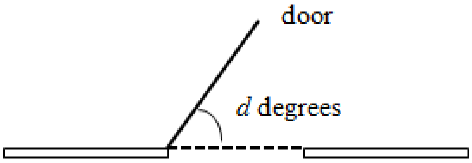
AP Calculus AB

Welcome!

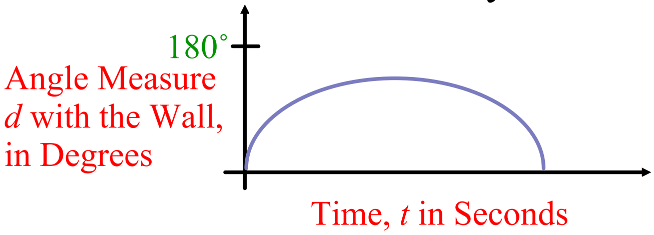
Let's start with the ...

Foerster Door Exercise

A door with a hydraulic closer opens and closes. At time t seconds, it forms an angle of $d(t)$ degrees with the wall, a

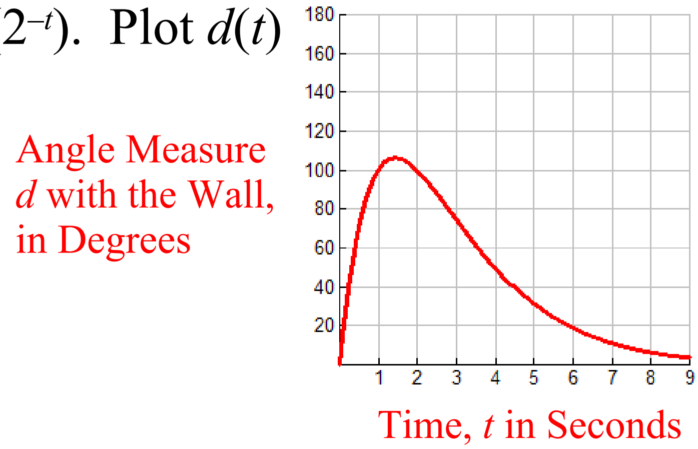


1. Sketch a reasonable graph with t as the independent variable and $d(t)$ as the dependent variable. Be sure to label your axes.



Any graph that starts at the origin, reaches a max $\leq 180^\circ$, and returns to the x-axis is acceptable.

2. Let $d(t) = 200t(2^{-t})$. Plot $d(t)$ on your calculator.



3. Make a table of values of $d(t)$ for each second from $t = 0$ to $t = 9$. Use 4 decimal places if necessary.

t	0	1	2	3	4	5	6	7	8	9
$d(t)$	0	100	100	75	50	31.25	18.75	10.9375	6.25	3.5156

4. At $t = 1$ second, does the door appear to be opening or closing? Opening

How can you tell? $d(t)$ is still increasing.

5. Find the average rate of change of d with $d(t) = 200t(2^{-t})$ for each of the following intervals:

- from $t = 1$ to 1.1 s:
- from $t = 1$ to 1.01 s:
- from $t = 1$ to 1.001 s:
- from $t = 1$ to 1.0001 s:

102.434 100 .1

```
(200*1.1*2^-1.1-200*1*2^-1
)/(1.1-1
                26.33629069
(200*1.01*2^-1.01-200*1*2^
-1)/(1.01-1
                30.23420391
(200*1.001*2^-1.001-200*1*
2^-1)/(1.001-1
                30.64000835
(200*1.0001*2^-1.0001-200*
1*2^-1)/(1.0001-1
                30.6807529
```

Calculator Tip: Use TBLSET (2ND WINDOW) to set your table to Ask so you can type in x-values and get y-values.

```
NORMAL FLOAT AUTO REAL RADIAN MP

TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

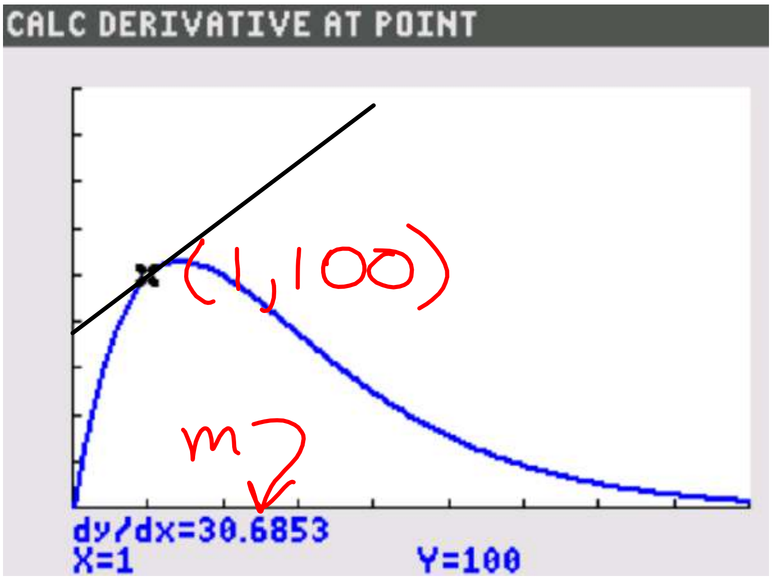
NORMAL FLOAT AUTO REAL RADIAN MP

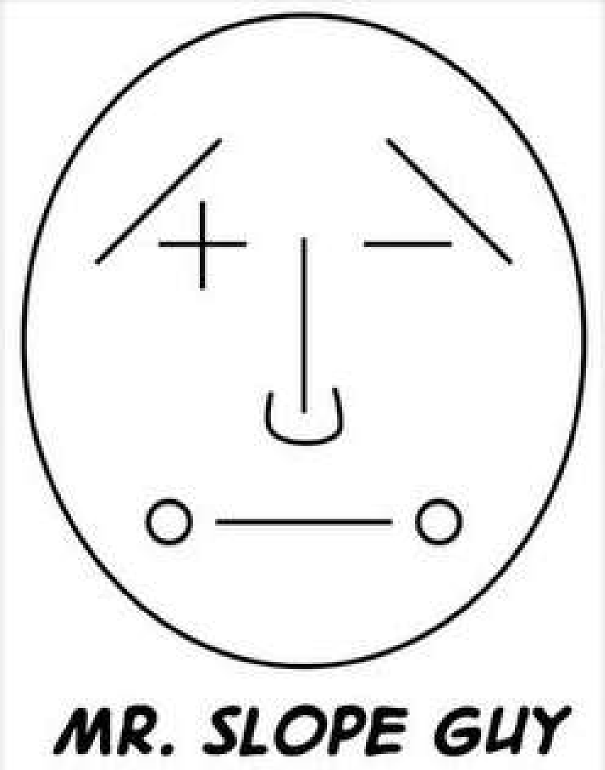
X	Y_1			
1.1	102.63			
1.01	100.3			
1.001	100.03			

6. Based on the calculations above, is the door opening or closing at $t = 1$ second? Explain.
It is opening because the instantaneous rate of change (slope) at that point approaches a positive number.

7. The instantaneous rate of change of $d(t)$ with respect to t is the limit of the average rates as the length of the time interval approaches 0. Make a conjecture about the approximate instantaneous rate of change at $t = 1$.
Conjecture: Instantaneous rate ≈ < any number slightly larger than 30.6807529 degrees/sec >. (Exact answer is about 30.68528194 deg/sec.)

The instantaneous rate of change is called the derivative of $d(t)$ with respect to t.
From the graph with window
[0, 9] by [0, 180]:
(2nd TRACE) CALC, 6: dy/dx, $x = 1$.





Review: Finding the equation of a line if you know the slope and a point on the line.

One way to do this is to substitute the slope for m and the point's x- and y-values for x and y in the slope - y-intercept equation $y = mx + b$, then solve for b and back substitute. However, in Calculus, you might find it more convenient to use the point - slope form of the equation of a line:

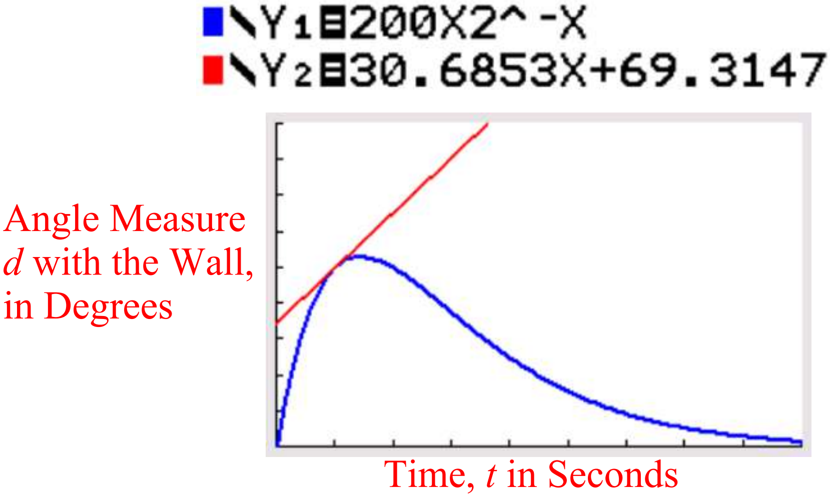
$$y - y_1 = m(x - x_1)$$

It works because $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y - y_1}{x - x_1}$.

Multiply both sides by $x - x_1$ to get point - slope form.

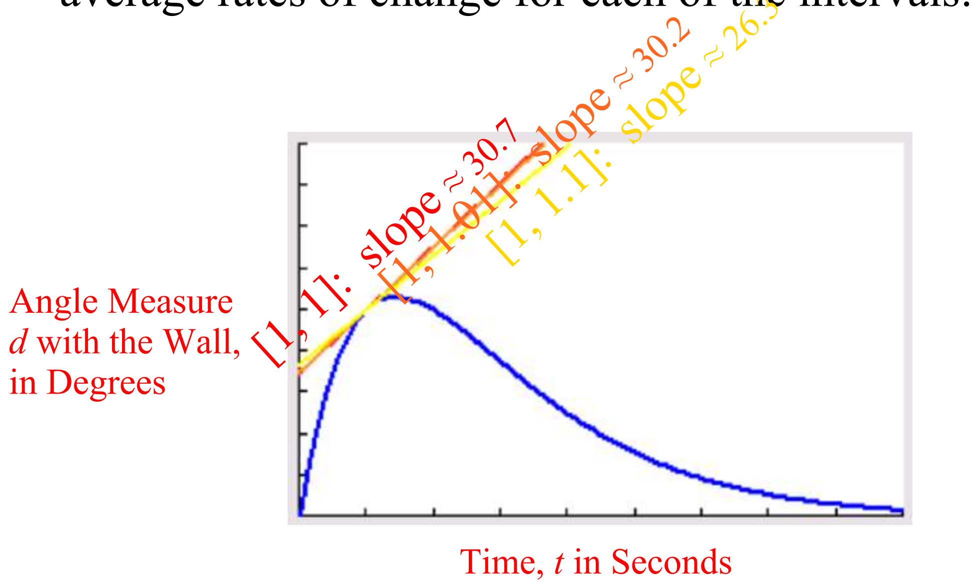
The derivative is the slope of a line tangent to the curve at *t*.

8. Use your estimate to find an equation of the line tangent to the curve at $t = 1$. Add that to your graph in # 2.



(Window: [0, 9] by [0, 180])

Let's review what we did when we found the average rates of change for each of the intervals:



(Window: [0, 9] by [0, 180])

We kept shrinking the interval till we found the instantaneous rate of change (slope) of the tangent line to the curve at $t = 1$.

What concepts did you review in this process?

- angle measures
- increasing (and decreasing) intervals
- average rates of change (slopes)
- y-intercepts
- finding the equation of a line
- graphing
- asymptotes

Let's summarize!

What is an instantaneous rate of change?

What is a limit?

What is a derivative?