Essential Questions

How do you identify, evaluate, add, subtract, and classify polynomials?

Academic Vocabulary

monomial
polynomial
degree of a monomial
degree of a polynomial
leading coefficient
binomial
trinomial
polynomial function

A **monomial** is a number or a product of numbers and variables with whole number exponents. A **polynomial** is a monomial or a sum or difference of monomials. Each monomial in a polynomial is a term. Because a monomial has only one term, it is the simplest type of polynomial.

Polynomials have no variables in denominators, no roots or absolute values of variables, and all variables have whole number exponents.

Polynomials: $3x^4$, $2z^{12} + 9z^3$, $\frac{1}{2}a^7$, $0.15x^{101}$, $3t^2 - t^3$

Not polynomials: 3^x, $|2b^3 - 6b|$, $\frac{8}{5y^2}$, $\frac{1}{2}$, $m^{0.75} - m$

The **degree of a monomial** is the sum of the exponents of the variables.

Example 1: Identifying the Degree of a Monomial

Identify the degree of each monomial.

A. z^6

z^6 *Identify the exponent.*

The degree is 6.

B. 5.6

$5.6 = 5.6x^0$ *Identify the exponent.*

The degree is 0.

C. $8xy^3$

$8x^1y^3$ *Add the exponents.*

The degree is 4.

D. a^2bc^3

$a^2b^1c^3$ *Add the exponents.*

The degree is 6.

Check It Out! Example 1

Identify the degree of each monomial.

a. x^3

x^3 *Identify the exponent.*

The degree is 3.

b. 7

$7 = 7x^0$ *Identify the exponent.*

The degree is 0.

c. $5x^3y^2$

$5x^3y^2$ *Add the exponents.*

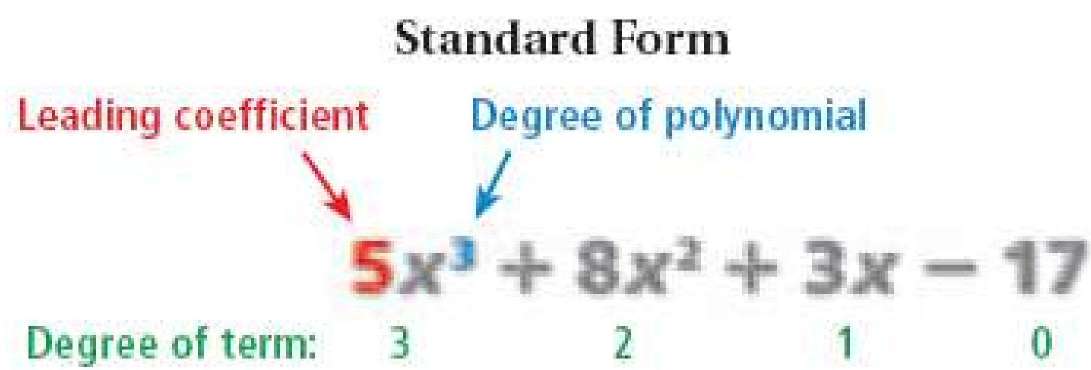
The degree is 5.

d. a^6bc^2

$a^6b^1c^2$ *Add the exponents.*

The degree is 9.

An **degree of a polynomial** is given by the term with the greatest degree. A polynomial with one variable is in standard form when its terms are written in descending order by degree. So, in standard form, the degree of the first term indicates the degree of the polynomial, and the **leading coefficient** is the coefficient of the first term.



A polynomial can be classified by its number of terms. A polynomial with two terms is called a **binomial**, and a polynomial with three terms is called a **trinomial**. A polynomial can also be classified by its degree.

Classifying Polynomials by Degree		
Name	Degree	Example
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

Example 2: Classifying Polynomials

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

A. $3 - 5x^2 + 4x$

Write terms in descending order by degree.

$-5x^2 + 4x + 3$

Leading coefficient: -5

Degree: 2

Terms: 3

Name: quadratic trinomial

B. $3x^2 - 4 + 8x^4$

Write terms in descending order by degree.

$8x^4 + 3x^2 - 4$

Leading coefficient: 8

Degree: 4

Terms: 3

Name: quartic trinomial

Check It Out! Example 2

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

a. $4x - 2x^2 + 2$

Write terms in descending order by degree.

$-2x^2 + 4x + 2$

Leading coefficient: -2

Degree: 2

Terms: 3

Name: quadratic trinomial

b. $-18x^2 + x^3 - 5 + 2x$

Write terms in descending order by degree.

$1x^3 - 18x^2 + 2x - 5$

Leading coefficient: 1

Degree: 3

Terms: 4

Name: cubic polynomial with 4 terms

To add or subtract polynomials, combine like terms. You can add or subtract horizontally or vertically.

Example 3: Adding and Subtracting Polynomials

Add or subtract. Write your answer in standard form.

A. $(2x^3 + 9 - x) + (5x^2 + 4 + 7x + x^3)$

Add vertically.

$(2x^3 + 9 - x) + (5x^2 + 4 + 7x + x^3)$

$2x^3 \qquad\qquad - x + 9$	*Write in standard form.*
$+x^3 + 5x^2 + 7x + 4$	*Align like terms.*
$3x^3 + 5x^2 + 6x + 13$	*Add.*

Example 3: Adding and Subtracting Polynomials

Add or subtract. Write your answer in standard form.

B. $(3 - 2x^2) - (x^2 + 6 - x)$

Add the opposite horizontally.

$(3 - 2x^2) - (x^2 + 6 - x)$

$(-2x^2 + 3) + (-x^2 + x - 6)$ *Write in standard form.*

$(-2x^2 - x^2) + (x) + (3 - 6)$ *Group like terms.*

$-3x^2 + x - 3$ *Add.*

Check It Out! Example 3a

Add or subtract. Write your answer in standard form.

$(-36x^2 + 6x - 11) + (6x^2 + 16x^3 - 5)$

Add vertically.

$(-36x^2 + 6x - 11) + (6x^2 + 16x^3 - 5)$

$$\begin{array}{rrrr} & -36x^2 & +\,6x & -\,11 \\ +16x^3 & +\,6x^2 & & -\,5 \\ \hline 16x^3 & -\,30x^2 & +\,6x & -\,16 \end{array}$$

Write in standard form.

Align like terms.

Add.

Check It Out! Example 3b

Add or subtract. Write your answer in standard form.

$$(5x^3 + 12 + 6x^2) - (15x^2 + 3x - 2)$$

Add the opposite horizontally.

$$(5x^3 + 12 + 6x^2) - (15x^2 + 3x - 2)$$

$(5x^3 + 6x^2 + 12) + (-15x^2 - 3x + 2)$ *Write in standard form.*

$(5x^3) + (6x^2 - 15x^2) + (-3x) + (12 + 2)$ *Group like terms.*

$5x^3 - 9x^2 - 3x + 14$ *Add.*

Example 4: Work Application

The cost of manufacturing a certain product can be approximated by $f(x) = 3x^3 – 18x + 45$, where x is the number of units of the product in hundreds. Evaluate $f(0)$ and $f(200)$ and describe what the values represent.

$f(0) = 3(0)^3 – 18(0) + 45 = 45$

$f(200) = 3(200)^3 – 18(200) + 45 = 23{,}996{,}445$

$f(0)$ represents the initial cost before manufacturing any products.

$f(200)$ represents the cost of manufacturing 20,000 units of the products.

Check It Out! Example 4

Cardiac output is the amount of blood pumped through the heart. The output is measured by a technique called dye dilution. For a patient, the dye dilution can be modeled by the function $f(t) = 0.000468t^4 – 0.016t^3 + 0.095t^2 + 0.806t$, where t represents time (in seconds) after injection and $f(t)$ represents the concentration of dye (in milligrams per liter). Evaluate $f(t)$ for $t = 4$ and $t = 17$, and describe what the values of the function represent.

$f(4) = 0.000468(4)^4 – 0.016(4)^3 + 0.095(4)^2$
$+ 0.806(4) = 3.8398$

$f(17) = 0.000468(17)^4 – 0.016(17)^3 + 0.095(4)^2$
$+ 0.806(17) = 1.6368$

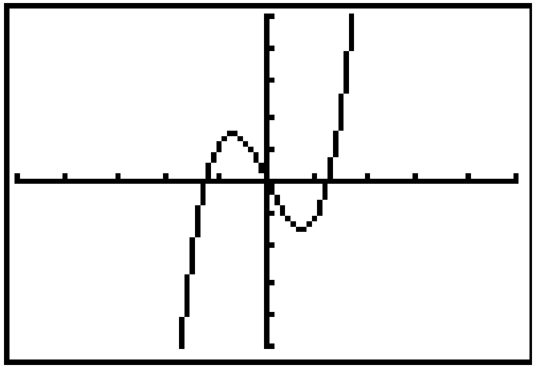
$f(4)$ represents the concentration of dye after 4s. $f(17)$ represents the concentration of dye after 17s.

Throughout this chapter, you will learn skills for analyzing, describing, and graphing higher-degree polynomials. Until then, the graphing calculator will be a useful tool.

Example 5: Graphing Higher-Degree Polynomials on a Calculator

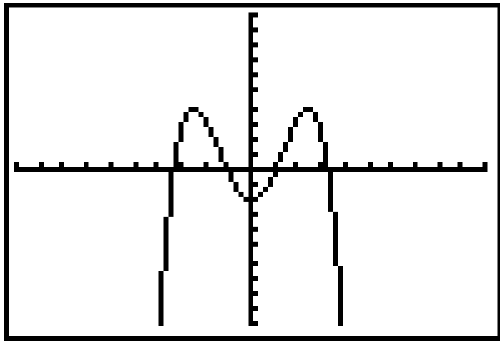
Given the graph of each polynomial function. Describe the graph and identify the number of real zeros.

A. $f(x) = 2x^3 - 3x$



From left to right, the graph increases, then decreases, and increases again. It crosses the *x*-axis 3 times, so there appear to be 3 real zeros.

B. $f(x) = -\frac{1}{6}x^4 + 2x^2 - 2$

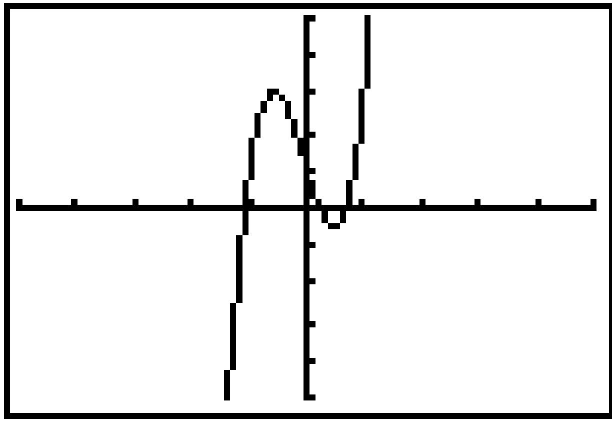


From left to right, the graph alternately increases and decreases, changing direction 3 times and crossing the *x*-axis 4 times; 4 real zeros.

Check It Out! Example 5

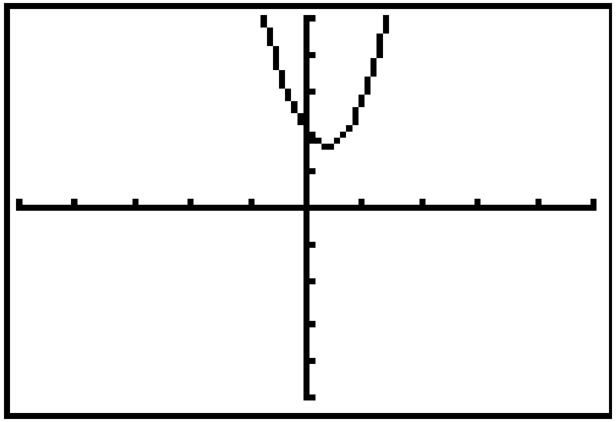
Graph each polynomial function on a calculator. Describe the graph and identify the number of real zeros.

a. $f(x) = 6x^3 + x^2 - 5x + 1$



From left to right, the graph increases, decreases slightly, and then increases again. It crosses the *x*-axis 3 times, so there appear to be 3 real zeros.

b. $f(x) = 3x^2 - 2x + 2$

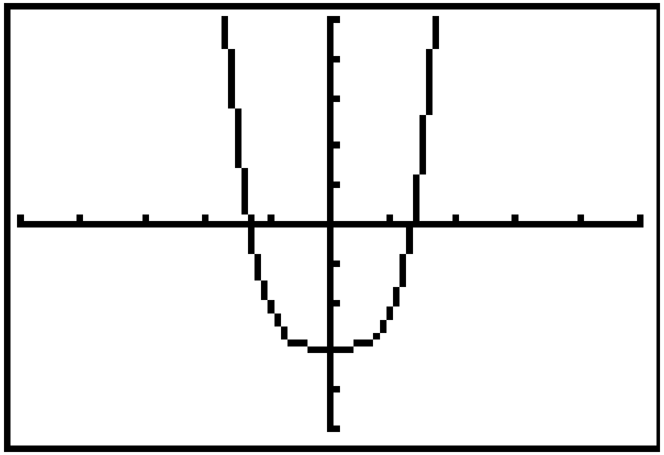


From right to left, the graph decreases and then increases. It does not cross the *x*-axis, so there are no real zeros.

Check It Out! Example 5

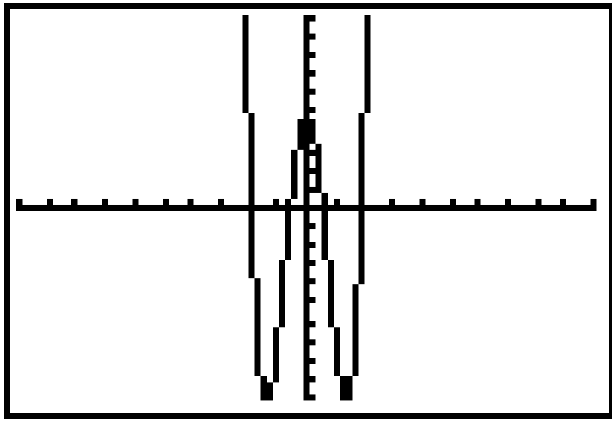
Graph each polynomial function on a calculator. Describe the graph and identify the number of real zeros.

c. $g(x) = x^4 - 3$



From left to right, the graph decreases and then increases. It crosses the *x*-axis twice, so there appear to be 2 real zeros.

d. $h(x) = 4x^4 - 16x^2 + 5$



From left to right, the graph alternately decreases and increases, changing direction 3 times. It crosses the *x*-axis 4 times, so there appear to be 4 real zeros.

Lesson Quiz

Rewrite in standard form. Identify the degree of the polynomial and the number of terms.

1. $9 - x^2 + 2x^5 - 7x$ $2x^5 - x^2 - 7x + 9$; 5; 4

2. $23 + 4x^3$ $4x^3 + 23$; 3; 2

3. Subtract $4x^5 - 8x + 2$ from $3x^4 + 10x - 9$. Write your answer in standard form.

$-4x^5 + 3x^4 + 18x - 11$

4. Evaluate $h(x) = 0.4x^2 - 1.2x + 7.5$ for $x = 0$ and $x = 3$. 7.5; 7.5

5. Describe the graph of $j(x) = 3x^2 - 6x + 6$ and identify the number of zeros.

From left to right, the graph decreases then increases, but it never crosses the *x*-axis; no real zeros.