TO: AP Calculus AB Students – 2018-2019
FROM: Mrs. Parham
SUBJECT: Summer Assignment
DATE: May 30, 2018

Welcome to AP Calculus AB! ***You are expected to complete the attached homework assignment during the summer.*** This is because of class time constraints and the amount of material that must be covered to properly prepare for the AP Calculus AB exam. I will check the assignment, and we will spend a few days reviewing the material at the beginning of the school year.

DUE DATE: On the Friday of the first week of school. Expect to work a minimum of about 8 hours on this assignment – plan accordingly!

SUMMER ASSIGNMENT: This assignment is challenging. You are being asked to review all of the algebra, geometry, and trigonometry skills that you have ever learned – and that's a lot! Please don't get too discouraged if you have some trouble; it is to be expected.

GRAPHING CALCULATORS: Are very helpful in Calculus. I will have TI N-spires available to use in class. TI-83/84's are usually the most popular and easiest to use. If you have one, please bring it, if not consider getting one. These are also used in college.

NOTEBOOK: You are required to keep up with your work and notes in whatever form works for you. We will pull from previous lessons as the year goes on. Please have pencils and paper every day. I expect high quality work!

TRIGONOMETRY: Trig functions are used extensively in calculus. You must have *memorized* exact values of sin(x), cos(x), and tan(x) for $x = 0, \pi/6, \pi/4, \pi/3, \pi/2, \pi$ and $3\pi/2$ radians, and be able to use these to find values in Quadrants II, III, and IV as well as for evaluating sec, csc, and cot functions. We will work exclusively in radians in calculus. The only trig identity you must memorize is $\sin^2 \theta + \cos^2 \theta = 1$.

This assignment is worth 100 points. I will randomly pick seven problems from the worksheet to grade on correctness and effort; I will also grade your assignment on overall neatness and thoroughness.

Please complete all work on a separate sheet of paper, and include the original problem with your work if you do not have space on the handout. All answers, except graphs and tables, should be boxed or circled. I expect high quality work!

I look forward to working with you next year, and remember ~ "Practice makes perfect."
Good Luck!

Complex Fractions

When simplifying complex fractions, there are different ways to simplify, two of which are shown below:

1. **work separately with the numerator and denominator, rewriting each with a common denominator, and then multiplying the numerator by the reciprocal of the denominator; or**
2. **multiply the entire complex fraction by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.**

Example:

1)

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7(x+1)-6}{x+1}}{\frac{5}{x+1}} = \frac{\frac{-7x-13}{x+1}}{\frac{5}{x+1}} = \frac{-7x-13}{x+1}\gamma\frac{x+1}{5} = \frac{-7x-13}{5}$$

OR:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\gamma\frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{(x-4)}}{5-\frac{1}{x-4}} = \frac{\frac{-2(x-4)+3x^2}{x(x-4)}}{\frac{5(x-4)-1}{x-4}} = \frac{3x^2-2x+8}{x(x-4)}\gamma\frac{x-4}{5x-21} = \frac{3x^2-2x+8}{(x)(5x-21)} = \frac{3x^2-2x+8}{5x^2-21x}$$

2)

OR:

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}}\gamma\frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Composite Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "***f*** of ***g*** of ***x***" means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. **Find each.**

6. $f(2) =$ ____________
7. $g(-3) =$ _____________
8. $f(t+1) =$ __________

9. $f(g(-2)) =$ __________
10. $g[f(m+2)] =$ ___________
11. $\dfrac{g(x+h) - g(x)}{h} =$ ______

Let *f(x) = sin x.* **Find each exactly**.

12. $f(\frac{\pi}{2}) =$ ________
13. $f(\frac{3\pi}{2}) =$ ________

Let $f(x) = x^2,\ g(x) = 2x + 5,\ and\ h(x) = x^2 - 1$. **Find each.**

14. $h(f(-2)) =$ _______
15. $f[g(x-1)] =$ _______
16. $g[h(x^3)] =$ _______

Find $\dfrac{f(x+h) - f(x)}{h}$ **for the given function *f.***

17. $f(x) = 5 - 2x$
18. *f(x) = 9x + 3*

Intercepts and Points of Intersection

To find the x-intercepts, also referred to as the zeros of the function, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$x-\text{int.}\ (Let\ y = 0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$y-\text{int.}\ (Let\ x = 0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

Use substitution or elimination method to solve the system of equations.
Example:

$x^2 + y^2 - 16x + 39 = 0$

$x^2 - y^2 - 9 = 0$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3\ and\ x = 5$

Plug $x = 3\ and\ x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\quad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\quad$ $16 = y^2$

$y = 0$ $\quad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ and $(3,0)$

Substitution Method
Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as previous example*)

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3\ or\ x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $\begin{array}{l} x+y=8 \\ 4x-y=7 \end{array}$

24. $\begin{array}{l} y=x^2+3x-4 \\ y=5x+11 \end{array}$.

25. $\begin{array}{l} x^2+y=6 \\ x+y=4 \end{array}$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	**Interval Notation**	**Graph**
$-2 < x \le 4$		
	$[-1,7)$	
		 **8**

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1 \ge 0$

28. $-4 \le 2x-3 < 4$

29. $\frac{x}{2}-\frac{x}{3} > 5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

34. $f(x)=\frac{4}{\sqrt{2x-5}}$

Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

35. $f(x)=2x+1$

36. $f(x)=\dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x))=g(f(x))=x$

Example:

If: $f(x)=\dfrac{x-9}{4}$ *and* $g(x)=4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x))=4\left(\frac{x-9}{4}\right)+9$$
$$=x-9+9$$
$$=x$$

$$g(f(x))=\frac{(4x+9)-9}{4}$$
$$=\frac{4x+9-9}{4}$$
$$=\frac{4x}{4}$$
$$=x$$

$f(g(x))=g(f(x))=x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

37. $f(x)=\frac{x^3}{2}$ $g(x)=\sqrt[3]{2x}$

38. $f(x)=9-x^2,\ x\ge 0$ $g(x)=\sqrt{9-x}$

Equation of a line

Slope intercept form: $y=mx+b$

Point-slope form: $y-y_1=m(x-x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

39. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

40. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

41. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

42. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

43. Find the equation of a line passing through the point (2, 8) and parallel to the line $y=\frac{5}{6}x-1$.

44. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

45. Find the equation of a line passing through the points (-3, 6) and (1, 2).

46. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

Note: In calculus, we always use radians, unless noted otherwise!

Use $\frac{180^\circ}{\pi\ radians}$ to convert from radians to degrees.

Use $\frac{\pi\ radians}{180^\circ}$ to convert from degrees to radians.

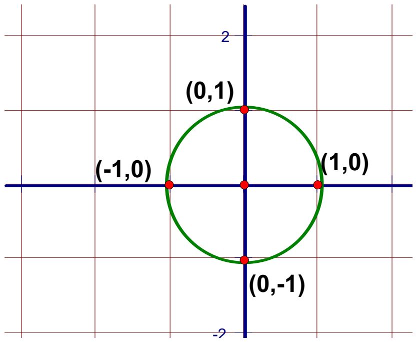
47. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

48. Convert to radians: a. $45°$ b. $-17°$ c. $237°$

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

Example: $\sin 90° = 1$ $\cos\frac{\pi}{2} = 0$



49. $a.)\ \sin 180°$ $b.)\ \cos 270°$ $c.)\ \sin(-90°)$ $d.)\ \sin\pi$ $e.)\ \cos 360°$ $f.)\ \cos(-\pi)$

50. Without a calculator, determine the exact value of each expression.

a) $\sin 0$ b) $\sin\frac{\pi}{2}$ c) $\sin\frac{3\pi}{4}$ d) $\cos\pi$ e) $\cos\frac{\pi}{3}$ f) $\cos\frac{3\pi}{4}$

g) $\tan\frac{7\pi}{4}$ h) $\tan\frac{\pi}{6}$ i) $\tan\frac{2\pi}{3}$ j) $\sec\frac{\pi}{3}$ k) $\csc\frac{5\pi}{4}$ l) $\cot\frac{\pi}{2}$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$.

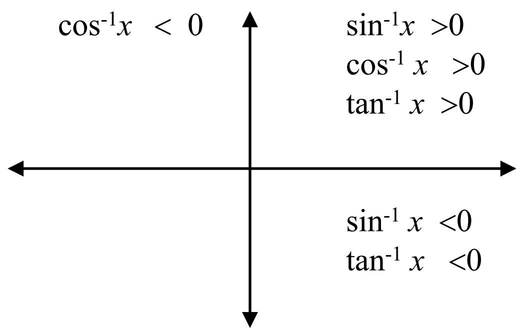
51. $\sin x = -\frac{1}{2}$ 52. $2\cos x = \sqrt{3}$ 53. $\sin^2 x = \frac{1}{2}$ 54. $2\cos^2 x - 1 - \cos x = 0$ 55. $4\cos^2 x - 3 = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

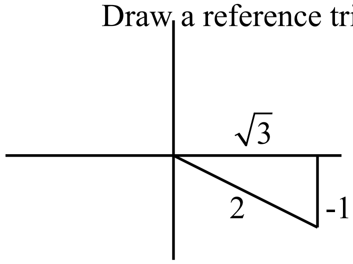
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan \frac{-1}{\sqrt{3}}$

Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = $-\frac{\pi}{6}$ so that it falls in the interval from $\frac{-\pi}{2} < y < \frac{\pi}{2}$

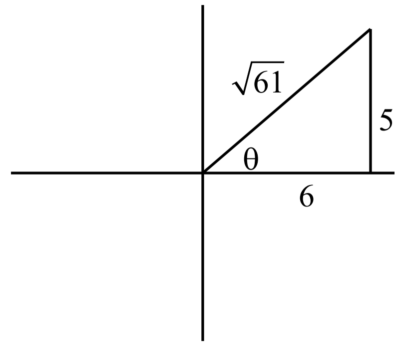
Answer: y = $-\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

56. y = $\arcsin\frac{1}{2}$ (or $\sin^{-1}\frac{1}{2}$) 57. y = $\arctan 1$ 58. $y = \arcsin\frac{-\sqrt{3}}{2}$ 59. $y = \arccos(-1)$

Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$



Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Theorem.

Find the ratio of the cosine of the reference triangle. $\cos\theta = \dfrac{6}{\sqrt{61}}$

For each of the following find the value without a calculator.

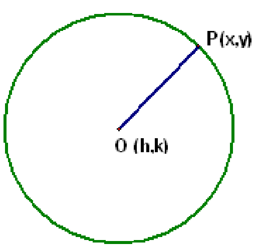
60. $\tan\left(\arccos\frac{2}{3}\right)$

61. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

62. $\sin\left(\arctan\frac{12}{5}\right)$

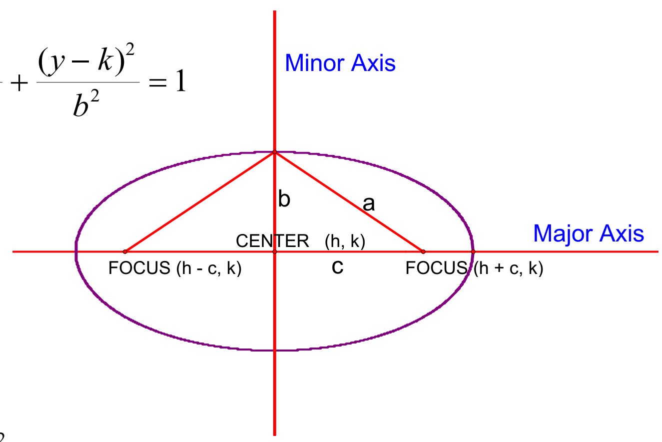
63. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

<u>Circles and Ellipses (No problems to do; just some information for you.)</u>



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

For an ellipse centered at the origin, the equation is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the *y*-axis. For our purposes in calculus, you will not need to locate the foci.

<u>Vertical Asymptotes</u>

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. If the numerator is not equal to zero at that x-value, that is the vertical asymptote.

64. $f(x)=\dfrac{1}{x^2}$ 65. $f(x)=\dfrac{x-2}{x^2-4}$ 66. $f(x)=\dfrac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the leading coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

67. $f(x)=\dfrac{x^2-2x+1}{x^3+x-7}$ 68. $f(x)=\dfrac{5x^3-2x^2+8}{4x-3x^3+5}$ 69. $f(x)=\dfrac{4x^5}{x^2-7}$

Logarithms and Exponentials

$$y=\log_b x \text{ is equivalent to } x=b^y$$

Product property: $\log_b mn=\log_b m+\log_b n$

Quotient property: $\log_b \dfrac{m}{n}=\log_b m-\log_b n$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m=\log_b n$, then m = n

Change of base formula: $\log_a n=\dfrac{\log_b n}{\log_b a}$

$\log_b 1=0$, $\ln 1=0$, $\log_b a=1$, $\ln e=1$

Because logarithms and exponentials are inverse functions of each other:

$\log_b(b^x)=x$ $\ln(e^x)=x$ $b^{\log_b x}=x$ $e^{\ln x}=x$

70. Solve each exponential or logarithmic equation.

$a)\quad 5^x = 125 \quad b)\quad 8^{x+1} = 16^x \quad c)\quad 81^{\frac{3}{4}} = x$

$d)\quad 8^{\frac{-2}{3}} = x \quad e)\quad \log_2 32 = x \quad f)\quad \log_x \frac{1}{9} = -2$

$g)\quad \log_4 x = 3 \quad h)\quad \log_3(x+7) = \log_3(2x-1) \quad$ i) $\log x + \log(x-3) = 1$

70. Expand each of the following using the properties of logs.

$a) \log_3 5x^2 \qquad b)\quad \ln \frac{5x}{y^2}$

71. Evaluate the following expressions.

a) $e^{\ln 3}$ b) $e^{(1+\ln x)}$ c) $\ln 1$ d) $\ln e^7$ e) $\log_3(1/3)$ f) $\log_{1/2} 8$ g) $e^{3\ln x}$

Series

72. Expand and evaluate.

a) $\sum_{n=0}^{4} \frac{n^2}{2}$ b) $\sum_{n=1}^{3} \frac{1}{n^3}$

Miscellaneous

73. Simplify. Show the work that leads to your answer.

a) $\frac{x-4}{x^2-3x-4}$ b) $\frac{5-x}{x^2-25}$ c) $\frac{x^2-2x-8}{x^3+x^2-2x}$ d) $\frac{\sqrt{x}}{x}$ e) $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

f) $(5a^{2/3})(4a^{3/2})$ g) $\frac{1}{x+h} - \frac{1}{x}$ h) $\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$ i) $\frac{\frac{1}{3+x} - \frac{1}{3}}{x}$ j) $\frac{(3x)^2 - (3+x)^2}{x}$

j) $\sqrt{x} \bullet \sqrt[3]{x} \bullet x^{\frac{1}{6}}$ k) $\frac{\sin^2 x + \cos^2 x}{\cot x}$ l) $\cot x \sec x$

74. Solve for z.

a) $4x + 10yz = 0$ b) $y^2 + 3yz = 4x + 8z$

75. Expand: $(x + y)^3$