AA,SSS and SAS similarity

Warm Up

Solve each proportion.

1. $\frac{6}{11} = \frac{8}{b}$

$b = \frac{44}{3}$ or $14\frac{2}{3}$

2. $\frac{5}{z} = \frac{z}{20}$

$z = \pm 10$

3. $\frac{3}{10} = \frac{6}{x+12}$

$x = 8$

4. If $\Delta QRS \sim \Delta XYZ$, identify the pairs of congruent angles and write 3 proportions using pairs of corresponding sides.

$\angle Q \cong \angle X$; $\angle R \cong \angle Y$; $\angle S \cong \angle Z$;

$\frac{QR}{XY} = \frac{RS}{YZ}$; $\frac{RS}{YZ} = \frac{QS}{XZ}$; $\frac{QS}{XZ} = \frac{QR}{XY}$

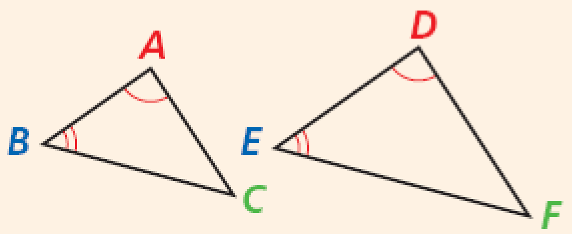
Objectives

Prove certain triangles are similar by using AA, SSS, and SAS.

Use triangle similarity to solve problems.

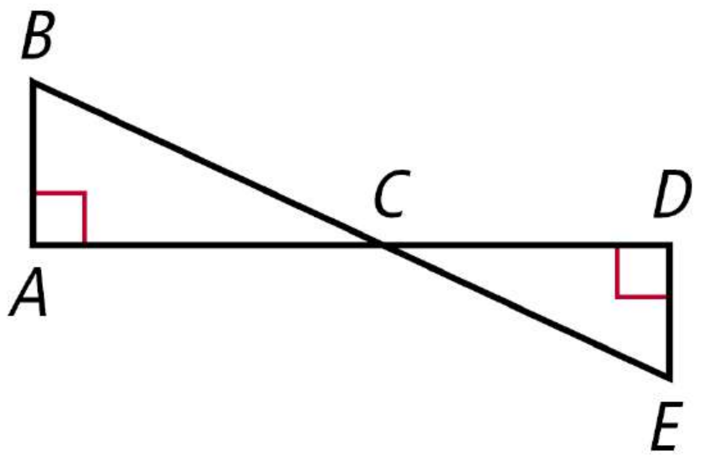
There are several ways to prove certain triangles are similar. The following postulate, as well as the SSS and SAS Similarity Theorems, will be used in proofs just as SSS, SAS, ASA, HL, and AAS were used to prove triangles congruent.

Postulate 7-3-1 Angle-Angle (AA) Similarity

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

Example 1: Using the AA Similarity Postulate

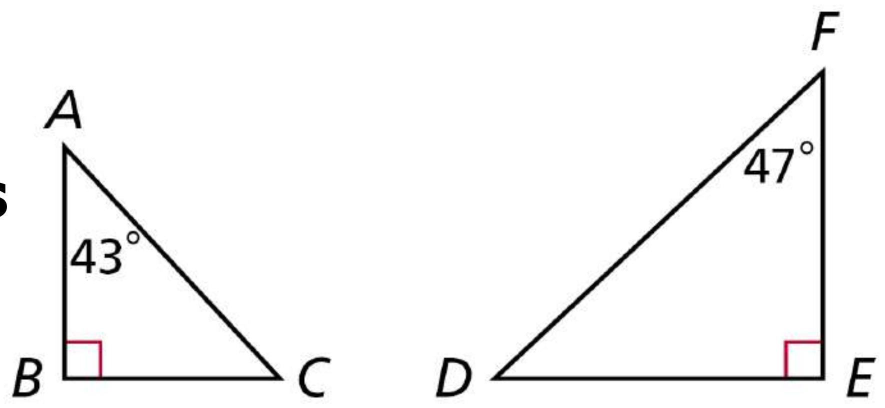
Explain why the triangles are similar and write a similarity statement.



Since $\overline{AC} \parallel \overline{DC}$, $\angle B \cong \angle E$ by the Alternate Interior Angles Theorem. Also, $\angle A \cong \angle D$ by the Right Angle Congruence Theorem. Therefore $\Delta ABC \sim \Delta DEC$ by AA~.

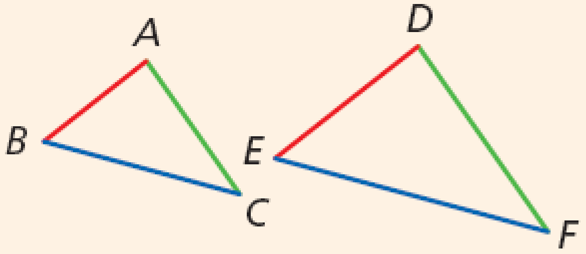
Check It Out! Example 1

Explain why the triangles are similar and write a similarity statement.

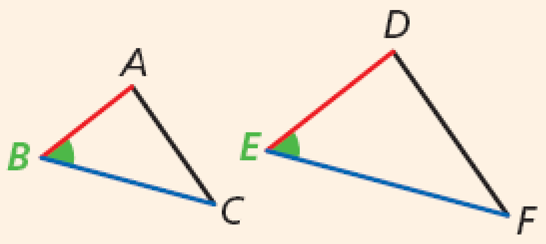


By the Triangle Sum Theorem, $m\angle C = 47°$, so $\angle C \cong \angle F$.
$\angle B \cong \angle E$ by the Right Angle Congruence Theorem.
Therefore, $\Delta ABC \sim \Delta DEF$ by AA $\sim$.

Theorem 7-3-2 Side-Side-Side (SSS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

Theorem 7-3-3 Side-Angle-Side (SAS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.	 $\angle B \cong \angle E$	$\triangle ABC \sim \triangle DEF$

Example 2A: Verifying Triangle Similarity

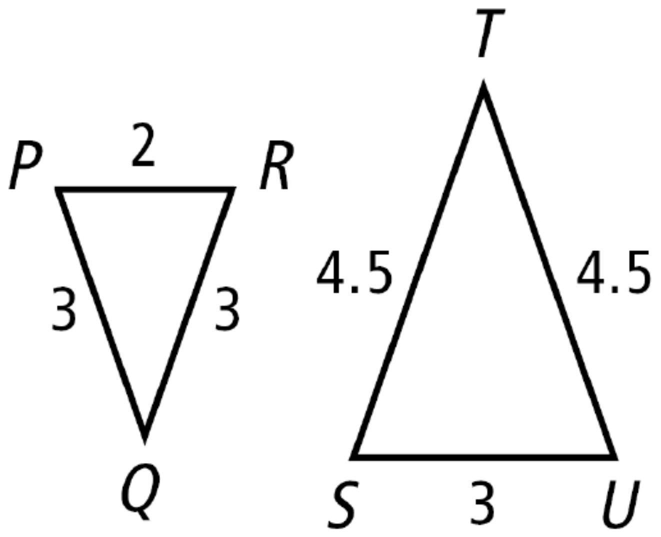
Verify that the triangles are similar.

ΔPQR and ΔSTU

$$\frac{PQ}{ST} = \frac{3}{4.5} = \frac{2}{3}$$

$$\frac{QR}{TU} = \frac{3}{4.5} = \frac{2}{3}$$

$$\frac{PR}{SU} = \frac{2}{3}$$

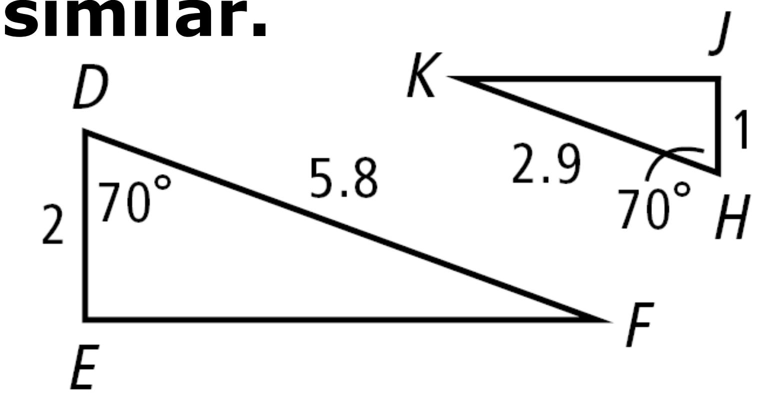


Therefore $\Delta PQR \sim \Delta STU$ by SSS $\sim$.

Example 2B: Verifying Triangle Similarity

Verify that the triangles are similar.

Δ*DEF* and Δ*HJK*



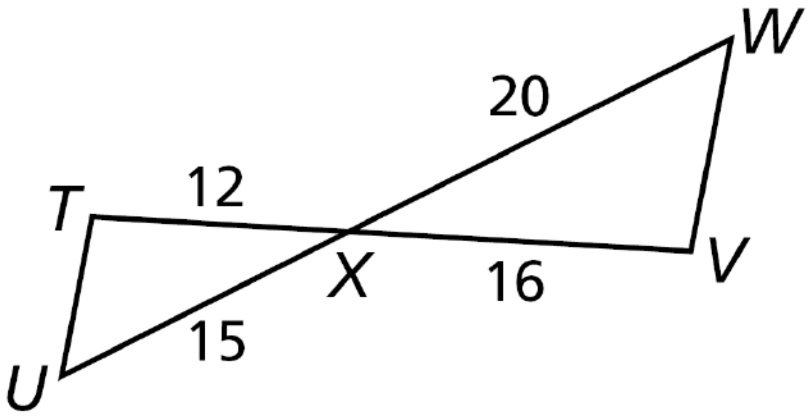
$\angle D \cong \angle H$ by the Definition of Congruent Angles.

$$\frac{DE}{HJ} = \frac{2}{1} = 2 \qquad \frac{DF}{HK} = \frac{5.8}{2.9} = 2$$

Therefore Δ*DEF* ~ Δ*HJK* by SAS ~.

Check It Out! Example 2

Verify that $\Delta TXU \sim \Delta VXW$.



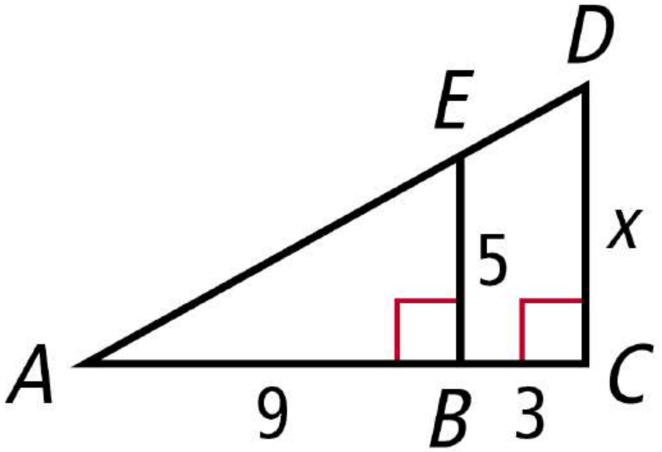
$\angle TXU \cong \angle VXW$ by the Vertical Angles Theorem.

$$\frac{TX}{VX} = \frac{12}{16} = \frac{3}{4} \qquad \frac{XU}{XW} = \frac{15}{20} = \frac{3}{4}$$

Therefore $\Delta TXU \sim \Delta VXW$ by SAS ~.

Example 3: Finding Lengths in Similar Triangles

Explain why $\Delta ABE \sim \Delta ACD$, and then find CD.



Step 1 Prove triangles are similar.

$\angle A \cong \angle A$ by Reflexive Property of $\cong$, and $\angle B \cong \angle C$ since they are both right angles.

Therefore $\Delta ABE \sim \Delta ACD$ by AA $\sim$.

Example 3 Continued

Step 2 Find *CD*.

$$\frac{CD}{BE} = \frac{CA}{BA} = \frac{CB + BA}{BA}$$ *Corr. sides are proportional. Seg. Add. Postulate.*

$$\frac{x}{5} = \frac{3+9}{9}$$ *Substitute x for CD, 5 for BE, 3 for CB, and 9 for BA.*

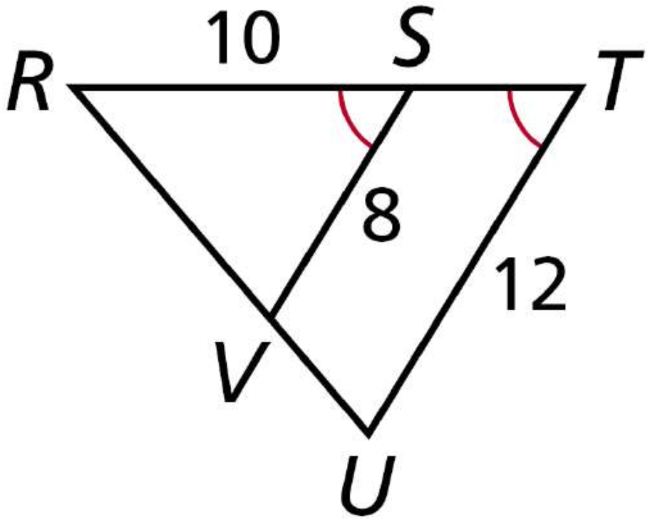
$x(9) = 5(3 + 9)$ *Cross Products Prop.*

$9x = 60$ *Simplify.*

$$x = \frac{60}{9} = 6\frac{2}{3}$$ *Divide both sides by 9.*

Check It Out! Example 3

Explain why ΔRSV ~ ΔRTU and then find RT.



Step 1 Prove triangles are similar.

It is given that ∠S ≅ ∠T.
∠R ≅ ∠R by Reflexive Property of ≅.

Therefore ΔRSV ~ ΔRTU by AA ~.

Check It Out! Example 3 Continued

Step 2 Find *RT*.

$$\frac{RT}{RS} = \frac{TU}{SV}$$ *Corr. sides are proportional.*

$$\frac{RT}{10} = \frac{12}{8}$$ *Substitute RS for 10, 12 for TU, 8 for SV.*

$$RT(8) = 10(12)$$ *Cross Products Prop.*

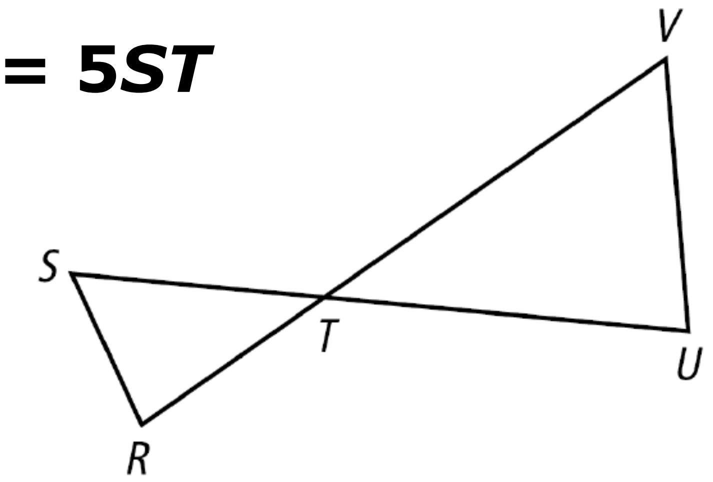
$$8RT = 120$$ *Simplify.*

$$RT = 15$$ *Divide both sides by 8.*

Example 4: Writing Proofs with Similar Triangles

Given: $3UT = 5RT$ **and** $3VT = 5ST$

Prove: $\Delta UVT \sim \Delta RST$

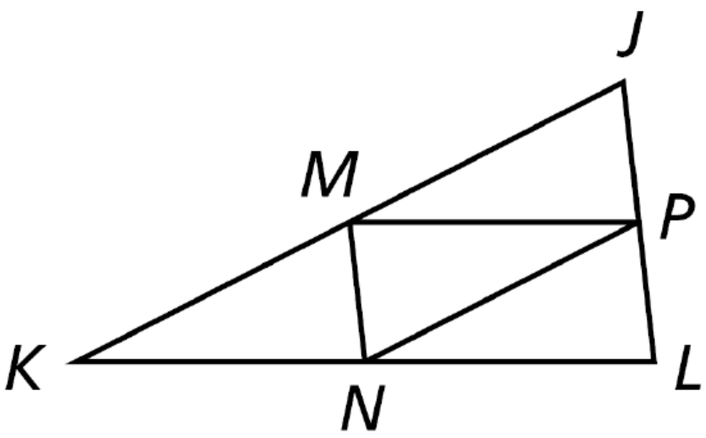


Example 4 Continued

Statements	Reasons
1. $3UT = 5RT$	**1.** Given
2. $\frac{UT}{RT} = \frac{5}{3}$	**2.** Divide both sides by $3RT$.
3. $3VT = 5ST$	**3.** Given.
4. $\frac{VT}{ST} = \frac{5}{3}$	**4.** Divide both sides by $3ST$.
5. $\angle RTS \cong \angle VTU$	**5.** Vert. $\angle$s Thm.
6. $\Delta UVT \sim \Delta RST$	**6.** SAS $\sim$ **Steps 2, 4, 5**

Check It Out! Example 4

Given: *M* is the midpoint of $\overline{JK}$. *N* is the midpoint of $\overline{KL}$, and *P* is the midpoint of $\overline{JL}$.

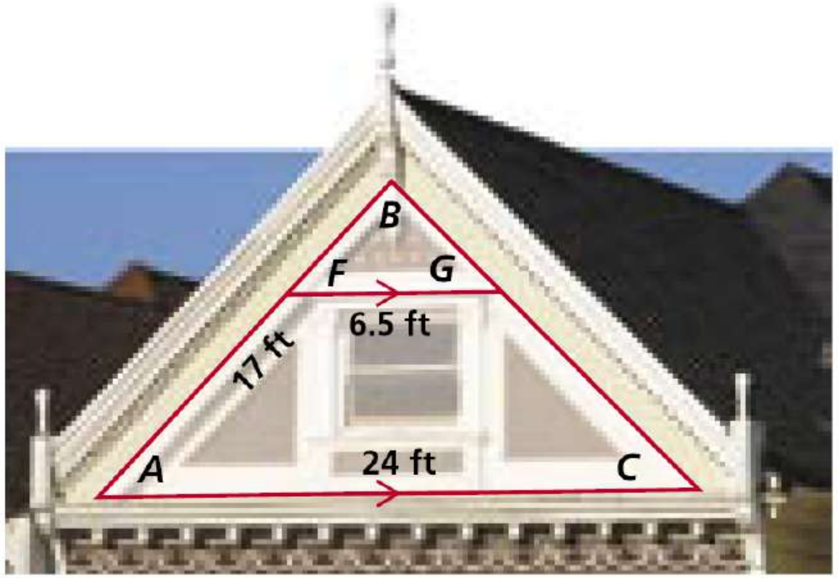


Check It Out! Example 4 Continued

Statements	Reasons
1. *M* is the mdpt. of $\overline{JK}$, *N* is the mdpt. of $\overline{KL}$, and *P* is the mdpt. of $\overline{JL}$.	**1.** Given
2. $MP = \frac{1}{2}KL,\ MN = \frac{1}{2}JL,\ NP = \frac{1}{2}KJ$	**2.** Δ Midsegs. Thm
3. $\frac{MP}{KL} = \frac{MN}{JL} = \frac{NP}{KJ} = \frac{1}{2}$	**3.** Div. Prop. of =.
4. $\Delta JKL \sim \Delta NPM$	**4.** SSS ~ **Step 3**

Example 5: Engineering Application

The photo shows a gable roof. $\overline{AC} \parallel \overline{FG}$. $\Delta ABC \sim \Delta FBG$. Find $\overline{BA}$ to the nearest tenth of a foot.



From p. 473, BF ≈ 4.6 ft.

$BA = BF + FA$

$\approx 6.3 + 17$

≈ 23.3 ft

Therefore, BA = 23.3 ft.

Check It Out! Example 5

What if…? If *AB* = 4*x*, *AC* = 5*x*, and *BF* = 4, find *FG*.

$$\frac{AB}{AC} = \frac{BF}{FG}$$ *Corr. sides are proportional.*

$$\frac{4x}{5x} = \frac{4}{FG}$$ *Substitute given quantities.*

$$4x(FG) = 4(5x)$$ *Cross Prod. Prop.*

$$FG = 5$$ *Simplify.*

You learned in Chapter 2 that the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. These properties also hold true for similarity of triangles.

Properties of Similarity

Reflexive Property of Similarity

$\triangle ABC \sim \triangle ABC$ (Reflex. Prop. of $\sim$)

Symmetric Property of Similarity

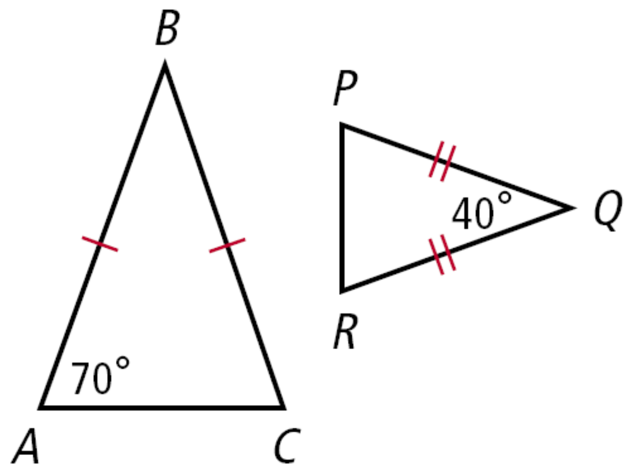
If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$. (Sym. Prop. of $\sim$)

Transitive Property of Similarity

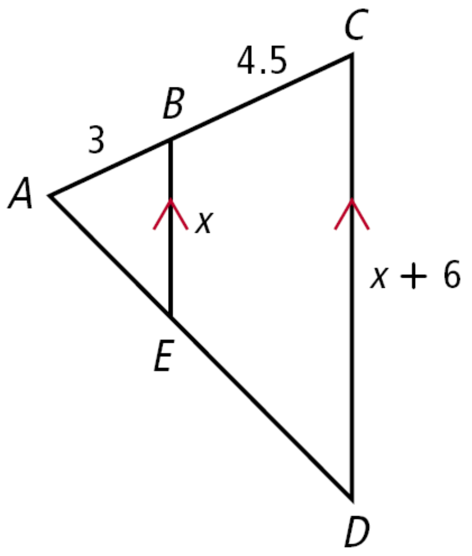
If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.
(Trans. Prop. of $\sim$)

Lesson Quiz

1. Explain why the triangles are similar and write a similarity statement.



2. Explain why the triangles are similar, then find *BE* and *CD*.



Lesson Quiz

1. By the Isosc. Δ Thm., $\angle A \cong \angle C$, so by the def. of ≅, $m\angle C = m\angle A$. Thus $m\angle C = 70°$ by subst. By the Δ Sum Thm., $m\angle B = 40°$. Apply the Isosc. Δ Thm. and the Δ Sum Thm. to ΔPQR. $m\angle R = m\angle P = 70°$. So by the def. of ≅, $\angle A \cong \angle P$, and $\angle C \cong \angle R$. Therefore $\Delta ABC \sim \Delta PQR$ by AA ~.

2. $\angle A \cong \angle A$ by the Reflex. Prop. of ≅. Since $BE \parallel CD$, $\angle ABE \cong \angle ACD$ by the Corr. ∠s Post. Therefore $\Delta ABE \sim \Delta ACD$ by AA ~. $BE = 4$ and $CD = 10$.