ILLUSTRATING INTEGERS

UTD The University of Texas at Dallas

INTRODUCTION TO INTEGERS

INTRODUCTION TO INTEGERS

Integers are positive and negative numbers.

…, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, …

Each negative number is paired with a positive number the same distance from 0 on a number line.

INTRODUCTION TO INTEGERS

We can represent integers using red and yellow counters.

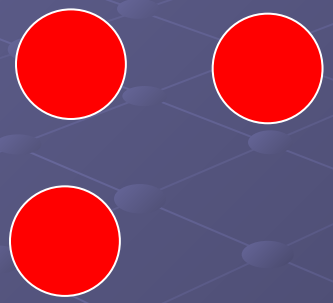
Red tiles will represent negative integers, and yellow tiles will represent positive integers.

Negative integer

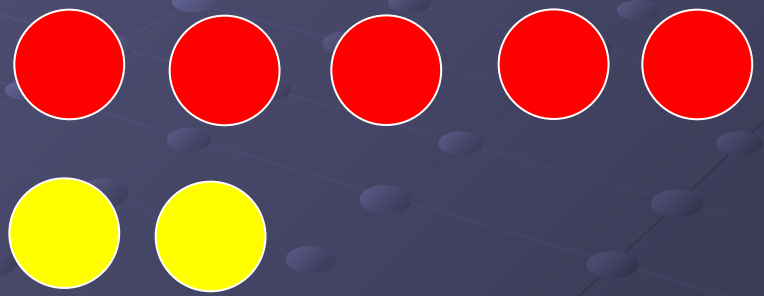
Positive integer

INTRODUCTION TO INTEGERS

The diagrams below show 2 ways to represent -3.



-3



-3

Represent -3 in 2 more ways.

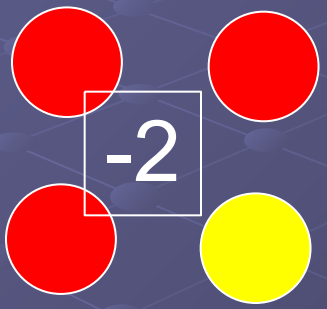
NOTE TO TUTOR

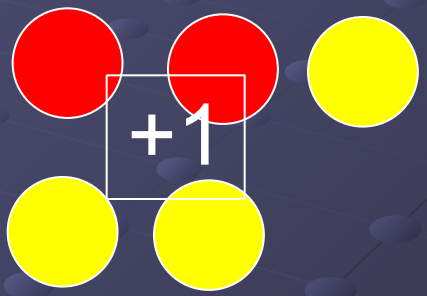
INTRODUCTION TO INTEGERS

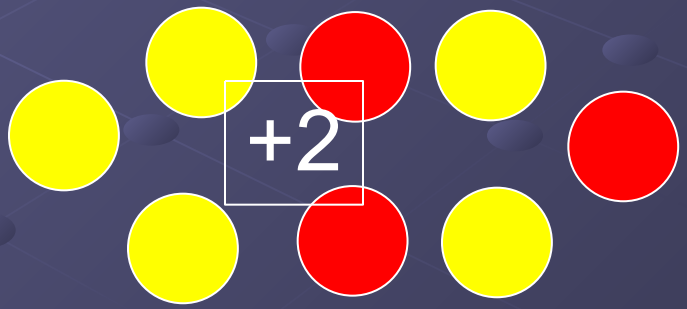
Students may represent -3 by using 3 red tiles and any number of groups of +1 and -1 tiles

INTRODUCTION TO INTEGERS

Tell which integer each group of tiles represents.



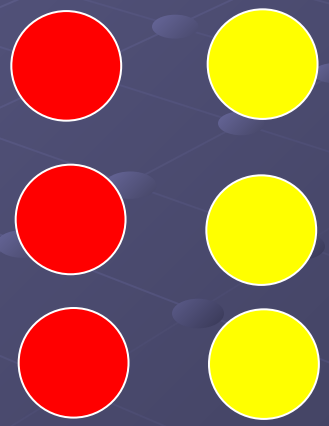




ANSWER

INTRODUCTION TO INTEGERS

If there are the same number of red tiles as yellow tiles, what number is represented?



It represents 0.

ADDITION AND SUBTRACTION

ADDING INTEGERS

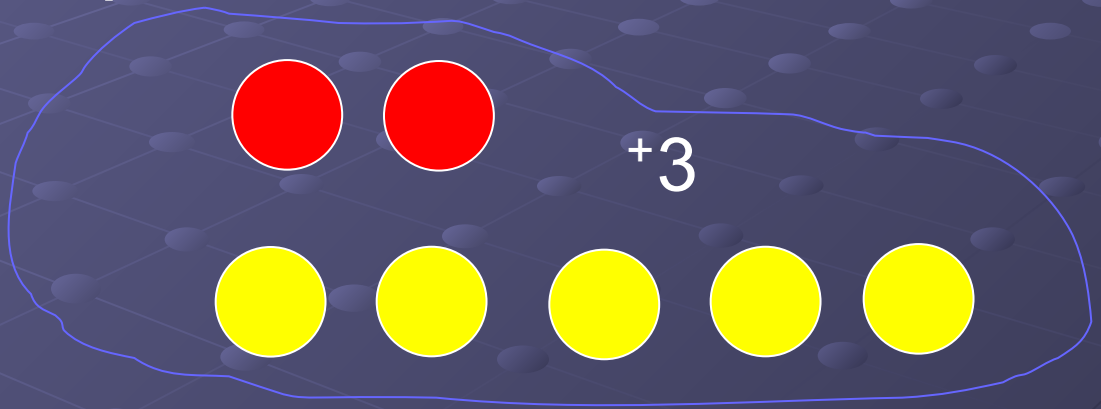
We can model integer addition with tiles.

Represent -2 with the fewest number of tiles

Represent +5 with the fewest number of tiles.

ADDING INTEGERS

What number is represented by combining the 2 groups of tiles?



Write the number sentence that is illustrated.

$-2 + {}^{+}5 = {}^{+}3$

ADDING INTEGERS

Use your red and yellow tiles to find each sum.

-2 + -3 = ?

ANSWER

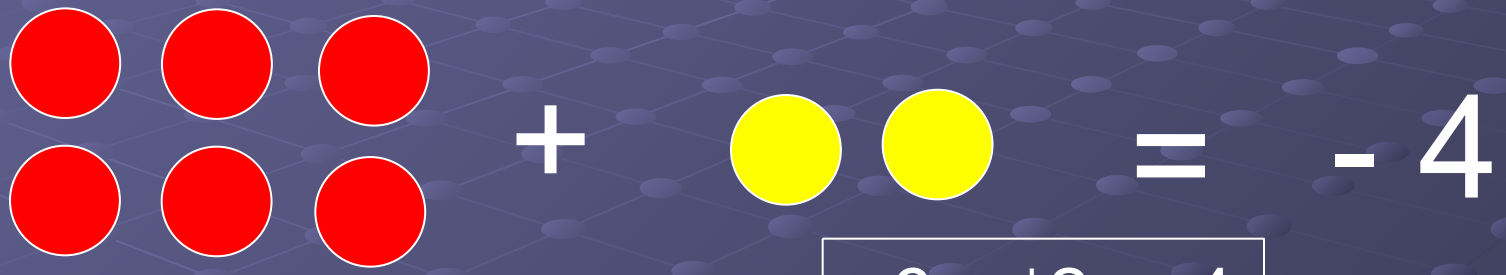
= -5

-2 + -3 = -5

ADDING INTEGERS

$^{-}6 + ^{+}2 = ?$

ANSWER



$^{-}6 + ^{+}2 = ^{-}4$

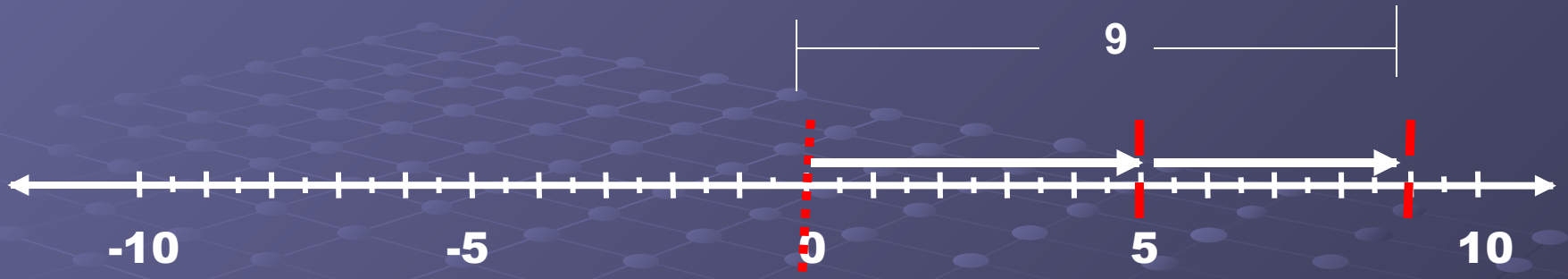
$^{-}3 + ^{+}4 = ?$

ANSWER

$^{-}3 + ^{+}4 = ^{+}1$

Adding Integers - Same Sign

We can show this same idea using a number line.



What is 5 + 4?

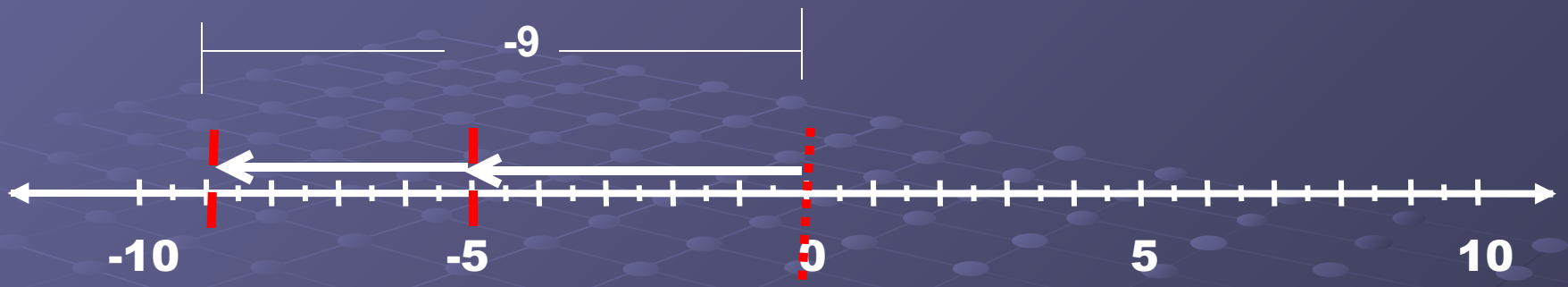
Move five (5) units to the right from zero.

Now move four more units to the right.

The final point is at 9 on the number line.

Therefore, 5 + 4 = 9.

Adding Integers - Same Sign



What is -5 + (-4)?

Move five (5) units to the left from zero.

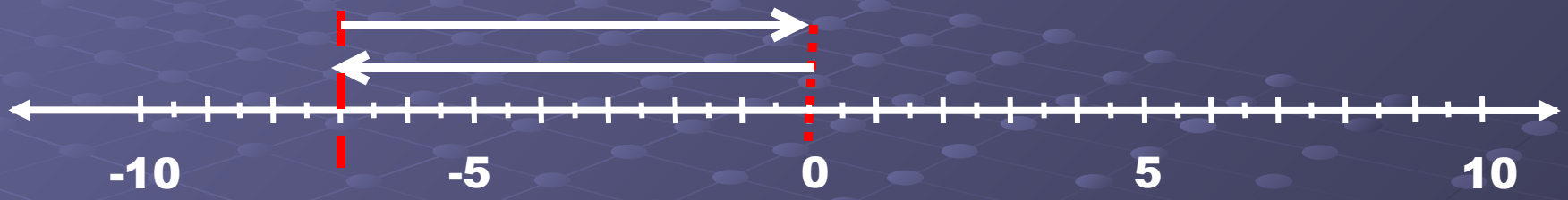
Now move four more units to the left.

The final point is at -9 on the number line.

Therefore, -5 + (-4) = -9.

Additive Inverse

What is (-7) + 7?



To show this, we can begin at zero and move seven units to the left.

Now, move seven units to the right.

Notice, we are back at zero (0).

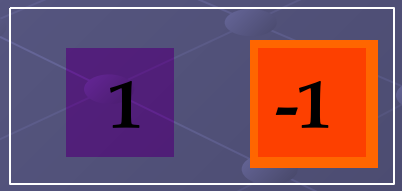
For every positive integer on the number line, there is a corresponding negative integer. These integer pairs are opposites or **additive inverses**.

Additive Inverse Property – For every number a, $a + (-a) = 0$

Additive Inverse

When using algebra chips, the additive inverses make what is called a zero pair.

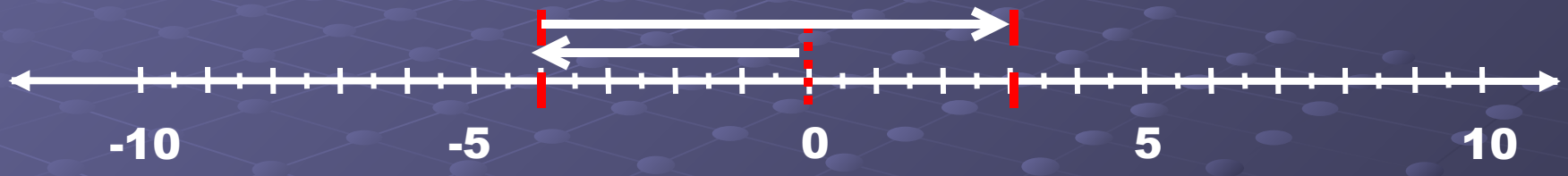
For example, the following is a zero pair.



1 + (-1) = 0.

Adding Integers - Different Signs

Add the following integers: (-4) + 7.



Start at zero and move four units to the left.

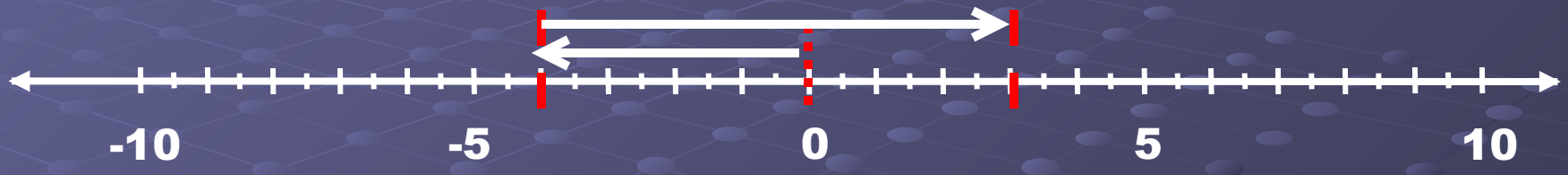
Now move seven units to the right.

The final position is at 3.

Therefore, (-4) + 7 = **3**.

Adding Integers - Different Signs

Add the following integers: (-4) + 7.



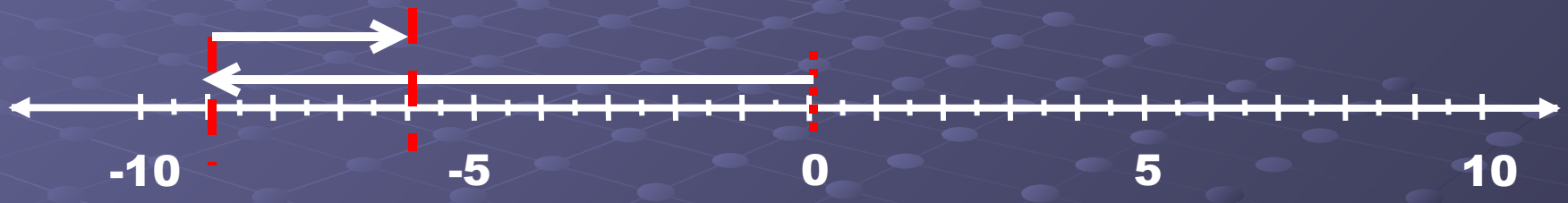
Notice that seven minus four also equals three.

In our example, the number with the larger absolute value was positive and our solution was positive.

Let's try another one.

Adding Integers - Different Signs

Add (-9) + 3



Start at zero and move nine places to the left.

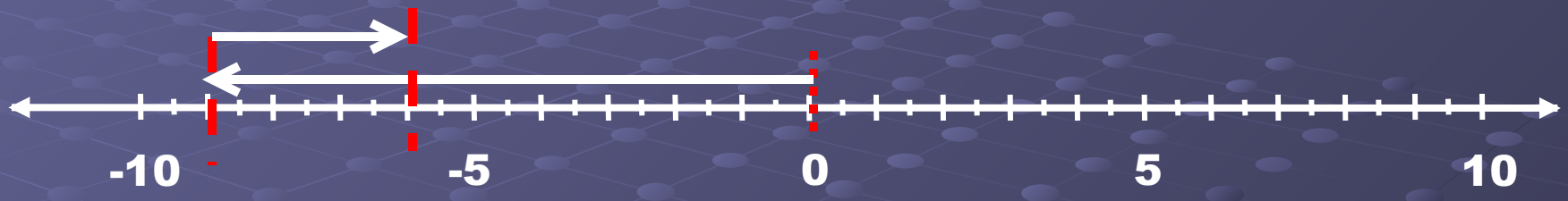
Now move three places to the right.

The final position is at negative six, (-6).

Therefore, (-9) + 3 = **-6**.

Adding Integers - Different Signs

Add (-9) + 3



In this example, the number with the larger absolute value is negative. The number with the smaller absolute value is positive.

We know that 9-3 = 6. However, (-9) + 3 = -6.

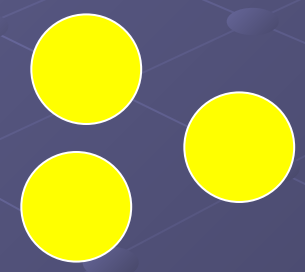
6 and –6 are opposites. Comparing these two examples shows us that the answer will have the same sign as the number with the larger absolute value.

SUBTRACTING INTEGERS

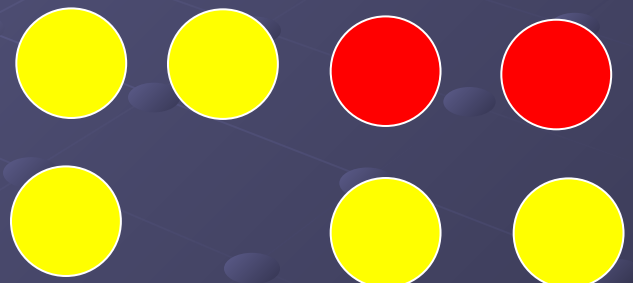
We often think of subtraction as a “take away” operation.

Which diagram could be used to compute

$^{+}3 - {}^{+}5 = ?$



$^{+}3$



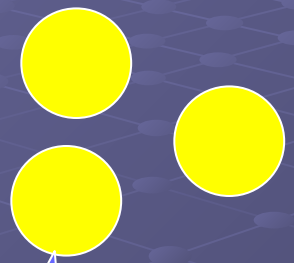
$^{+}3$

Adding Integers Rule

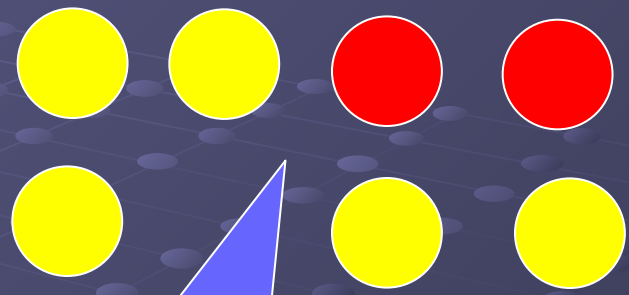
Sang to the tune of Row Row Your Boat

- Same Sign add and keep
- Different signs subtract
- Take the sign of the larger number
- Then you'll be exact

SUBTRACTING INTEGERS



We can't take away 5 yellow tiles from this diagram. There is not enough tiles to take away!!



This diagram also represents +3, and we can take away +5.

- When we take 5 yellow tiles away, we have 2 red tiles left.

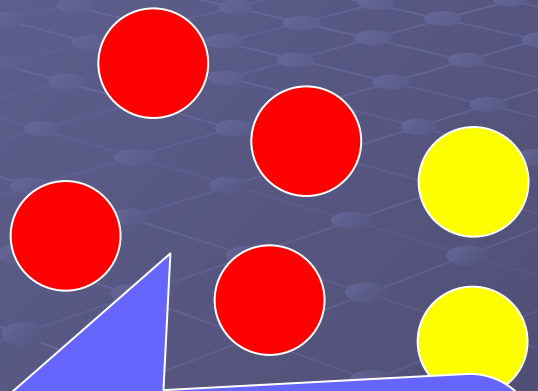
SUBTRACTING INTEGERS

Use your red and yellow tiles to model each subtraction problem.

$-2 - -4 = ?$

ANSWER

SUBTRACTING INT



This representation of $^{-}2$ doesn't have enough tiles to take away $^{-}4$.

$-2 - -4 = +2$

Now you can take away 4 red tiles.

2 yellow tiles are left, so the answer is…

reds
tiles
uld
have a total of 4 red tiles and the tiles still represent $^{-}2$.

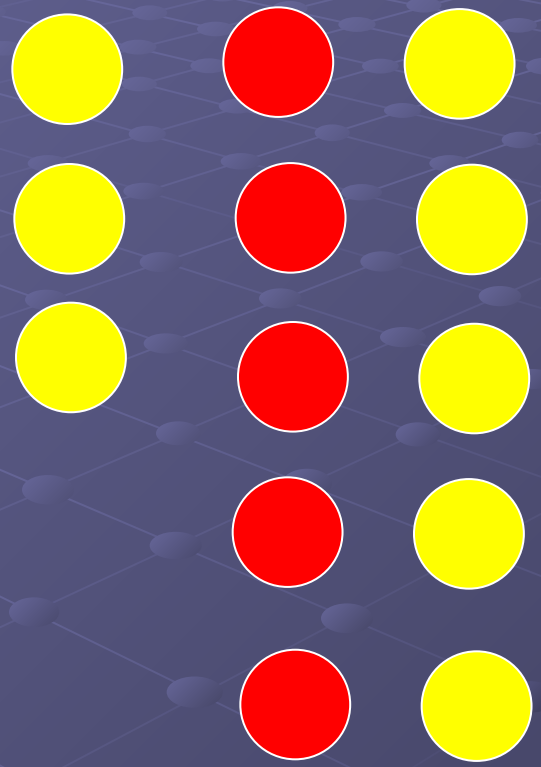
SUBTRACTING INTEGERS

Work this problem.

$^{+}3 - {}^{-}5 = ?$

ANSWER

SUBTRACTING INTEGERS



- Add enough red and yellow pairs so you can take away 5 red tiles.
- Take away 5 red tiles, you have 8 yellow tiles left.

$$^{+}3 - {}^{-}5 = {}^{+}8$$

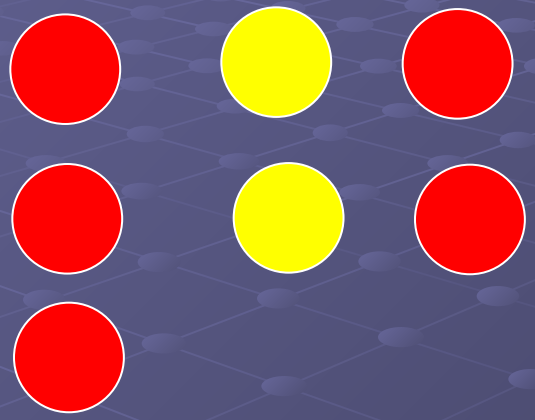
SUBTRACTING INTEGERS

Work this problem.

$^{-}3 - {}^{+}2 = ?$

ANSWER

SUBTRACTING INTEGERS



- Add two pairs of red and yellow tiles so you can take away 2 yellow tiles.
- Take away 2 yellow tiles, you have 5 red tiles left.

$$^{-}3 - {}^{+}2 = {}^{-}5$$

SUBTRACTING INTEGERS

A fact family gives 4 true equations using the same 3 numbers.

For example:

$7 + 8 = 15$
$8 + 7 = 15$
$15 - 7 = 8$
$15 - 8 = 7$

All of these statements are true.

SUBTRACTING INTEGERS

We can also use fact family with integers.

Use your red and yellow tiles to verify this fact family:

$^{-}3 + {}^{+}8 = {}^{+}5$

$^{+}8 + {}^{-}3 = {}^{+}5$

$^{+}5 - {}^{+}8 = {}^{-}3$

$^{+}5 - {}^{-}3 = {}^{+}8$

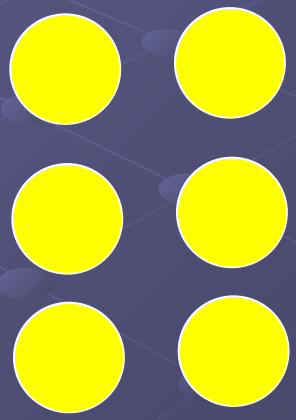
INTEGERS AND MULTIPLICATION

MULTIPLICATION

Red and yellow tiles can be used to model multiplication.

Remember that multiplication can be described as repeated addition.

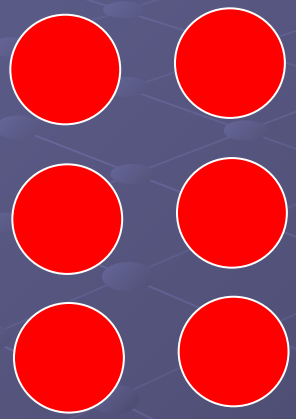
So 2 x 3 = ?



2 groups of 3 tiles = 6 tiles

MULTIPLICATION

2 x $^{-}3$ means 2 groups of $^{-}3$



$$2 \times {}^{-}3 = {}^{-}6$$

MULTIPLICATION

Since $2 \times 3 = 6$ and $3 \times 2 = 6$, does it make sense that $^{-}3 \times 2 = {}^{-}6$?

$^{-}3 \times 2 = {}^{-}6$ is true.

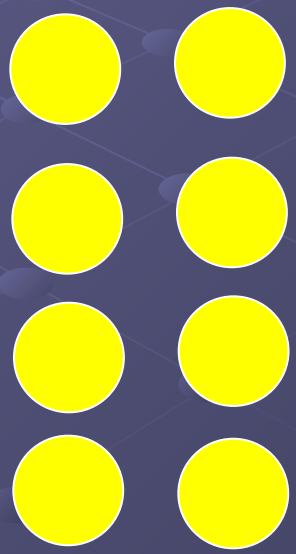
$^{+}2 \times {}^{-}3 = {}^{-}6$ and $^{-}3 \times {}^{+}2 = {}^{-}6$ belong to a fact family:

$$^{+}2 \times {}^{-}3 = {}^{-}6$$
$$^{-}3 \times {}^{+}2 = {}^{-}6$$
$$^{-}6 \div {}^{+}2 = {}^{-}3$$
$$^{-}6 \div {}^{-}3 = {}^{+}2$$

MULTIPLICATION

Use your tiles to model each multiplication problem.

$^{+}2 \times {}^{+}4 = ?$



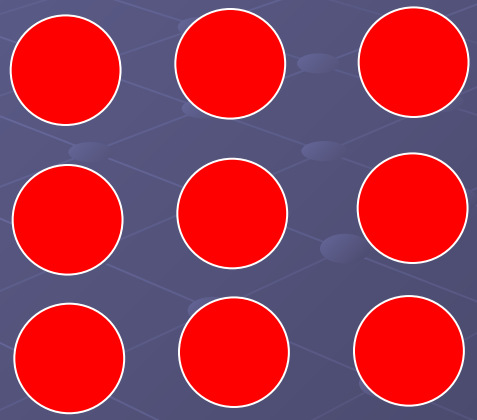
2 groups of $^{+}4$

$^{+}2 \times {}^{+}4 = {}^{+}8$

ANSWER

MULTIPLICATION

$^{+}3 \times {}^{-}3 = ?$



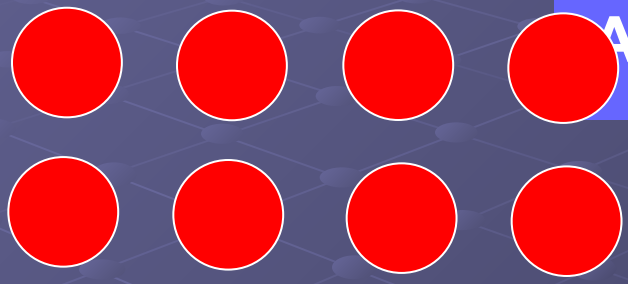
ANSWER

3 groups of $^{-}3$

$^{+}3 \times {}^{-}3 = {}^{-}9$

MULTIPLICATION

$^{-}2 \times {}^{+}4 = ?$



ANSWER 4 groups of $^{-}2$

$^{-}2 \times {}^{+}4 = {}^{-}8$

Use the fact family for

$^{-}2 \times {}^{+}4 = ?$ ← We can't show $^{-}2$ groups of $^{+}4$

$^{+}4 \times {}^{-}2 = ?$ ← we can show 4 groups of $^{-}2$

MULTIPLICATION

$^{+}1$, $^{-}1$ are opposites

$^{+}1 \times {}^{+}3 = {}^{+}3$
$^{-}1 \times {}^{+}3 = {}^{-}3$

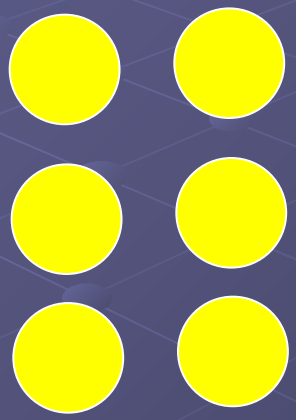
the products are opposite

Since $^{+}2$ and $^{-}2$ are opposites of each other,

$^{+}2 \times {}^{-}3$ and $^{-}2 \times {}^{-}3$ have opposite products.

MULTIPLICATION

To model $^{-}2 \times {}^{-}3$ use 2 groups of the opposite of $^{-}3$

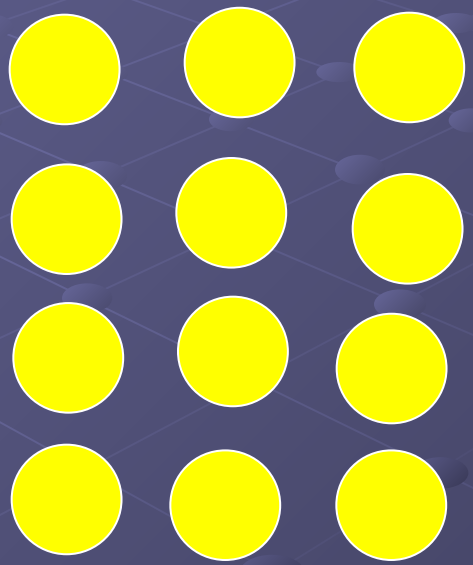


$$^{-}2 \times {}^{-}3 = {}^{+}6$$

MULTIPLICATION

Use tiles to illustrate $^{-}3 \times {}^{-}4 = ?$

ANSWER



3 groups of $^{+}4$ (the opposite of $^{-}4$)

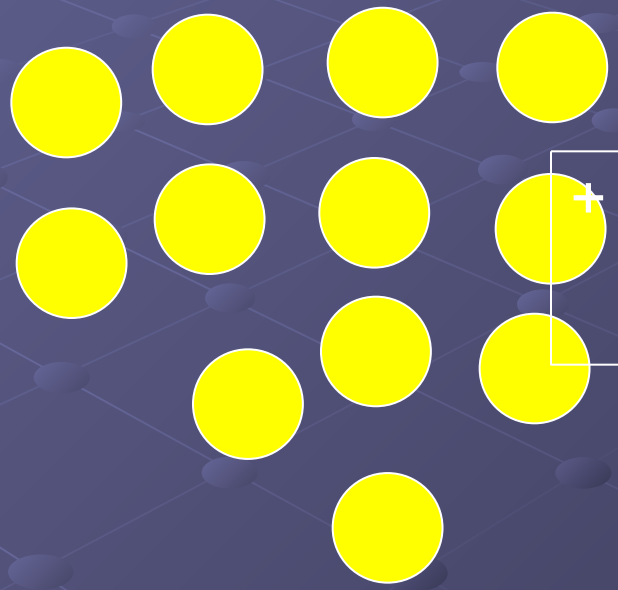
$$^{-}3 \times {}^{-}4 = {}^{+}12$$

INTEGERS AND DIVISION

DIVISION

Use tiles to model $^{+}12 \div {}^{+}3 = ?$

4 yellow tiles in each group.

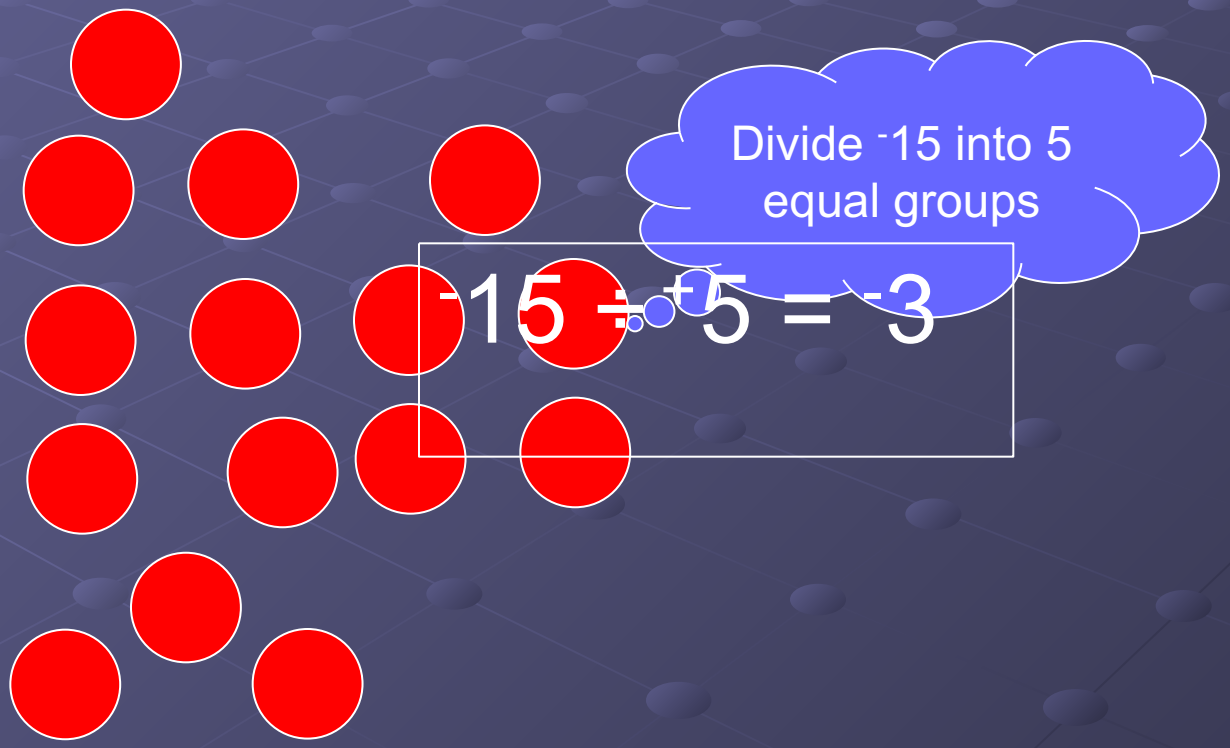


Divide 12 yellow tiles into 3 equal groups

$^{+}12 \div {}^{+}3 = {}^{+}4$

DIVISION

Use tiles to model $^{-}15 \div {}^{+}5 = ?$



DIVISION

Write the fact family

$^{-}4 \times {}^{+}2 = {}^{-}8$

Compare your answer.

$^{+}2 \times {}^{-}4 = {}^{-}8$

$^{-}8 \div {}^{+}2 = {}^{-}4$

$^{-}8 \div {}^{-}4 = {}^{+}2$

DIVISION

Now try these! Use red and yellow tiles to find each answer.

1) $^{-}2 + {}^{-}8$

2) $^{+}6 + {}^{-}4$

3) $^{-}5 - {}^{+}4$

4) $^{-}6 - {}^{-}7$

5) $^{+}3 \times {}^{-}4$

6) $^{-}5 \times {}^{-}2$

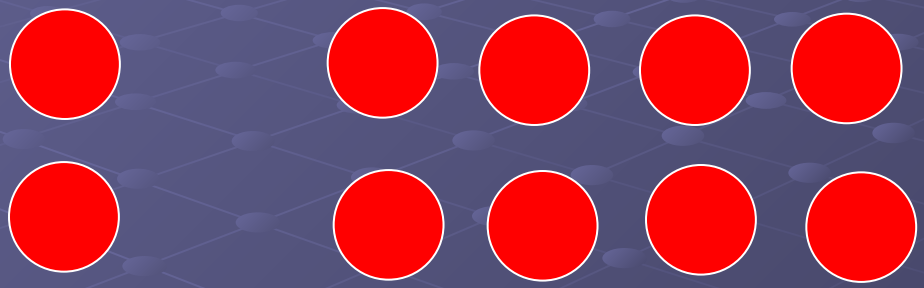
7) $^{-}12 \div {}^{+}6$

8) $^{-}8 \div {}^{+}2$

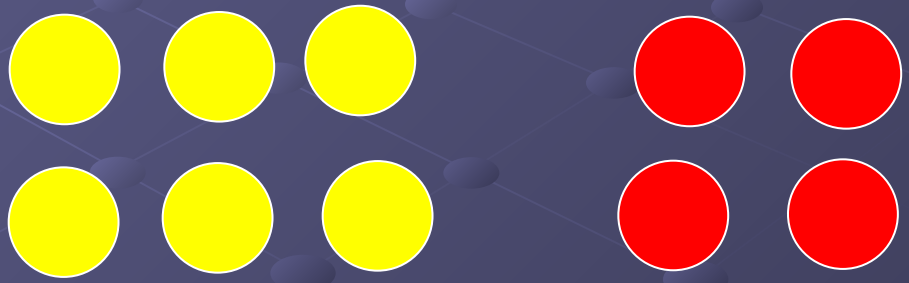
ANSWER

DIVISION

1) $^{-}2 + {}^{-}8 = {}^{-}10$

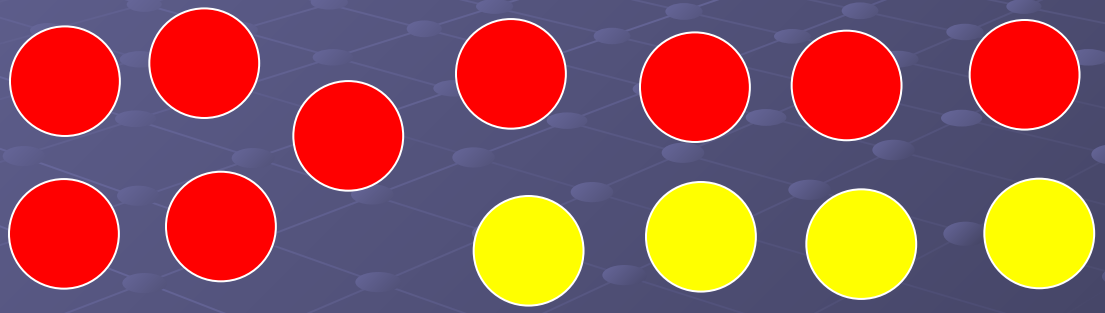


2) $^{+}6 + {}^{-}4 = {}^{+}2$

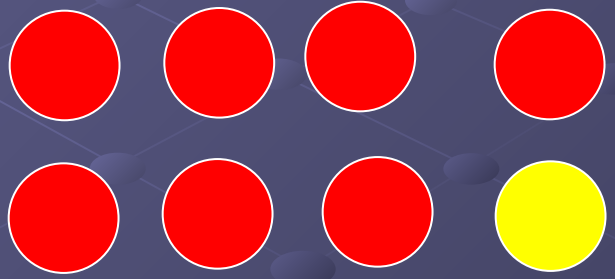


DIVISION

3) $^{-}5$ - $^{+}4$ = $^{-}9$

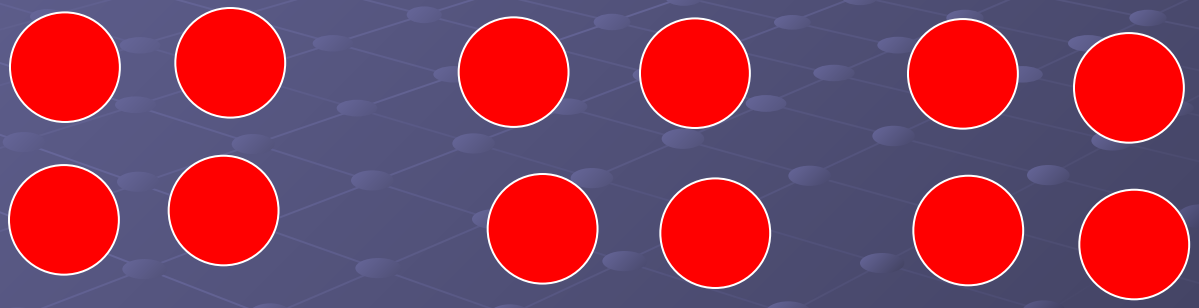


4) $^{-}6$ - $^{-}7$ = $^{+}1$

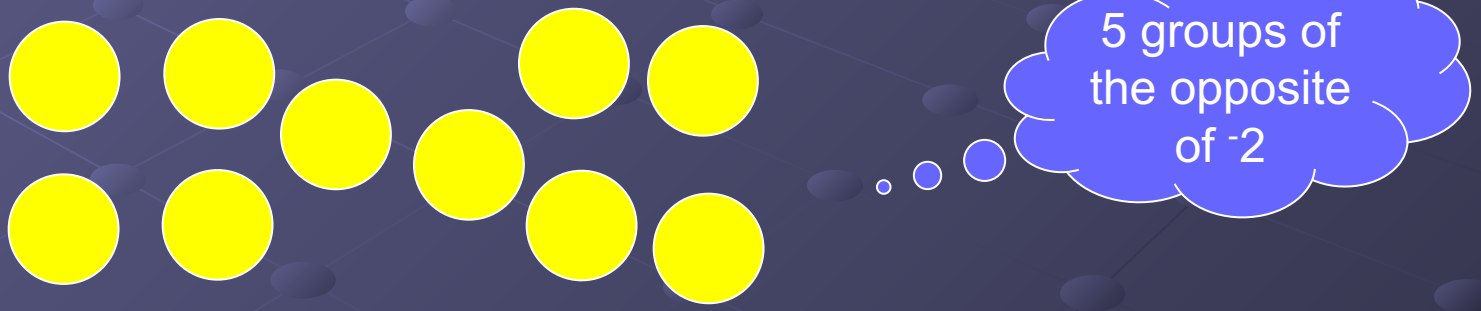


DIVISION

5) $^{+}3 \times {}^{-}4 = {}^{-}12$



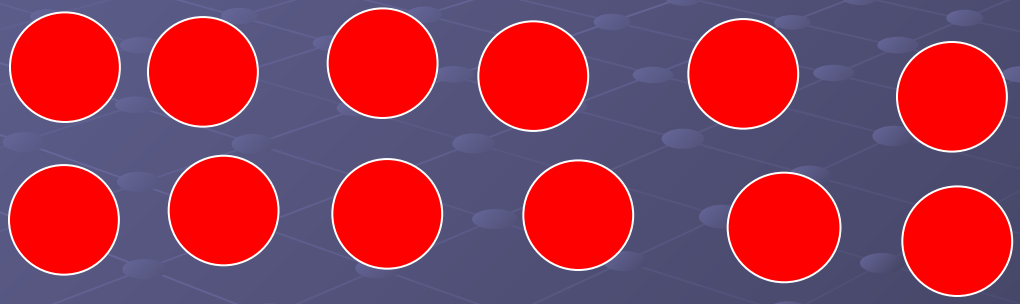
6) $^{-}5 \times {}^{-}2 = {}^{+}10$



DIVISION

7) $^{-}12 \div {}^{+}6 = {}^{-}2$

-12 divided into 6 equal groups



8) $^{-}8 \div {}^{+}2 = {}^{-}4$

-8 divided into 2 equal groups