the figure.



Guided Practice: Example 3, *continued*

1. **Identify the similar triangles.**

$\triangle ABC$ is a right triangle.

The altitude of $\triangle ABC$ is drawn from right $\angle ACB$ to the opposite side, creating two smaller similar triangles.

$$\triangle ABC \sim \triangle ACD \sim \triangle CBD$$

Guided Practice: Example 3, *continued*

2. **Use corresponding sides to write a proportion containing *c*.**

$$\frac{\text{hypotenuse of } \triangle ACD}{\text{hypotenuse of } \triangle ABC} = \frac{\text{shorter leg of } \triangle ACD}{\text{shorter leg of } \triangle ABC}$$

$\frac{8}{c} = \frac{4.8}{6}$	Substitute values for each side.
$(8)(6) = (c)(4.8)$	Find the cross products.
$48 = 4.8c$	Simplify.
$c = 10$	Solve for *c*.

Guided Practice: Example 3, *continued*

3. **Use corresponding sides to write a proportion containing *e*.**

$$\frac{\text{longer leg of } \triangle ACD}{\text{longer leg of } \triangle ABC} = \frac{\text{shorter leg of } \triangle ACD}{\text{shorter leg of } \triangle ABC}$$

$\frac{e}{8} = \frac{4.8}{6}$ Substitute values for each side.

$(e)(6) = (8)(4.8)$ Find the cross products.

$6e = 38.4$ Simplify.

$e = 6.4$ Solve for *e*.

Guided Practice: Example 3, *continued*

4. Use corresponding sides to write a proportion containing *f*.

$$\frac{\text{longer leg of } \triangle CBD}{\text{longer leg of } \triangle ABC} = \frac{\text{shorter leg of } \triangle CBD}{\text{shorter leg of } \triangle ABC}$$

$\dfrac{4.8}{8} = \dfrac{f}{6}$ Substitute values for each side.

$(4.8)(6) = (8)(f)$ Find the cross products.

$28.8 = 8f$ Simplify.

$f = 3.6$ Solve for *f*.

Guided Practice: Example 3, *continued*

5. Summarize your findings.

The length of c is 10 units.

The length of e is 6.4 units.

The length of f is 3.6 units.

Guided Practice: Example 3, *continued*