Review of First Semester Topics – AP Calculus AB 2016-17

Name: ______________________________________ **Date:** __________________ **Period:** _____

Exercises - Rate of Change

Multiple Choice Questions

1. The traffic flow at a particular intersection is modeled by the function f defined by $f(t) = 25 + 6\cos(\frac{x}{3})$ for $0 \le t \le 120$. What is the average rate of change of the traffic flow over the time interval $30 \le t \le 40$.

   (A) 0.743 (B) 0.851 (C) 0.935 (D) 1.176

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2. The rate of change of the altitude of a hot air balloon rising from the ground is given by $y(t) = t^3 - 3t^2 + 3t$ for $0 \le t \le 10$. What is the average rate of change in altitude of the balloon over the time interval $0 \le t \le 10$.

   (A) 56 (B) 73 (C) 85 (D) 94

Free Response Questions

t (sec)	0	10	20	30	40	50	60	70	80	90
$f(t)$ (ft/sec)	0	28	43	67	82	85	74	58	42	35

3. The table above shows the velocity of a car moving on a straight road. The car's velocity v is measured in feet per second.

   (a) Find the average velocity of the car from $t = 60$ to $t = 90$.

   (b) The instantaneous rate of change of f (See Ch. 2.1 for an explanation of instantaneous rate of change) with respect to x at $x = a$ can be approximated by finding the average rate of change of f near $x = a$. Approximate the instantaneous rate of change of f at $x = 40$ using two points, $x = 30$ and $x = 50$.

Exercises - The Limit of a Function and One Sided Limits

Multiple Choice Questions

1. $\lim_{x\to\frac{\pi}{6}} \sec^2 x =$

(A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{4}{3}$ (D) $\frac{2\sqrt{3}}{3}$

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2. If $f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 1, & x = 1 \end{cases}$, then $\lim_{x\to 1} f(x) =$

(A) 1 (B) 2 (C) 3 (D) 4

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3. $\lim_{x\to 1} \frac{|x-1|}{1-x} =$

(A) -2 (B) -1 (C) 1 (D) nonexistent

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4. Let f be a function given by $f(x) = \begin{cases} 3-x^2, & \text{if } x<0 \\ 2-x, & \text{if } 0 \leq x < 2 \\ \sqrt{x-2}, & \text{if } x > 2 \end{cases}$.

Which of the following statements are true about f ?

I. $\lim_{x\to 0} f(x) = 2$

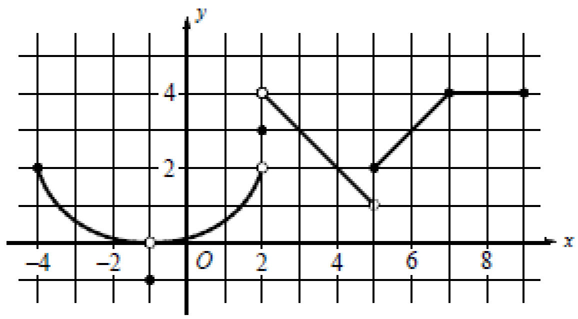
II. $\lim_{x\to 2} f(x) = 0$

III. $\lim_{x\to 1} f(x) = \lim_{x\to 6} f(x)$

(A) I only (B) II only (C) II and III only (D) I , II, and III

Free Response Questions

Questions 5-11 refer to the following graph.



The figure above shows the graph of $y = f(x)$ on the closed interval $[-4, 9]$.

5. Find $\lim_{x \to -1} \cos(f(x))$.

6. Find $\lim_{x \to 2^-} f(x)$.

7. Find $\lim_{x \to 2^+} f(x)$.

8. Find $\lim_{x \to 2} f(x)$.

9. Find $f(2)$.

10. Find $\lim_{x \to 5^-} \arctan(f(x))$.

11. Find $\lim_{x \to 5^+} [x f(x)]$.

Exercises - Calculating Limits Using the Limit Laws

Multiple Choice Questions

1. $\lim_{x\to\pi/3}\frac{\sin(\frac{\pi}{3}-x)}{\frac{\pi}{3}-x}=$

(A) -1 (B) 0 (C) $\frac{\sqrt{3}}{2}$ (D) 1

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2. $\lim_{x\to 0}\frac{\sin 3x}{\sin 2x}=$

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) nonexistent

---

3. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}=$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) nonexistent

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4. $\lim_{x\to 1}\frac{\sqrt{3+x}-2}{x^3-1}=$

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\sqrt{3}$ (D) nonexistent

5. $\lim_{\theta \to 0} \frac{\theta + \theta \cos\theta}{\sin\theta \cos\theta} =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

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6. $\lim_{x \to 0} \frac{\tan 3x}{x} =$

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3

---

7. $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} =$

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) -9 (D) 9

Free Response Questions

8. If $\lim_{x \to 0} \frac{\sqrt{2 + ax} - \sqrt{2}}{x} = \sqrt{2}$ what is the value of a ?

9. Find $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, if $f(x)=\sqrt{2x+1}$.

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10. Find $\lim_{x\to 0}\dfrac{f(x)-g(x)}{\sqrt{g(x)+7}}$, if $\lim_{x\to 0} f(x)=2$ and $\lim_{x\to 0} g(x)=-3$.

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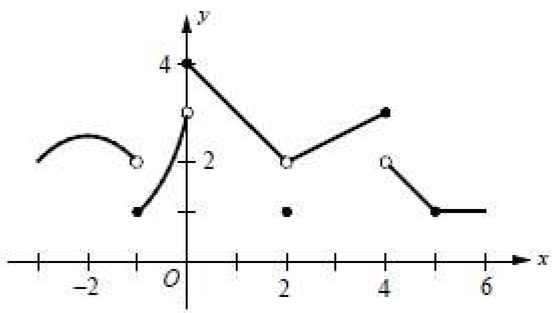
11. Find $\lim_{x\to\sqrt{3}} g(x)$, if $\lim_{x\to\sqrt{3}} \dfrac{1}{x^2+g(x)}=\dfrac{1}{5}$.

Multiple Choice Questions

1. Let f be a function defined by $f(x)=\begin{cases}\dfrac{x^2-a^2}{x-a}, & \text{if } x\neq a\\ 4, & \text{if } x=a\end{cases}$. If f is continuous for all real numbers x, what is the value of a?

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2



2. The graph of a function f is shown above. If $\lim_{x\to a} f(x)$ exists and f is not continuous at $x=a$, then $a=$

(A) −1 (B) 0 (C) 2 (D) 4

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3. If $f(x)=\begin{cases}\dfrac{\sqrt{3x-1}-\sqrt{2x}}{x-1}, & \text{for } x\neq 1\\ a, & \text{for } x=1\end{cases}$, and if f is continuous at $x=1$, then $a=$

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\sqrt{2}$ (D) 2

4. Let f be a continuous function on the closed interval $[-2,7]$. If $f(-2)=5$ and $f(7)=-3$, then the Intermediate Value Theorem guarantees that

(A) $f'(c)=0$ for at least one c between -2 and 7

(B) $f'(c)=0$ for at least one c between -3 and 5

(C) $f(c)=0$ for at least one c between -3 and 5

(D) $f(c)=0$ for at least one c between -2 and 7

Free Response Questions

5. Let g be a function defined by $g(x)=\begin{cases} \dfrac{\pi \sin x}{x}, & \text{if } x<0 \\ a-bx, & \text{if } 0 \le x < 1. \\ \arctan x, & \text{if } x \ge 1 \end{cases}$

If g is continuous for all real numbers x, what are the values of a and b?

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6. Evaluate $\lim\limits_{a\to 0} \dfrac{-1+\sqrt{1+a}}{a}$.

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7. What is the value of a, if $\lim\limits_{x\to 0} \dfrac{\sqrt{ax+9}-3}{x}=1$?

Exercises - Limits and Asymptotes

Multiple Choice Questions

1. $\lim\limits_{x\to\infty}\dfrac{3+2x^2-x^4}{3x^4-5}=$

(A) -2 (B) $-\dfrac{1}{3}$ (C) $\dfrac{1}{5}$ (D) 1

---

2. What is $\lim\limits_{x\to-\infty}\dfrac{x^3+x-8}{2x^3+3x-1}=$

(A) $-\dfrac{1}{2}$ (B) 0 (C) $\dfrac{1}{2}$ (D) 2

---

3. Which of the following lines is an asymptote of the graph of $f(x)=\dfrac{x^2+5x+6}{x^2-x-12}$?

I. $x=-3$

II. $x=4$

III. $y=1$

(A) II only (B) III only (C) II and III only (D) I, II, and III

4. If the horizontal line $y=1$ is an asymptote for the graph of the function f, which of the following statements must be true?

(A) $\lim_{x\to\infty} f(x)=1$

(B) $\lim_{x\to 1} f(x)=\infty$

(C) $f(1)$ is undefined

(D) $f(x)=1$ for all x

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5. If $x=1$ is the vertical asymptote and $y=-3$ is the horizontal asymptote for the graph of the function f, which of the following could be the equation of the curve?

(A) $f(x)=\dfrac{-3x^2}{x-1}$

(B) $f(x)=\dfrac{-3(x-1)}{x+3}$

(C) $f(x)=\dfrac{-3(x^2-1)}{x-1}$

(D) $f(x)=\dfrac{-3(x^2-1)}{(x-1)^2}$

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6. What are all horizontal asymptotes of the graph of $y=\dfrac{6+3e^x}{3-3e^x}$ in the xy-plane?

(A) $y=-1$ only

(B) $y=2$ only

(C) $y=-1$ and $y=2$

(D) $y=0$ and $y=2$

Free Response Questions

7. Let $f(x)=\dfrac{3x-1}{x^3-8}$.

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.

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8. Let $f(x)=\dfrac{\sin x}{x^2+2x}$.

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.

Multiple Choice Questions

1. $\lim_{h \to 0} \frac{\sqrt[3]{8+h}-2}{h} =$

(A) $\frac{1}{12}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt[3]{2}}{2}$ (D) $\sqrt[3]{2}$ (E) 2

---

2. $\lim_{h \to 0} \frac{(2+h)^5 - 32}{h}$ is

(A) $f'(5)$, where $f(x) = x^2$

(B) $f'(2)$, where $f(x) = x^5$

(C) $f'(5)$, where $f(x) = 2^x$

(D) $f'(2)$, where $f(x) = 2^x$

$$f(x) = \begin{cases} 1-2x, & \text{if } x \le 1 \\ -x^2, & \text{if } x > 1 \end{cases}$$

3. Let f be the function given above. Which of the following must be true?

I. $\lim_{x \to 1} f(x)$ exists.

II. f is continuous at $x = 1$.

III. f is differentiable at $x = 1$.

(A) I only

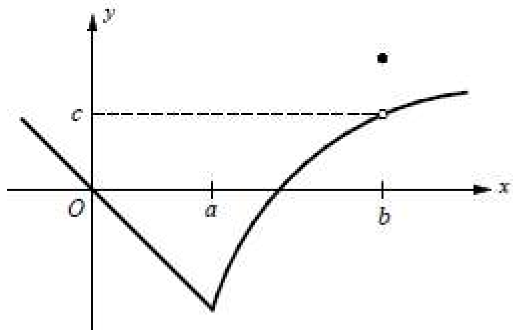
(B) I and II only

(C) II and III only

(D) I, II, and III

4. What is the instantaneous rate of change at $x = -1$ of the function $f(x) = -\sqrt[3]{x^2}$?

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$



Graph of f

5. The graph of a function f is shown in the figure above. Which of the following statements must be false?

(A) $f(x)$ is defined for $0 \le x \le b$.

(B) $f(b)$ exists.

(C) $f'(b)$ exists.

(D) $\lim_{x \to a^-} f'(x)$ exists.

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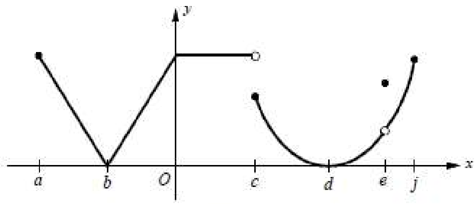
6. If f is a differentiable function, then $f'(1)$ is given by which of the following?

I. $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

II. $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$

III. $\lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only



Graph of f

7. The graph of a function f is shown in the figure above. At how many points in the interval $a < x < j$ is f' not defined?

(A) 3 (B) 4 (C) 5 (D) 6

Free Response Questions

8. Let f be the function defined by $f(x) = \begin{cases} mx^2 - 2 & \text{if } x \le 1 \\ k\sqrt{x} & \text{if } x > 1 \end{cases}$. If f is differentiable at $x = 1$, what are the values of k and m?

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9. Let f be a function that is differentiable throughout its domain and that has the following properties.

(1) $f(x+y) = f(x) + x^3 y - xy^3 - f(y)$

(2) $\lim_{x \to 0} \frac{f(x)}{x} = 1$

Use the definition of the derivative to show that $f'(x) = x^3 - 1$.

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10. Let f be the function defined by

$$f(x) = \begin{cases} x+2 & \text{for } x \le 0 \\ \frac{1}{2}(x+2)^2 & \text{for } x > 0. \end{cases}$$

(a) Find the left-hand derivative of f at $x = 0$.

(b) Find the right-hand derivative of f at $x = 0$.

(c) Is the function f differentiable at $x = 0$? Explain why or why not.

(d) Suppose the function g is defined by

$$g(x) = \begin{cases} x+2 & \text{for } x \le 0 \\ a(x+b)^2 & \text{for } x > 0, \end{cases}$$

where a and b are constants. If g is differentiable at $x = 0$, what are the values of a and b?

Exercises - The Product and Quotient Rules and Higher Derivatives

Multiple Choice Questions

1. If $f(x)=(x^3-2x+5)(x^{-2}+x^{-1})$, then $f'(1)=$

(A) -10 (B) -6 (C) $-\frac{9}{2}$ (D) $\frac{7}{2}$

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2. If $f(x)=\dfrac{\sqrt{x}-1}{\sqrt{x}+1}$ then $f'(x)=$

(A) $\dfrac{\sqrt{x}}{(\sqrt{x}+1)^2}$

(B) $\dfrac{x}{(\sqrt{x}+1)^2}$

(C) $\dfrac{1}{\sqrt{x}(\sqrt{x}+1)^2}$

(D) $\dfrac{\sqrt{x}-1}{\sqrt{x}(\sqrt{x}+1)^2}$

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3. If $g(2)=3$ and $g'(2)=-1$, what is the value of $\dfrac{d}{dx}\left(\dfrac{g(x)}{x^2}\right)$ at $x=2$?

(A) -3 (B) -1 (C) 0 (D) 2

4. If $f(x)=\dfrac{x}{x-\dfrac{a}{x}}$ and $f'(1)=\dfrac{1}{2}$, what is the value of a?

(A) $-\dfrac{5}{2}$ (B) -1 (C) $\dfrac{1}{2}$ (D) 2

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5. If $y=4\sqrt{x}-16\sqrt[4]{x}$, then $y''=$

(A) $\sqrt[4]{x}-3$ (B) $-3\sqrt{x}+3$ (C) $\dfrac{-\sqrt[4]{x}+3}{x\sqrt[4]{x^3}}$ (D) $\dfrac{\sqrt{x}-3}{x\sqrt[4]{x}}$

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6. If $y=x^2\cdot f(x)$, then $y''=$

(A) $x^2 f''(x)+x f'(x)+2f(x)$

(B) $x^2 f''(x)+x f'(x)+f(x)$

(C) $x^2 f''(x)+2x f'(x)+f(x)$

(D) $x^2 f''(x)+4x f'(x)+2f(x)$

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7. Let $f(x)=\dfrac{1}{2}x^6-10x^3+12x$. What is the value of $f(x)$, when $f'''(x)=0$?

(A) $-\dfrac{23}{4}$ (B) $-\dfrac{3}{2}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{5}{2}$

Free Response Questions

8. Let $h(x) = x \cdot f(x) \cdot g(x)$. Find $h'(1)$, if $f(1) = -2$, $g(1) = 3$, $f'(1) = 1$, and $g'(1) = \frac{1}{2}$.

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9. Let $g(x) = \frac{x}{\sqrt{x} - 1}$. Find $g''(4)$.

Exercises - The Chain Rule and the Composite Functions

Multiple Choice Questions

1. If $f(x)=\sqrt{x+\sqrt{x}}$, then $f'(x)=$

(A) $\dfrac{1}{2\sqrt{x+\sqrt{x}}}$ (B) $\dfrac{\sqrt{x}+1}{2\sqrt{x+\sqrt{x}}}$ (C) $\dfrac{2\sqrt{x}}{4\sqrt{x+\sqrt{x}}}$ (D) $\dfrac{2\sqrt{x}+1}{4\sqrt{x^2+x\sqrt{x}}}$

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2. If $f(x)=(x^2-3x)^{3/2}$, then $f'(4)=$

(A) $\dfrac{15}{2}$ (B) 9 (C) $\dfrac{21}{2}$ (D) 15

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3. If f, g, and h are functions that is everywhere differentiable, then the derivative of $\dfrac{f}{g\cdot h}$ is

(A) $\dfrac{g\,h\,f'-f\,g'\,h'}{g\,h}$

(B) $\dfrac{g\,h\,f'-f\,g\,h'-f\,h\,g'}{g\,h}$

(C) $\dfrac{g\,h\,f'-f\,g\,h'-f\,g'h}{g^2h^2}$

(D) $\dfrac{g\,h\,f'-f\,g\,h'+f\,h\,g'}{g^2h^2}$

4. If $f(x)=(3-\sqrt{x})^{-1}$, then $f''(4)=$

(A) $\frac{3}{32}$ (B) $\frac{3}{16}$ (C) $\frac{3}{4}$ (D) $\frac{9}{4}$

Free Response Questions

Questions 5-9 refer to the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	1	-1
2	-2	1	-1	3
3	1	4	2	3
4	5	2	1	-2

The table above gives values of f, f', g, and g' at selected values of x.

5. Find $h'(1)$, if $h(x)=f(g(x))$.

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6. Find $h'(2)$, if $h(x)=x\,f(x^2)$.

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7. Find $h'(3)$, if $h(x)=\dfrac{f(x)}{\sqrt{g(x)}}$.

8. Find $h'(2)$, if $h(x) = [f(2x)]^2$.

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9. Find $h'(1)$, if $h(x) = (x^9 + f(x))^{-2}$.

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10. Let f and g be differentiable functions such that $f(g(x)) = 2x$ and $f'(x) = 1 + [f(x)]^2$.

(a) Show that $g'(x) = \dfrac{2}{f'(g(x))}$.

(b) Show that $g'(x) = \dfrac{2}{1+4x^2}$.

Exercises - Derivatives of Trigonometric Functions

Multiple Choice Questions

1. $\lim_{h\to 0}\frac{\cos(\frac{\pi}{3}+h)-\frac{1}{2}}{h}=$

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

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2. $\lim_{h\to 0}\frac{\sin 2(x+h)-\sin 2x}{h}=$

(A) $2\sin 2x$ (B) $-2\sin 2x$ (C) $2\cos 2x$ (D) $-2\cos 2x$

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3. If $f(x)=\sin(\cos 2x)$, then $f'(\frac{\pi}{4})=$

(A) 0 (B) -1 (C) 1 (D) -2

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4. If $y=a\sin x+b\cos x$, then $y+y''=$

(A) 0 (B) $2a\sin x$ (C) $2b\cos x$ (D) $-2a\sin x$

5. $\frac{d}{dx}\sec^2(\sqrt{x}) =$

(A) $\frac{2\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(B) $\frac{2\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(C) $\frac{\sec^2(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

(D) $\frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}$

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6. $\frac{d}{dx}\left[x^2 \cos 2x\right] =$

(A) $-2x\sin 2x$

(B) $2x(-x\sin 2x + \cos 2x)$

(C) $2x(x\sin 2x - \cos 2x)$

(D) $2x(x\sin 2x - \cos 2x)$

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7. If $f(\theta) = \cos\pi - \frac{1}{2\cos\theta} + \frac{1}{3\tan\theta}$, then $f'(\frac{\pi}{6}) =$

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{4}{\sqrt{3}}$ (D) $2\sqrt{3}$

Free Response Questions

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	$-1/2$	$3/2$	4	$\sqrt{2}$
$\pi/4$	-2	1	2	3

8. The table above gives values of f, f', g, and g' at selected values of x.

   Find $h'(\frac{\pi}{4})$, if $h(x) = f(x) \cdot g(\tan x)$.

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9. Find the value of the constants a and b for which the function

   $f(x) = \begin{cases} \sin x, & x < \pi \\ ax + b, & x \geq \pi \end{cases}$ is differentiable at $x = \pi$.

Exercises - Derivatives of Exponential and Logarithmic Functions

Multiple Choice Questions

1. $\lim_{h \to 0} \dfrac{\frac{1}{2}[\ln(e+h)-1]}{h}$ is

(A) $f'(1)$, where $f(x) = \ln\sqrt{x}$

(B) $f'(1)$, where $f(x) = \ln\sqrt{x+e}$

(C) $f'(e)$, where $f(x) = \ln\sqrt{x}$

(D) $f'(e)$, where $f(x) = \ln(\frac{x}{2})$

---

2. If $f(x) = e^{\tan x}$, then $f'(\frac{\pi}{4}) =$

(A) $\frac{e}{2}$ (B) e (C) $2e$ (D) $\frac{e^2}{2}$

---

3. If $y = \ln(\cos x)$, then $y' =$

(A) $-\tan x$ (B) $\tan x$ (C) $-\cot x$ (D) $\csc x$

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4. If $y = x^x$, then $y' =$

(A) $x^x \ln x$ (B) $x^x(1+\ln x)$ (C) $x^x(x+\ln x)$ (D) $\frac{x^x \ln x}{x}$

5. If $y = e^{\sqrt{x^2+1}}$, then $y' =$

(A) $\sqrt{x^2+1}\, e^{\sqrt{x^2+1}}$

(B) $2x\sqrt{x^2+1}\, e^{\sqrt{x^2+1}}$

(C) $\dfrac{e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

(D) $\dfrac{xe^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$

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6. If $y = (\sin x)^{1/x}$, then $y' =$

(A) $(\sin x)^{\frac{1}{x}}\left[\dfrac{\ln(\sin x)}{x}\right]$

(B) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x-\ln(\sin x)}{x^2}\right]$

(C) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x\sin x-\ln(\sin x)}{x^2}\right]$

(D) $(\sin x)^{\frac{1}{x}}\left[\dfrac{x\cot x-\ln(\sin x)}{x^2}\right]$

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7. If $f(x) = \ln\left[\sec(\ln x)\right]$, then $f'(e) =$

(A) $\dfrac{\cos 1}{e}$ (B) $\dfrac{\sin 1}{e}$ (C) $\dfrac{\tan 1}{e}$ (D) $\dfrac{\cot 1}{e}$

8. If $y = x^{\ln\sqrt{x}}$, then $y' =$

(A) $\dfrac{x^{\ln\sqrt{x}}\ln x}{2x}$

(B) $\dfrac{x^{\ln\sqrt{x}}\ln x}{x}$

(C) $\dfrac{2x^{\ln\sqrt{x}}\ln x}{x}$

(D) $\dfrac{x^{\ln\sqrt{x}}(1+\ln x)}{x}$

Free Response Questions

9. Let $f(x) = xe^x$ and $f^{(n)}(x)$ be the nth derivative of f with respect to x. If $f^{(10)}(x) = (x+n)\,e^x$, what is the value of n?

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10. Let f and h be twice differentiable functions such that $h(x) = e^{f(x)}$. If $h''(x) = e^{f(x)}\left[1+x^2\right]$, then $f'(x) =$

Exercises - Tangent Lines and Normal Lines

Multiple Choice Questions

1. The equation of the line tangent to the graph of $y = x\sqrt{3+x^2}$ at the point $(1, 2)$ is

(A) $y = \frac{3}{2}x - \frac{1}{2}$ (B) $y = 2x + \frac{1}{2}$ (C) $y = \frac{5}{2}x - \frac{1}{2}$ (D) $y = \frac{5}{2}x + \frac{1}{2}$

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2. Which of the following is an equation of the line tangent to the graph of $f(x) = x^2 - x$ at the point where $f'(x) = 3$?

(A) $y = 3x - 2$

(B) $y = 3x + 2$

(C) $y = 3x - 4$

(D) $y = 3x + 4$

---

3. A curve has slope $2x + x^{-2}$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 3)$?

(A) $y = 2x^2 + \frac{1}{x}$

(B) $y = x^2 - \frac{1}{x} + 3$

(C) $y = x^2 + \frac{1}{x} + 1$

(D) $y = x^2 - \frac{2}{x^2} + 4$

4. An equation of the line normal to the graph of $y = \tan x$, at the point $(\frac{\pi}{6}, \frac{1}{\sqrt{3}})$ is

(A) $y - \frac{1}{\sqrt{3}} = -\frac{1}{4}(x - \frac{\pi}{6})$

(B) $y - \frac{1}{\sqrt{3}} = \frac{1}{4}(x - \frac{\pi}{6})$

(C) $y - \frac{1}{\sqrt{3}} = -\frac{3}{4}(x - \frac{\pi}{6})$

(D) $y - \frac{1}{\sqrt{3}} = \frac{3}{4}(x - \frac{\pi}{6})$

---

5. If $2x + 3y = 4$ is an equation of the line normal to the graph of f at the point $(-1, 2)$, then $f'(-1) =$

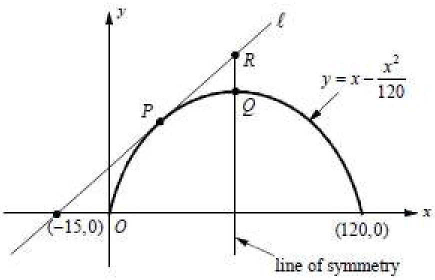
(A) $-\frac{2}{3}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $\frac{3}{2}$

---

6. If $2x - y = k$ is an equation of the line normal to the graph of $f(x) = x^4 - x$, then $k =$

(A) $\frac{23}{16}$ (B) $\frac{13}{18}$ (C) $\frac{15}{16}$ (D) $\frac{9}{8}$

Free Response Questions



7. Line ℓ is tangent to the graph of $y = x - \frac{x^2}{120}$ at the point P and intersects x-axis at $(-15, 0)$ as shown in the figure above.

(a) Find the x-coordinates of point P.

(b) Write an equation for line ℓ.

(c) If the line of symmetry for the curve $y = x - \frac{x^2}{120}$ intersects line ℓ at point R, what is the length of $\overline{QR}$?

Exercises - Implicit Differentiation

Multiple Choice Questions

1. If $3xy + x^2 - 2y^2 = 2$, then the value of $\frac{dy}{dx}$ at the point $(1,1)$ is

(A) 5 (B) $\frac{7}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{7}{2}$

---

2. If $3x^4 - x^2 - y^2 = 0$, then the value of $\frac{dy}{dx}$ at the point $(1,\sqrt{2})$ is

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{3\sqrt{2}}{2}$ (C) $\frac{5\sqrt{2}}{2}$ (D) $\frac{7\sqrt{2}}{2}$

---

3. If $x^2 y + 2xy^2 = 5x$, then $\frac{dy}{dx} =$

(A) $\frac{5 - 4xy - 4y}{x^2 + 4xy}$

(B) $\frac{5 - 2xy - 2y^2}{x^2 + 4xy}$

(C) $\frac{5 - 2xy - y^2}{x^2 + 2xy}$

(D) $\frac{5 - xy - 2y}{x^2 - 2xy}$

4. If $xy + \tan(xy) = \pi$, then $\dfrac{dy}{dx} =$

(A) $-y\sec^2(xy)$ (B) $-y\cos^2(xy)$ (C) $-x\sec^2(xy)$ (D) $-\dfrac{y}{x}$

---

5. An equation of the line tangent to the graph of $3y^2 - x^3 - xy^2 = 7$ at the point $(1, 2)$ is

(A) $y = \dfrac{3}{4}x - \dfrac{3}{8}$ (B) $y = \dfrac{3}{4}x + \dfrac{1}{2}$ (C) $y = -\dfrac{7}{8}x + \dfrac{3}{2}$ (D) $y = \dfrac{7}{8}x + \dfrac{9}{8}$

---

6. An equation of the line normal to the graph of $2x^2 + 3y^2 = 5$ at the point $(1,1)$ is

(A) $y = \dfrac{3}{2}x + 1$ (B) $y = \dfrac{3}{2}x - \dfrac{1}{2}$ (C) $y = -\dfrac{2}{3}x + \dfrac{5}{3}$ (D) $y = -\dfrac{2}{3}x + \dfrac{3}{2}$

---

7. If $x + \sin y = y + 3$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{-\sin y}{(1-\cos y)^2}$ (B) $\dfrac{-\sin y}{(1+\cos y)^2}$ (C) $\dfrac{-\sin y}{(1-\cos y)^3}$ (D) $\dfrac{-\sin y}{(1+\cos y)^3}$

Free Response Questions

8. Consider the curve given by $x^3 - xy + y^2 = 3$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is horizontal.

---

9. Consider the curve $x^2 + y^2 - xy = 7$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find all points on the curve whose x-coordinate is 2, and write an equation for the tangent line at each of these points.

   (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Exercises - Derivatives of an Inverse Function

Multiple Choice Questions

1. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(3)=4$ and $f'(4)=\frac{3}{2}$, then $g'(3)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

---

2. If $f(-3)=2$ and $f'(-3)=\frac{3}{4}$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{4}$

---

3. If $f(x)=x^3-x+2$, then $(f^{-1})'(2)=$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 4 (D) 6

---

4. If $f(x)=\sin x$, then $(f^{-1})'(\frac{\sqrt{3}}{2})=$

(A) $\frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 2

5. If $f(x) = 1 + \ln x$, then $(f^{-1})'(2) =$

(A) $-\frac{1}{e}$ (B) $\frac{1}{e}$ (C) $-e$ (D) e

Free Response Questions

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	3	-2	2	6
0	-2	-1	0	-3
1	0	1	-1	2
2	-1	4	3	-1

6. The functions f and g are differentiable for all real numbers. The table above gives the values of the functions and their first derivatives at selected values of x.

(a) If f^{-1} is the inverse function of f, write an equation for the line tangent to the graph of $y = f^{-1}(x)$ at $x = -1$.

(b) Let h be the function given by $h(x) = f(g(x))$. Find $h(1)$ and $h'(1)$.

(c) Find $(h^{-1})'(3)$, if h^{-1} is the inverse function of h.

Exercises - Derivatives of Inverse Trigonometric Functions

Multiple Choice Questions

1. $\frac{d}{dx}(\arcsin x^2) =$

(A) $-\frac{2x}{\sqrt{1-x^2}}$ (B) $\frac{2x}{\sqrt{x^2-1}}$ (C) $\frac{2x}{\sqrt{x^4-1}}$ (D) $\frac{2x}{\sqrt{1-x^4}}$

---

2. If $f(x) = \arctan(e^{-x})$, then $f'(-1) =$

(A) $\frac{-e}{1+e}$ (B) $\frac{e}{1+e}$ (C) $\frac{-e}{1+e^2}$ (D) $\frac{-1}{1+e^2}$

---

3. If $f(x) = \arctan(\sin x)$, then $f'(\frac{\pi}{3}) =$

(A) $\frac{2}{7}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{\sqrt{3}}{3}$

4. If $y = \cos(\sin^{-1} x)$, then $y' =$

(A) $-\frac{1}{\sqrt{1-x^2}}$ (B) $-\frac{x}{\sqrt{1-x^2}}$ (C) $\frac{2x}{\sqrt{1-x^2}}$ (D) $-\frac{2x}{\sqrt{x^2-1}}$

Free Response Questions

5. Let f be the function given by $f(x) = x^{\tan^{-1} x}$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

Exercises - Approximating a Derivative

Multiple Choice Questions

1. Some values of differentiable function f are shown in the table below.
   What is the approximation value of $f'(3.5)$?

x	3.0	3.3	3.8	4.2	4.9
$f(x)$	21.8	26.1	32.5	38.2	48.7

(A) 8 (B) 10 (C) 13 (D) 16

Free Response Questions

Month	1	2	3	4	5	6
Temperature	−8	0	25	50	72	88

2. The normal daily maximum temperature F for a certain city is shown in the table above.

   (a) Use data in the table to find the average rate of change in temperature from $t=1$ to $t=6$.

   (b) Use data in the table to estimate the rate of change in maximum temperature at $t=4$.

   (c) The rate at which the maximum temperature changes for $1 \le t \le 6$ is modeled by $F(t)=40-52\sin(\frac{\pi t}{6}-5)$ degrees per minute. Find $F'(4)$ using the given model.

Exercises - Related Rates

Multiple Choice Questions

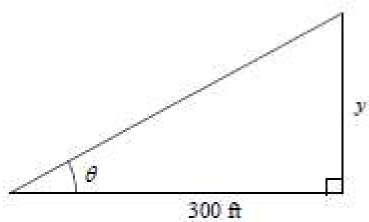
1. The radius of a circle is changing at the rate of $1/\pi$ inches per second. At what rate, in square inches per second, is the circle's area changing when $r = 5$ in ?

(A) $\frac{5}{\pi}$ (B) 10 (C) $\frac{10}{\pi}$ (D) 15

---

2. The volume of a cube is increasing at the rate of $12\ \text{in}^3/\text{min}$. How fast is the surface area increasing, in square inches per minute, when the length of an edge is 20 in?

(A) 1 (B) $\frac{6}{5}$ (C) $\frac{4}{3}$ (D) $\frac{12}{5}$



3. In the figure shown above, a hot air balloon rising straight up from the ground is tracked by a television camera 300 ft from the liftoff point. At the moment the camera's elevation angle is $\pi/6$, the balloon is rising at the rate of 80 ft/min . At what rate is the angle of elevation changing at that moment?

(A) 0.12 radian per minute
(B) 0.16 radian per minute
(C) 0.2 radian per minute
(D) 0.4 radian per minute

4. A car is approaching a right-angled intersection from the north at 70 mph and a truck is traveling to the east at 60 mph. When the car is 1.5 miles north of the intersection and the truck is 2 miles to the east, at what rate, in miles per hour, is the distance between the car and truck is changing?

(A) Decreasing 15 miles per hour

(B) Decreasing 9 miles per hour

(C) Increasing 6 miles per hour

(D) Increasing 12 miles per hour

---

5. The radius r of a sphere is increasing at a constant rate. At the time when the surface area and the radius of sphere are increasing at the same numerical rate, what is the radius of the sphere? (The surface area of a sphere is $S = 4\pi r^2$.)

(A) $\frac{1}{8\pi}$ (B) $\frac{1}{4\pi}$ (C) $\frac{1}{3\pi}$ (D) $\frac{\pi}{8}$

---

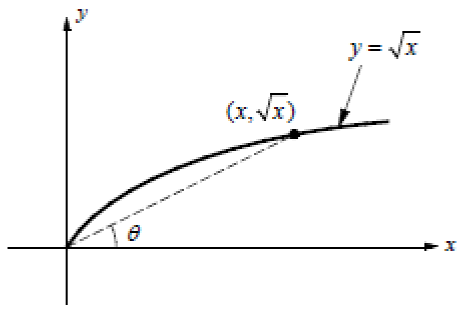
6. If the radius r of a cone is decreasing at a rate of 2 centimeters per minute while its height h is increasing at a rate of 4 centimeters per minute, which of the following must be true about the volume V of the cone? ($V = \frac{1}{3}\pi r^2 h$)

(A) V is always decreasing.

(B) V is always increasing.

(C) V is increasing only when $r > h$.

(D) V is increasing only when $r < h$.

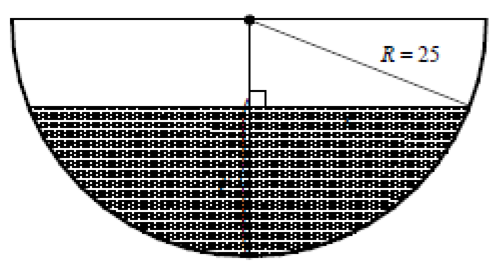


7. A particle moves along the curve $y=\sqrt{x}$. When $y=2$ the x-component of its position is increasing at the rate of 4 units per second.

(a) What is the value of $\frac{dy}{dt}$ when $y=2$?

(b) How fast is the distance from the particle to the origin changing when $y=2$?

(c) What is the value of $\frac{d\theta}{dt}$ when $y=2$?

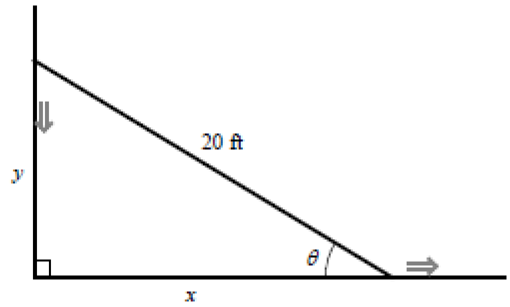


8. As shown in the figure above, water is draining at the rate of 12 ft^3/min from a hemispherical bowl of radius 25 feet. The volume of water in a hemispherical bowl of radius R when the depth of the water is y meters is given as $V=\frac{\pi}{3}y^2(3R-y)$.

(a) Find the rate at which the depth of water is decreasing when the water is 18 meters deep. Indicate units of measure.

(b) Find the radius r of the water's surface when the water is y feet deep.

(c) At what rate is the radius r changing when the water is 18 meters deep. Indicate units of measure.



9. In the figure shown above, the top of a 20-foot ladder is sliding down a vertical wall at a constant rate of 2 feet per second.

   (a) When the top of the ladder is 12 feet from the ground, how fast is the bottom of the ladder moving away from the wall?

   (b) The triangle is formed by the wall, the ladder and the ground. At what rate is the area of the triangle is changing when the top of the ladder is 12 feet from the ground?

   (c) At what rate is the angle θ between the ladder and the ground is changing when the top of the ladder is 12 feet from the ground?

---

10. Consider the curve given by $2y^2 + 3xy = 1$.

    (a) Find $\frac{dy}{dx}$.

    (b) Find all points (x, y) on the curve where the line tangent to the curve has a slope of $-\frac{3}{4}$.

    (c) Let x and y be functions of time t that are related by the equation $2y^2 + 3xy = 1$. At time $t = 3$, the value of y is 2 and $\frac{dy}{dt} = -2$. Find the value of $\frac{dx}{dt}$ at time $t = 3$.

Exercises - Position, Velocity, and Acceleration

Multiple Choice Questions

1. A particle moves along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = -\frac{1}{2}\cos t - 3t$. What is the acceleration of the particle when $t = \frac{\pi}{3}$?

   (A) $-\frac{\sqrt{3}}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{\sqrt{3}}{4}$

---

2. A point moves along the x-axis so that at any time t, its position is given by $x(t) = \sqrt{x}\ln x$. For what values of t is the particle at rest?

   (A) No values (B) $\frac{1}{e^2}$ (C) $\frac{1}{e}$ (D) e

---

3. A particle moves along the x-axis so that at any time t, its position is given by $x(t) = 3\sin t + t^2 + 7$. What is velocity of the particle when its acceleration is zero?

   (A) 1.504 (B) 1.847 (C) 2.965 (D) 3.696

---

4. Two particles start at the origin and move along the x-axis. For $0 \leq t \leq 8$, their respective position functions are given by $x_1(t) = \sin^2 t$ and $x_2(t) = e^{-t}$. For how many values of t do the particles have the same velocity?

   (A) 3 (B) 4 (C) 5 (D) 6

5. A particle moves along a line so that at time t, where $0 \le t \le 5$, its velocity is given by $v(t) = -t^3 + 6t^2 - 15t + 10$. What is the minimum acceleration of the particle on the interval?

(A) -30 (B) -15 (C) -3 (D) 0

---

6. A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = -t^3 e^{-t}$. At what value of t does v attain its minimum?

(A) $3\sqrt[3]{e}$ (B) 3 (C) 0 (D) $\sqrt[3]{e}$

---

7. The position of a particle moving along a line is given by $s(t) = t^3 - 12t^2 + 21t + 10$ for $t \ge 0$. For what value of t is the speed of the particle increasing?

(A) $1 < t < 7$ only

(B) $4 < t < 7$ only

(C) $0 < t < 1$ and $4 < t < 7$

(D) $1 < t < 4$ and $t > 7$

Free Response Questions

8. A particle moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = (t-2)^3(t-6)$.

(a) Find the velocity and acceleration of the particle at any time $t \ge 0$.

(b) Find the value of t when the particle is moving and the acceleration is zero.

(c) When is the particle moving to the right?

(d) When is the velocity of the particle decreasing?

(e) When is the speed of the particle increasing?

Exercises - Roll's Theorem and the Mean Value Theorem

Multiple Choice Questions

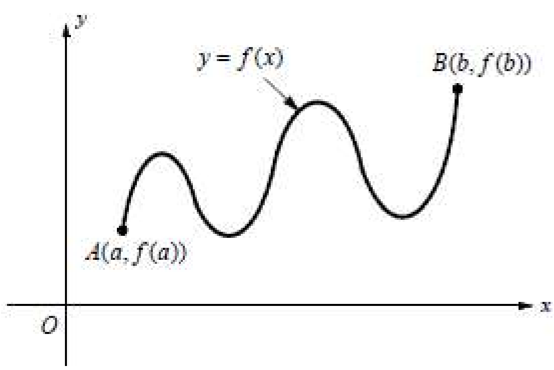
1. Let f be the function given by $f(x)=\sin(\pi x)$. What are the values of c that satisfy Roll's Theorem on the closed interval $[0,2]$?

(A) $\frac{1}{4}$ only (B) $\frac{1}{2}$ only (C) $\frac{1}{4}$ and $\frac{1}{2}$ (D) $\frac{1}{2}$ and $\frac{3}{2}$

---

2. Let f be the function given by $f(x)=-x^3+3x+2$. What are the values of c that satisfy Mean Value Theorem on the closed interval $[0,3]$?

(A) $-\sqrt{3}$ only (B) $-\sqrt{3}$ and $\sqrt{3}$ (C) $\sqrt{3}$ only (D) 1.5 and $\sqrt{3}$

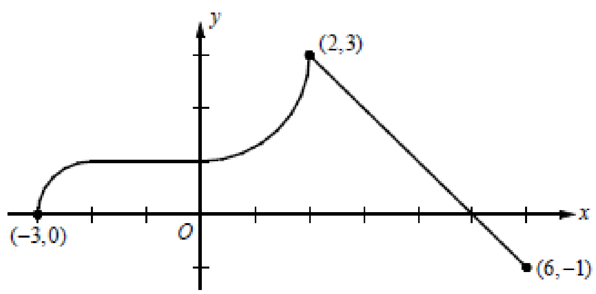


3. The figure above shows the graph of f. On the closed interval $[a,b]$, how many values of c satisfy the conclusion of the Mean Value Theorem?

(A) 2 (B) 3 (C) 4 (D) 5

4. Let f be the function given by $f(x) = \dfrac{x}{x+2}$. What are the values of c that satisfy the Mean Value Theorem on the closed interval $[-1, 2]$?

(A) -4 only (B) 0 only (C) 0 and $\dfrac{3}{2}$ (D) -4 and 0



Graph of f

5. The continuous function f is defined on the interval $-3 \le x \le 6$. The graph of f consists of two quarter circles and two line segments, as shown in the figure above. Which of the following statements must be true?

I. The average rate of change of f on the interval $-3 \le x \le 6$ is $-\dfrac{1}{9}$.

II. There is a point c on the interval $-3 < x < 6$, for which $f'(c)$ is equal to the average rate of change of f on the interval $-3 \le x \le 6$.

III. If h is the function given by $h(x) = f(\frac{1}{2}x)$, then $h'(6) = -\dfrac{1}{2}$.

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III

Free Response Questions

t (min)	0	5	10	15	20	25	30	35	40	45	50
$v(t)$ (km/min)	1.5	1.8	2.3	2.4	1.8	1.3	0.8	0.3	0	−0.4	−1.2

6. A car drives on a straight road with positive velocity $v(t)$, in kilometers per minute at time t minutes. The table above gives selected values of $v(t)$ for $0 \le t \le 50$. The function $v(t)$ is a twice-differentiable function of t.

   (a) For $0 < t < 50$, must there be a time t when $v(t) = -1$? Justify your answer.

   (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the car could equal zero in the open interval $0 < t < 50$? Justify your answer.

Exercises - The First Derivative Test and the Extreme Values of Functions

Multiple Choice Questions

1. At what values of x does $f(x)=(x-1)^3(3-x)$ have the absolute maximum?

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$

---

2. At what values of x does $f(x)=x-2x^{2/3}$ have a relative minimum?

(A) $\frac{64}{27}$ (B) $\frac{16}{9}$ (C) $\frac{4}{3}$ (D) 2

---

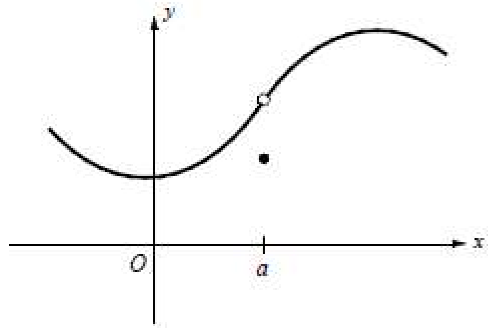
3. What is the minimum value of $f(x)=x^2\ln x$?

(A) $-e$

(B) $-\frac{1}{2e}$

(C) $-\frac{1}{e}$

(D) $-\frac{1}{\sqrt{e}}$



4. The graph of a function f is shown above. Which of the following statements about f are true?

I. $\lim_{x\to a} f(x)$ exists.
II. $x = a$ is the domain of f.
III. f has a relative minimum at $x = a$.

(A) I only
(B) I and II only
(C) I and III only
(D) I, II, and III

---

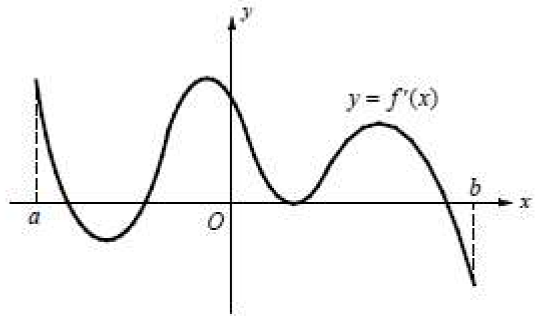
5. A polynomial $f(x)$ has a relative minimum at $(-4, 2)$, a relative maximum at $(-1, 5)$, a relative minimum at $(3, -3)$ and no other critical points. How many zeros does $f(x)$ have?

(A) one (B) two (C) three (D) four

---

6. At $x = 2$, which of the following is true of the function f defined by $f(x) = x^2e^{-x}$?

(A) f has a relative maximum.
(B) f has a relative minimum.
(C) f is increasing.
(D) f is decreasing.



7. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

(A) One relative maximum and two relative minima

(B) Two relative maxima and one relative minimum

(C) Two relative maxima and two relative minima

(D) Three relative maxima and two relative minim

---

8. The first derivative of a function f is given by $f'(x)=\dfrac{3\sin(2x)}{x^2}$. How many critical values does f have on the open interval $(0,10)$?

(A) four (B) five (C) six (D) seven

---

9. The function f is continuous on the closed interval $[-1,5]$ and differentiable on the open interval $(-1,5)$. If $f(-1)=4$ and $f(5)=-2$, which of the following statements could be false?

(A) There exist c, on $[-1,5]$, such that $f(c)\le f(x)$ for all x on the closed interval $[-1,5]$.

(B) There exist c, on $(-1,5)$, such that $f(c)=0$.

(C) There exist c, on $(-1,5)$, such that $f'(c)=0$.

(D) There exist c, on $(-1,5)$, such that $f(c)=2$.

x	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$	-1	-2	0	1	2	1	0	-2	-3	-1

10. The derivative, f', of a function f is continuous and has exactly two zeros on $[-4, 5]$. Selected values of $f'(x)$ are given in the table above. On which of the following intervals is f increasing?

(A) $-3 \leq x \leq 0$ or $4 \leq x \leq 5$

(B) $-2 \leq x \leq 0$ or $4 \leq x \leq 5$

(C) $-3 \leq x \leq 2$ only

(D) $-2 \leq x \leq 2$ only

---

11. The height h, in meters, of an object at time t is given by $h(t) = t^3 - 6t^2 + 20t$. What is the height of the object, in meters, at the instant it reaches its maximum upward velocity?

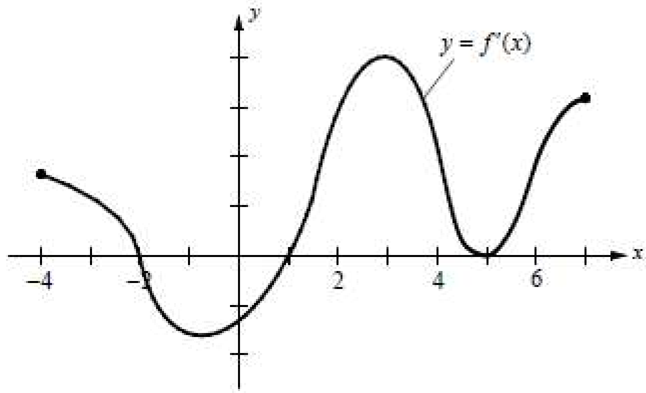
(A) 24 (B) 28 (C) 33 (D) 42

---

12. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = x^2 + c$, where c is a constant?

(A) $y = -\frac{1}{x}$ (B) $y = -x^2$ (C) $y = \frac{1}{\ln x}$ (D) $y = -\frac{1}{2}\ln x$

Free Response Questions



13. The figure above shows the graph of f', the derivative of the function f, for $-4 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -1$, $x = 3$, and $x = 5$.

(a) Find all values of x, for $-4 \le x \le 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-4 \le x \le 7$, at which f attains a relative maximum. Justify your answer.

(c) At what value of x, for $-4 \le x \le 7$, does f attain its absolute maximum. Justify your answer.

Exercises - The Second Derivative Test

Multiple Choice Questions

1. The graph of $y = x^4 - 2x^3$ has a point of inflection at

(A) $(0,0)$ only

(B) $(0,0)$ and $(1,-1)$

(C) $(1,-1)$ only

(D) $(0,0)$ and $(\frac{3}{2},-\frac{27}{16})$

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2. If the graph of $y = ax^3 - 6x^2 + bx - 4$ has a point of inflection at $(2,-2)$, what is the value of $a+b$?

(A) -2 (B) 3 (C) 6 (D) 10

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3. At what value of x does the graph of $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$ have a point of inflection?

(A) $\frac{1}{2}$ (B) 1 (C) 3 (D) $\frac{7}{2}$

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4. The graph of $y = 3x^5 - 40x^3 - 21x$ is concave up for

(A) $x < 0$

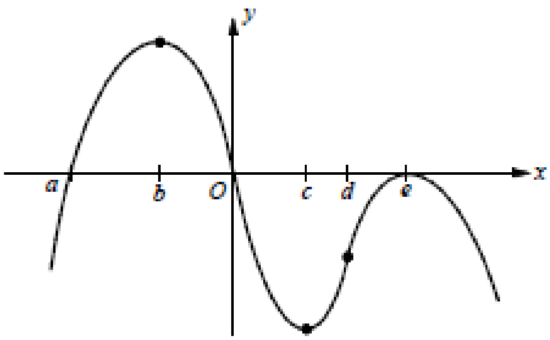
(B) $x > 2$

(C) $x < 0$ or $0 < x < 2$

(D) $-2 < x < 0$ or $x > 2$

5. Let f be a twice differentiable function such that $f(1) = 7$ and $f(3) = 12$. If $f'(x) > 0$ and $f''(x) < 0$ for all real numbers x, which of the following is a possible value for $f(5)$?

(A) 16 (B) 17 (C) 18 (D) 19



Graph of f''

6. The second derivative of the function f is given by $f''(x) = x(x+a)(x-e)^2$ and the graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) b and c (B) b, c and e (C) b, c and d (D) a and 0

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7. The first derivative of the function f is given by $f'(x) = (x^3 + 2)\, e^x$. What is the x-coordinate of the inflection point of the graph of f?

(A) −3.196 (B) −1.260 (C) −1 (D) 0

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8. Let f be a twice differentiable function with $f'(x) > 0$ and $f''(x) > 0$ for all x, in the closed interval $[2, 8]$. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	−1
4	3
6	6
8	8

(B)

x	$f(x)$
2	−1
4	2
6	5
8	8

(C)

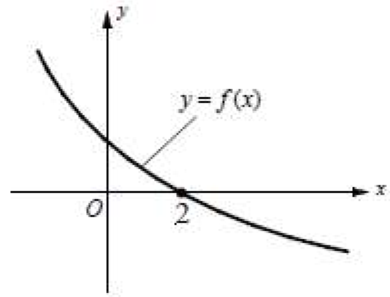
x	$f(x)$
2	−1
4	1
6	4
8	8

(D)

x	$f(x)$
2	8
4	4
6	1
8	−1

9. Let f be the function given by $f(x) = 3\sin(\frac{2x}{3}) - 4\cos(\frac{3x}{4})$. For $0 \le x \le 7$, f is increasing most rapidly when $x =$

(A) 0.823 (B) 1.424 (C) 1.571 (D) 3.206



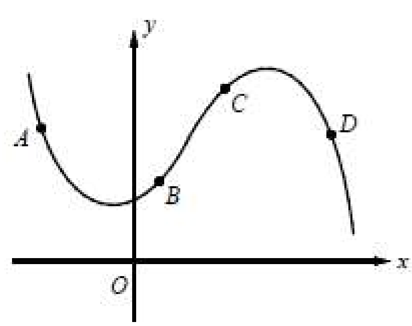
10. The graph of a twice differentiable function f is shown in the figure above. Which of the following is true?

(A) $f''(2) < f(2) < f'(2)$

(B) $f'(2) < f''(2) < f(2)$

(C) $f'(2) < f(2) < f''(2)$

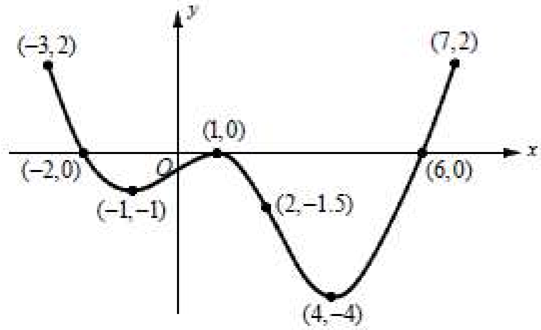
(D) $f(2) < f'(2) < f''(2)$



11. At which of the five points on the graph in the figure above is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$?

(A) A (B) B (C) C (D) D

Free Response Questions



Graph of f'

12. The figure above shows the graph of f', the derivative of the function f, on the closed interval $[-3, 7]$. The graph of f' has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 4$. The function f is twice differentiable and $f(-2) = \frac{1}{2}$.

(a) Find the x-coordinates of each of the points of inflection of the graph of f. Justify your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $[-3, 7]$.

(c) Let h be the function defined by $h(x) = x^2 f(x)$. Find an equation for the line tangent to the graph of h at $x = -2$.

13. Let f be a twice differentiable function with $f(1)=-1$, $f'(1)=2$, and $f''(1)=0$. Let g be a function whose derivative is given by $g'(x)=x^2\left[2f(x)+f'(x)\right]$ for all x.

(a) Write an equation for the line tangent to the graph of f at $x=1$.

(b) Does the graph of f have a point of inflection when $x=1$? Explain.

(c) Given that $g(1)=3$, write an equation for the line tangent to the graph of g at $x=1$.

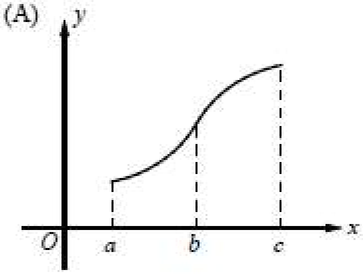
(d) Show that $g''(x)=4xf(x)+2x(x+1)f'(x)+x^2f''(x)$. Does g have a local maximum or minimum at $x=1$? Explain your reasoning.

Exercises - Curves of f , f', f'' and Curve Sketching

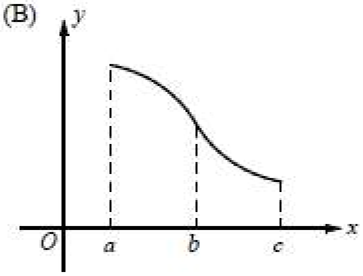
Multiple Choice Questions

1. If f is a function such that $f' > 0$ for $a < x < c$, $f'' < 0$ for $a < x < b$, and $f'' > 0$ for $b < x < c$ which of the following could be the graph of f ?

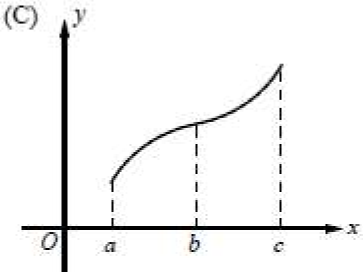
(A)



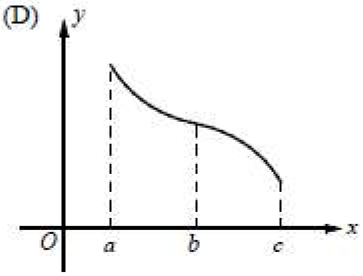
(B)



(C)



(D)



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2. The graph of $f(x) = xe^{-x^2}$ is symmetric about which of the following

I. The x-axis

II. The y-axis

III. The origin

(A) I only (B) II only (C) III only (D) II and III only

3. Let f be the function given by $f(x)=\dfrac{-3x^2}{\sqrt{3x^4+1}}$. Which of the following is the equation of horizontal asymptote of the graph of f ?

(A) $y=-3$ (B) $y=-\sqrt{3}$ (C) $y=\sqrt{3}$ (D) $y=3$

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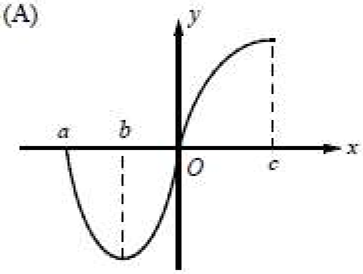
4. Let f be a function that is continuous on the closed interval $[a,c]$, such that the derivative of function f has the properties indicated on the table below.

x	$a<x<b$	b	$b<x<0$	0	$0<x<c$
$f'(x)$	$-$	0	$+$	3	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$

Which of the following could be the graph of f ?

(A)



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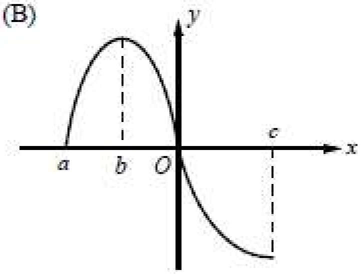
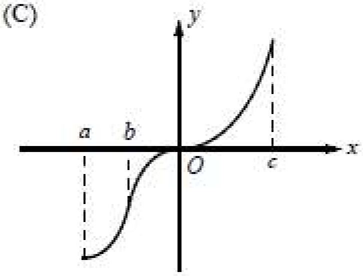
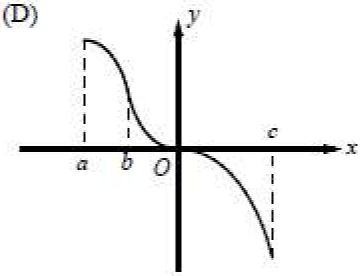
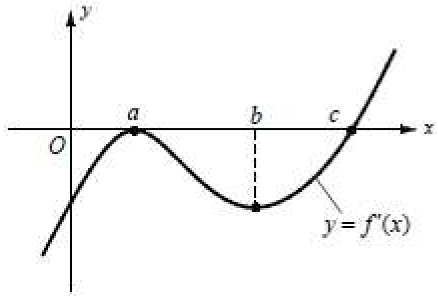


(C)



(D)





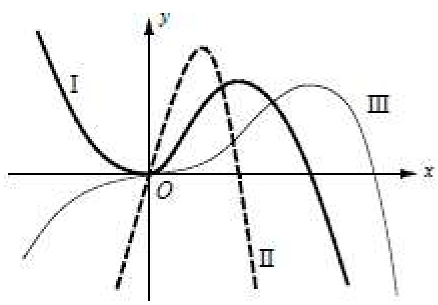
5. The graph of f', the derivative of function f, is shown above. If f is a twice differentiable function, which of the following statements must be true?

I. $f(c) > f(a)$

II. The graph of f is concave up on the interval $b < x < c$.

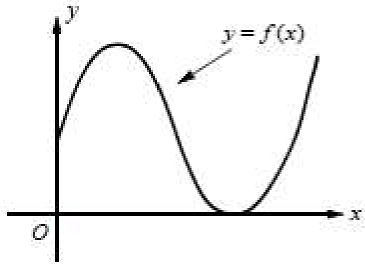
III. f has a relative minimum at $x = c$.

(A) I only (B) II only (C) III only (D) II and III only



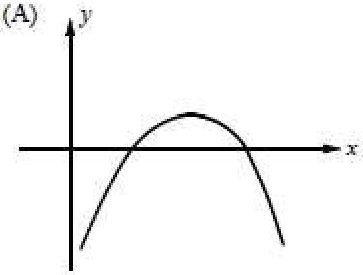
6. Three graphs labeled I, II, and III are shown above. They are the graphs of f, f', and f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	III	I	II
(D)	I	III	II

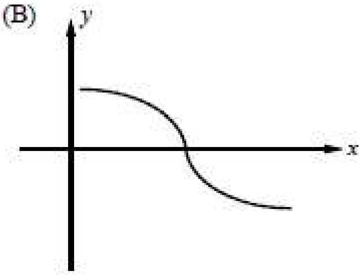


7. The graph of f is shown in the figure above. Which of the following could be the graph of f'?

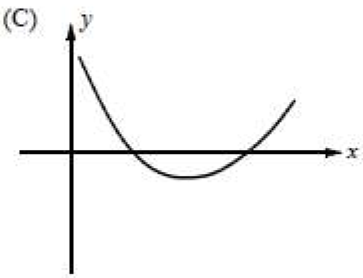
(A)



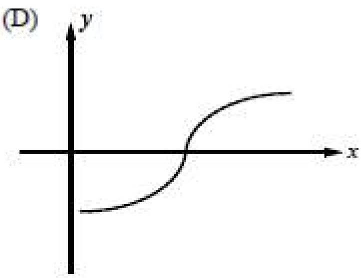
(B)



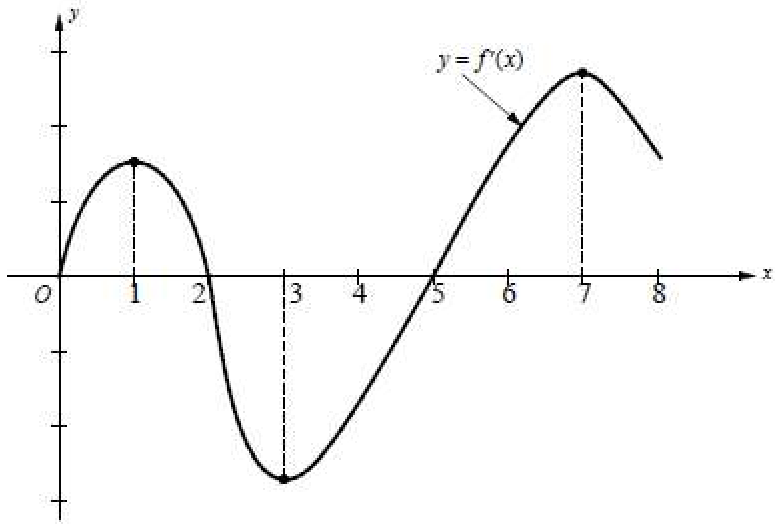
(C)



(D)



Free Response Questions



8. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $0 \le x \le 8$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) On what intervals is f increasing?

(c) On what intervals is f concave upward?

(d) For what values of x does the graph of f have a relative maximum?

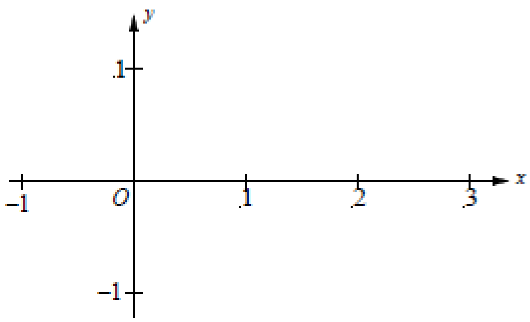
(e) Find the x-coordinate of each inflection point on the graph of f.

x	-1	$-1 < x < 0$	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	+	0	−	−1	−	0	+
$f'(x)$	−4	−	0	−	DNE	+	1	+
$f''(x)$	2	+	0	−	DNE	−	0	+

9. Let f be a function that is continuous on the interval $-1 \le x < 3$. The function is twice differentiable except at $x = 1$. The function f and its derivatives have the properties indicated in the table above.

(a) For $-1 < x < 3$, find all values of x at which f has a relative extrema. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axis provided, sketch the graph of a function that has all the given characteristics of f.



(c) Let h be the function defined by $h'(x) = f(x)$ on the open interval $-1 < x < 3$. For $-1 < x < 3$, find all values of x at which h has a relative extremum. Determine whether h has a relative maximum or a relative minimum at each of these values. Justify your answer.

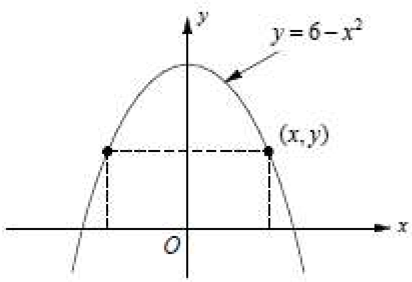
(d) For the function h, find all values of x, for $-1 < x < 3$, at which h has a point of inflection. Justify your answer.

Exercises - Optimization Problems

Multiple Choice Questions

1. The point on the curve $y = 2 - x^2$ nearest to $(3, 2)$ is

(A) $(0, 2)$ (B) $(\frac{1}{2}, \frac{7}{4})$ (C) $(\frac{3}{4}, \frac{23}{16})$ (D) $(1, 1)$



2. What is the area of the largest rectangle that has its base on the x-axis and its other two vertices on the parabola $y = 6 - x^2$?

(A) $8\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{3}$ (D) $3\sqrt{2}$

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3. If $y = \frac{1}{\sqrt{x}} - \sqrt{x}$, what is the maximum value of the product of xy ?

(A) $\frac{1}{9}$ (B) $\frac{\sqrt{3}}{9}$ (C) $\frac{2\sqrt{3}}{9}$ (D) $\frac{2}{3}$

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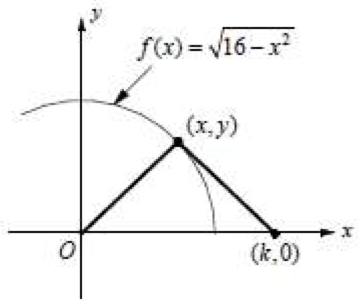
4. If the maximum value of the function $y = \frac{\cos x - m}{\sin x}$ is at $x = \frac{\pi}{4}$, what the value of m ?

(A) $-\sqrt{2}$ (B) $\sqrt{2}$ (C) -1 (D) 1

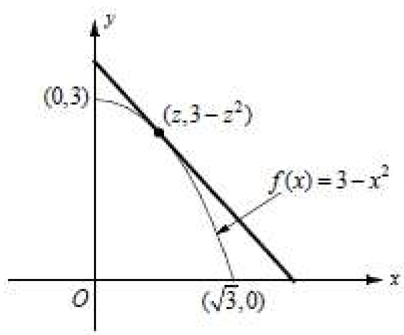
Free Response Questions



5. Let $f(x)=\sqrt{16-x^2}$. An isosceles triangle, whose base is the line segment from $(0,0)$ to $(k,0)$, where $k>0$, has its vertex on the graph of f as shown in the figure above.

(a) Find the area of the triangle in terms of k.

(b) For what values of k does the triangle have a maximum area?



6. The figure above shows the graph of the function $f(x)=3-x^2$. For $0<z<\sqrt{3}$, let $A(z)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(z,3-z^2)$.

(a) Find the equation of the line tangent to the graph of f at the point $(z,3-z^2)$.

(b) For what values of z does the triangle bounded by the coordinate axis and tangent line have a minimum area?

Exercises - Tangent Line Approximation and Differentials

Multiple Choice Questions

1. For small values of h, the function $h(x)=\sqrt[3]{8+h}$ is best approximated by which of the following?

(A) $\frac{h}{12}$ (B) $2-\frac{h}{12}$ (C) $2+\frac{h}{12}$ (D) $3+\frac{h}{12}$

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2. The approximate value of $y=\sqrt{1-\sin x}$ at $x=-0.1$, obtained from the line tangent to the graph at $x=0$, is

(A) 0.9 (B) 0.95 (C) 1.01 (D) 1.05

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3. Let $y=x^2\ln x$. When $x=e$ and $dx=0.1$, the value of dy is

(A) $\frac{e}{10}$ (B) $\frac{e}{5}$ (C) $\frac{3e}{10}$ (D) $\frac{2e}{5}$

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4. Let f be a differentiable function such that $f(2)=\frac{5}{2}$ and $f'(2)=\frac{1}{2}$. If the line tangent to the graph of f at $x=2$ is used to find an approximation of a zero of f, that approximation is

(A) -3 (B) -2.4 (C) -1.8 (D) -1.2

5. The approximate value of $y=\dfrac{1}{\sqrt{x}}$ at $x=4.1$, obtained from the line tangent to the graph at $x=4$ is

(A) $\dfrac{39}{80}$ (B) $\dfrac{79}{160}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{81}{160}$

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6. Let f be the function given by $f(x)=x^2-4x+5$. If the line tangent to the graph of f at $x=1$ is used to find an approximate value of f, which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 1.5 (B) 1.6 (C) 1.7 (D) 1.8

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7. The linear approximation to the function f at $x=a$ is $y=\dfrac{1}{2}x-3$. What is the value of $f(a)+f'(a)$ in terms of a?

(A) $a-4$

(B) $a-\dfrac{5}{2}$

(C) $\dfrac{1}{2}a-4$

(D) $\dfrac{1}{2}a-\dfrac{5}{2}$

<u>Free Response Questions</u>

8. Let f be the function given by $f(x)=\dfrac{2}{e^{\sin x}+1}$.

(a) Write an equation for the line tangent to the graph of f at $x=0$.

(b) Using the tangent line to the graph of f at $x=0$, approximate $f(0.1)$.

(c) Find $f^{-1}(x)$.

x	-2	0	1	3	6
$f(x)$	-1	-4	-3	0	7

9. Let f be a twice differentiable function such that $f'(3) = \frac{9}{5}$. The table above gives values of f for selected points in the closed interval $-2 \le x \le 6$.

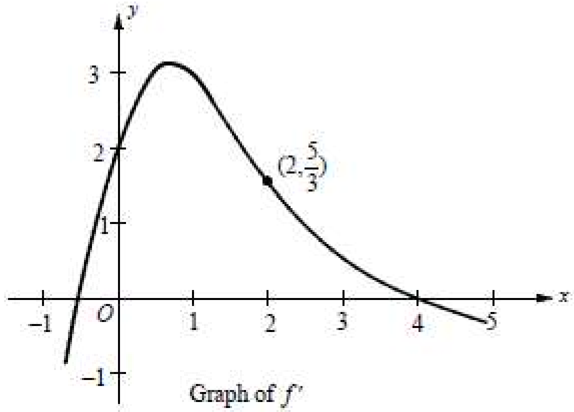
(a) Estimate $f'(0)$. Show the work that leads to your answer.

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

(c) Write an equation of the secant line for the graph of f on $1 \le x \le 6$.

(d) Suppose $f''(x) > 0$ for all x in the closed interval $1 \le x \le 6$. Use the line tangent to the graph of f at $x = 3$ to show $f(5) \ge \frac{18}{5}$.

(e) Suppose $f''(x) > 0$ for all x in the closed interval $1 \le x \le 6$. Use the secant line for the graph of f on $1 \le x \le 6$ to show $f(5) \le 5$.



Graph of f'

10. Let f be twice differentiable function on the interval $-1 < x < 5$ with $f(1) = 0$ and $f(2) = 3$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -0.5$ and $x = 4$. Let h be the function given by $h(x) = f(\sqrt{x+1})$.

(a) Write an equation for the line tangent to the graph of h at $x = 3$.

(b) The second derivative of h is $h''(x) = \frac{1}{4}\left[\frac{\sqrt{x+1}f''(\sqrt{x+1}) - f'(\sqrt{x+1})}{(x+1)^{3/2}}\right]$. Is $h''(3)$ positive, negative, or zero? Justify your answer.

(c) Suppose $h''(x) < 0$ for all x in the closed interval $0 \le x \le 3$. Use the line tangent to the graph of h at $x = 3$ to show $h(2) \le \frac{31}{12}$. Use the secant line for the graph of h on $0 \le x \le 3$ to show $h(2) \ge 2$.