Plan for Algebra 2 Unit 6: Trigonometric Functions

Relevant Unit(s) to review: Geometry Unit 4: Right Triangle Trigonometry,
Geometry Unit 7: Circles

Essential prior concepts to engage with this unit	• Pythagorean Theorem • Right Triangle Trigonometry • Radians
Brief narrative of approach	This unit connects right triangle trigonometry and the study of circles from geometry to build the unit circle. Once the unit circle is established, cosine, sine, and tangent are extended to be functions with an infinite domain. If the Check Your Readiness shows that students do not remember right triangle trigonometry it will be necessary to add activities from a previous course to teach that content. If the Check Your Readiness shows that students are not fluent with radians there is an optional activity in the unit or additional activities from a previous course to teach that content as well.

Lessons to Add	Lessons to Remove or Modify
1. Geometry Unit 4 Activity 1.2 Geometry Unit 4 Activities 4.1, 4.2 2. Geometry Unit 4 Lesson 6 3. Geometry Unit 7 Lesson 11	1. Remove Alg2.6.8. Introduces a variety of models of periodicity, a lesson that is nice but not necessary 2. Remove Alg2.6.17. A lesson consisting primarily of an Info Gap that reinforces graphing and writing equations of periodic functions. 3. Remove Alg2.6.19. A lesson applying the ideas of the unit. It could be reformatted as a modeling prompt.
Lessons added: 3	Lessons removed: 3

Modified Plan for Algebra 2 Unit 4

Day	IM lesson	Notes
	Alg2.6 Check Your Readiness assessment	Note that the Check Your Readiness assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in Alg2.6.
1	Alg2.6.1	
2	Geo.4.1.2 Geo.4.4.1 Geo.4.4.2	If the initial assessment and Lesson 1 show that students are not familiar with the Pythagorean Theorem, include Geometry, Unit 4, Lesson 1, Activity 2 before continuing with grade-level content. If the initial assessment shows that students are not familiar with the right triangle trigonometry, include Geometry, Unit 4, Lesson 4, Activities 1 and 2 before continuing with grade-level content.
3	Geo.4.6	If the initial assessment shows that students are not familiar with right triangle trigonometry, include this lesson before continuing with grade-level content.
4	Alg2.6.2	
5	Alg2.6.3	
6	Geo.7.11	If the initial assessment and Lesson 3 show that students are not familiar with radians, include this lesson before continuing with grade-level content.
7	Alg2.6.4	
8	Alg2.6.5	
9	Alg2.6.6	
10	Alg2.6.7	
11	Alg2.6.9	

12	Alg2.6.10	
13	Alg2.6.11	
14	Alg2.6.12	
15	Alg2.6.13	
16	Alg2.6.14	
17	Alg2.6.15	
18	Alg2.6.16	
19	Alg2.6.18	
20	Alg2.6 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
Alg2.6.1	+	E	This lesson introduces periodic functions and the term period.
Alg2.6.2	+	D	This lesson begins connecting right triangle trigonometry to a circle on coordinate axes.
Alg2.6.3	+	E	This lesson explores the unit circle.
Alg2.6.4	+	D	This lesson connects radian angles and the coordinates associated with those angles on the unit circle.

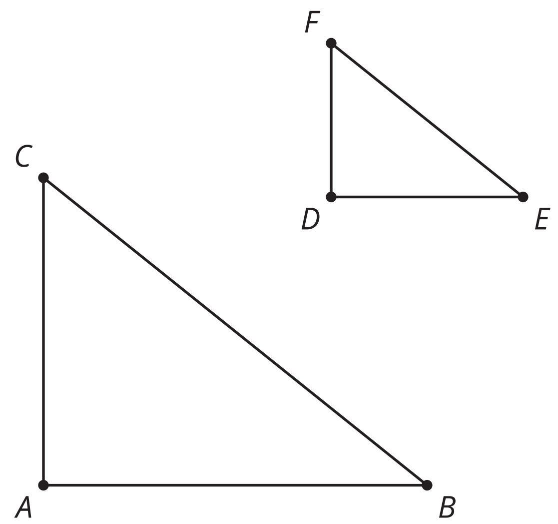
Alg2.6.5	+	D	This lesson establishes the Pythagorean Identity.
Alg2.6.6	0	A	This lesson expands on the previous lesson to make connections between cosine, sine, and tangent for an angle on the unit circle.
Alg2.6.7	0	A	This lesson provides opportunities to model with periodic functions.
Alg2.6.8	-	E	This lesson expands the idea of a periodic function to include any function in which the output values repeat at regular intervals.
Alg2.6.9	+	E	This lesson expands the understanding of cosine and sine as functions that have an angle measure as an input. Note: Activity 9.2 is the high priority activity. Activity 9.3 includes the sum and product of functions, a topic which may have been removed from the previous unit.
Alg2.6.10	0	D	This lesson extends the understanding of the cosine and sine functions to inputs greater than 2π radians.
Alg2.6.11	0	D	This lesson extends the domain of trigonometric functions to negative values.
Alg2.6.12	0	A	This lesson introduces tangent as a function.
Alg2.6.13	0	E	This lesson introduces the midline and amplitude of trigonometric functions.
Alg2.6.14	0	D	This lesson explores the effect of changes to both the amplitude and midline in the same function. This lesson introduces horizontal translations.
Alg2.6.15	0	D	This lesson introduces horizontal scale factor.
Alg2.6.16	0	A	This lesson offers opportunities to practice identifying important features of trigonometric functions when starting from equations or graphs.
Alg2.6.17	-	A	This lesson consists primarily of an Info Gap that reinforces graphing and writing

			equations of periodic functions.
Alg2.6.18	-	A	This lesson applies trigonometric functions to situations involving circular motion.
Alg2.6.19	-	A	This lesson applies trigonometric functions to model other periodic data.

Lesson 1: Angles and Steepness

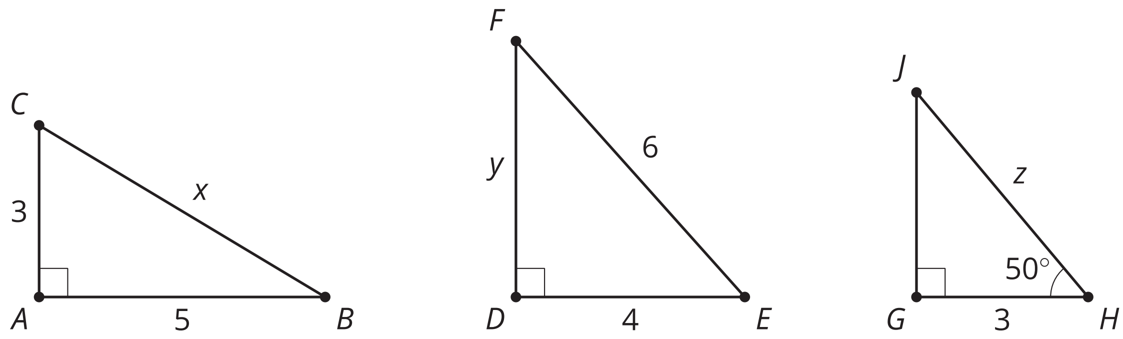
1.1: Ratios Galore

Triangle ABC is similar to triangle DEF. Write as many equations as you can to describe the relationships between the sides and angles of the 2 triangles.



1.2: Can You Calculate?

Find the values of $x, y,$ and z. If there is not enough information, what else do you need to know?



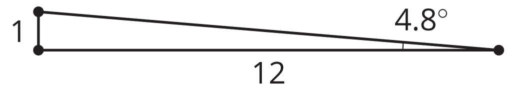
1.3: Is it Accessible?

1. Some buildings offer ramps in addition to stairs so people in wheelchairs have access to the building. What characteristics make a ramp safe?

2. A school has 4 steps to the front door. Each step is 7 inches tall. Design a ramp for the school.

3. Your teacher will give you the Americans with Disabilities Act (ADA) guidelines. Does your design follow the rules of this law? If not, draw a new design that does.

Are you ready for more?



A ramp with a length to height ratio of $12 : 1$ forms a right triangle with a 4.8 degree angle.

1. What is the angle measurement if the base is only 6 units long and the height is 1 unit tall?

2. When the length is half as long does that make the angle half as big?

3. What is the angle measurement if the base is 6 units long and the height is increased to 2 units tall?

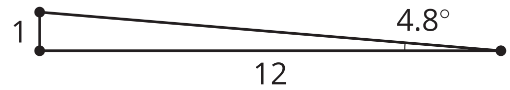
4. When the height is twice as tall does that make the angle twice as big?

Lesson 1 Summary

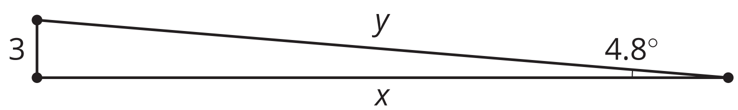
Because of the Pythagorean Theorem, if we know any 2 sides of a right triangle, we can calculate the length of the third side. But what if we know a side and an angle rather than 2 sides?

All right triangles with one pair of congruent acute angles are similar by the Angle-Angle Triangle Similarity Theorem. Knowing just one side length in addition to those angle measures is enough to uniquely define the triangle.

The Americans with Disabilities Act includes guidelines for safe and accessible wheelchair ramps. Ramps must form a maximum 4.8 degree angle with the ground, which creates a maximum $1 : 12$ ratio for the legs of the right triangle.



Let's assume we are building a ramp for a 3 inch threshold.



To find length x we can use similarity. By corresponding sides, $\frac{1}{12} = \frac{3}{x}$ so x is 36 units. To find length y we can use the Pythagorean Theorem, $3^2 + 36^2 = y^2$. So $y = \sqrt{1,305}$ or about 36.1 units. To build a ramp that goes up 3 inches we need to start 36 inches, or 3 feet, out from the edge of the threshold and use a board that's about 36.1 inches long.

Americans with Disabilities Act Ramp Guidelines

Ramps are required for any change in level greater than $\frac{1}{2}$ inch.

Ramps have a maximum slope of 1:12 (4.8 degrees)

Ramps have a minimum width of 36 inches.

There must be a minimum 5 feet by 5 feet flat area at the top and bottom of the ramp for turning.

Ramps may be no longer than 30 feet horizontally between flat rest or turn platforms.

Source: https://www.access-board.gov/guidelines-and-standards/buildings-and-sites/about-the-ada-standards/guide-to-the-ada-standards

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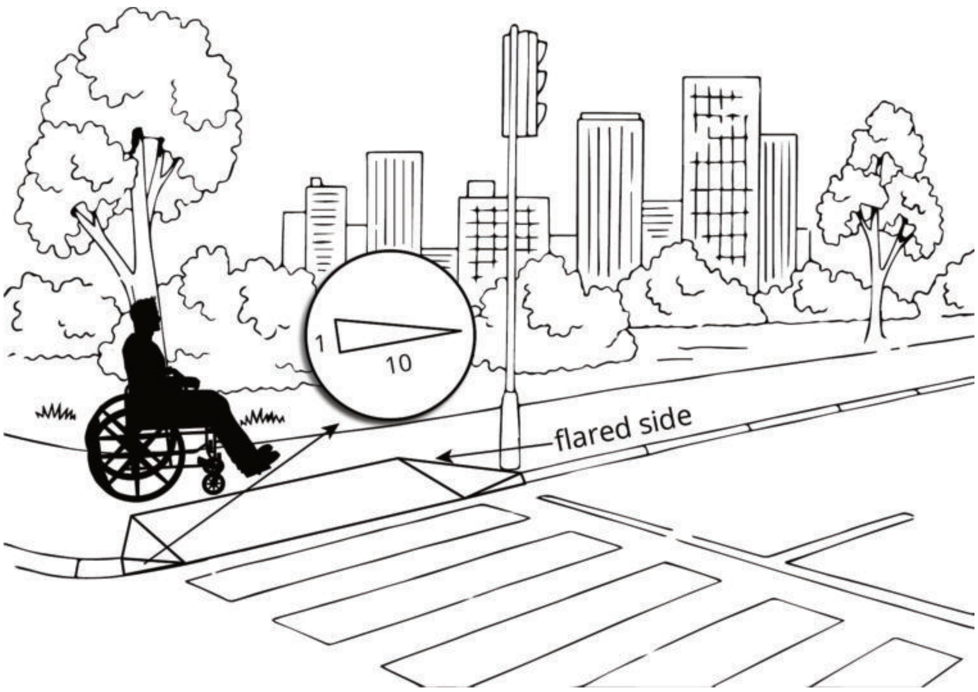
Lesson 1: Angles and Steepness

Cool Down: Sidewalk Ramp

A curb is 4 inches high. The ramp from the sidewalk to the street follows the same guidelines as other ramps (no more than 4.8 degrees, so a $1 : 12$ ratio of vertical to horizontal). What is the horizontal distance for a ramp with a 4 inch vertical distance?

Image not drawn to scale.

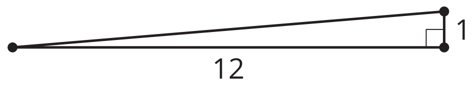


Flared sides smoothly connect the ramp to the sidewalk. The guidelines say the flared sides must be no more than 5.7 degrees ($1 : 10$ ratio of vertical to horizontal). What is the horizontal distance for flared sides with a 4 inch vertical distance?

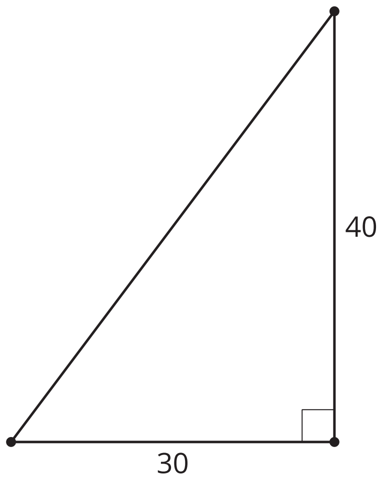
Unit 4 Lesson 1 Cumulative Practice Problems

1. The Americans with Disabilities Act states that ramps must have an angle less than or equal to 4.8 degrees. Remember, a 4.8 degree angle in a right triangle has a $1:12$ ratio for the legs. Select **all** ramps that meet the Americans with Disabilities Act requirements.

Triangle A

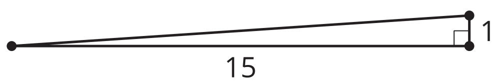


Triangle B

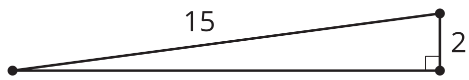


Triangle C

Triangle D



Triangle E



A. Triangle A

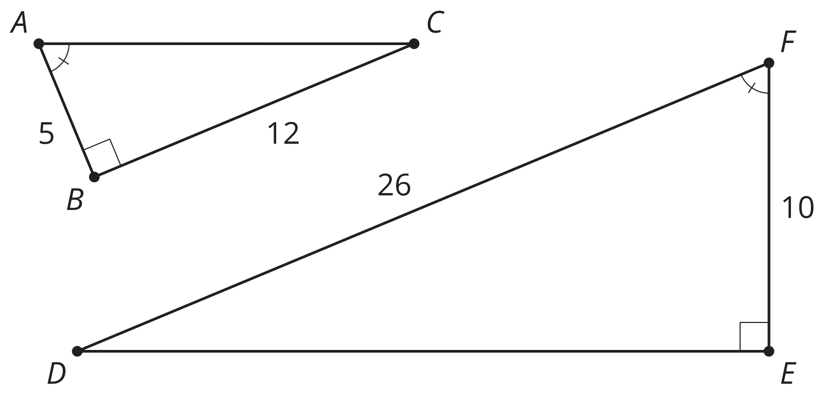
B. Triangle B

C. Triangle C

D. Triangle D

E. Triangle E

2. Find the missing side in each triangle using any method. Check your answers using a different method.



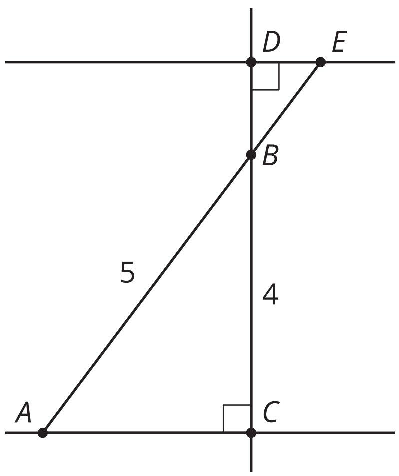
3. The Americans with Disabilities Act states that ramps must have an angle less than or equal to 4.8 degrees. Remember, a 4.8 degree angle in a right triangle has a $1:12$ ratio for the legs. Design 2 ramps that meet the Americans with Disabilities Act requirements.

4. Kiran is visiting the Statue of Liberty. He wants to test the mirror method of indirect measurement for calculating heights. He is 5.8 feet tall and knows that the Statue of Liberty is 305 feet tall. Sketch a diagram showing where a mirror could be placed to use similar triangles to verify the height of the Statue of Liberty. Make sure to include the distance from Kiran to the mirror and the distance from the mirror to the Statue of Liberty.

(From Unit 3, Lesson 16.)

5. In this diagram, lines AC and DE are parallel, and line DC is perpendicular to each of them. What is a reasonable estimate for the length of side DE?

$AC \parallel DE$, $DC \perp DE$, $DC \perp AC$



A. $\frac{1}{3}$

B. 1

C. 3

D. 6

(From Unit 3, Lesson 15.)

6. Lin says she has memorized the lengths of a few right triangles, for example, 3, 4, and 5. She is trying to compile a list of several right triangles but needs your help. Find the lengths of at least 2 triangles that are right.

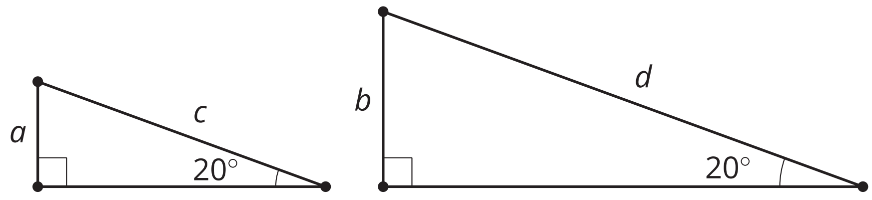
(From Unit 3, Lesson 14.)

7. In triangle ABC, the measure of angle A is 35° and the measure of angle B is 20°. In triangle DEF, the measure of angle D is 35° and the measure of angle F is 125°. Are triangles ABC and DEF similar? Explain or show your reasoning.

(From Unit 3, Lesson 9.)

Lesson 4: Ratios in Right Triangles

4.1: Ratio Rivalry



Consider $\frac{a}{c}$ and $\frac{b}{d}$. Which is greater, or are they equal? Explain how you know.

4.2: Tons of Triangles

Your teacher will give you some angles.

1. Draw several right triangles using the angles you receive.
2. Precisely measure the side lengths of the triangles.
3. Complete the tables by computing 3 quotients for the acute angles in each triangle:
    a. The length of the leg adjacent to your angle divided by the length of the hypotenuse
    b. The length of the leg opposite from your angle divided by the length of the hypotenuse
    c. The length of the leg opposite from your angle divided by the length of the leg adjacent to your angle
4. Find the mean of each column in your table.
5. What do you notice about your table? What do you wonder about your table?

4.3: Tons of Ratios

1. Compare the row for 20 degrees and the row for 70 degrees in the right triangle table. What is the same? What is different?

2. The row for 55 degrees is given here. Complete the row for 35 degrees.

angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
35°			
55°	0.574	0.819	1.428

3. What do you know about a triangle with an adjacent leg to hypotenuse ratio value of 0.839?

Are you ready for more?

1. What is the range for the possible ratios of each of the following ratios?
    a. adjacent leg ÷ hypotenuse

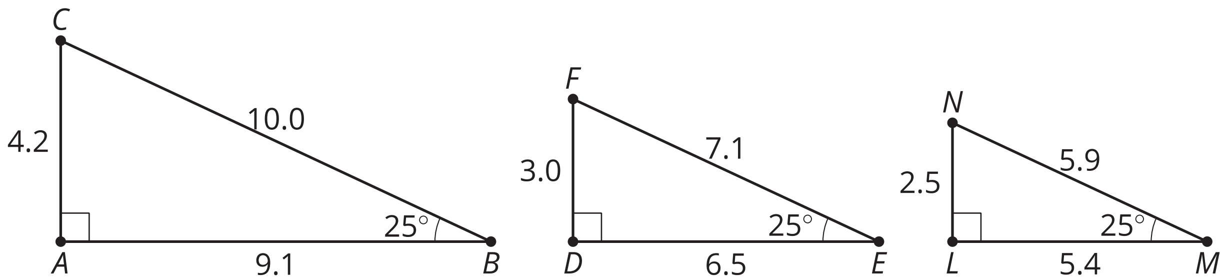
    b. opposite leg ÷ hypotenuse

    c. opposite leg ÷ adjacent leg

2. What would the triangle look like if the adjacent leg ÷ hypotenuse ratio was 1? Greater than 1?

Lesson 4 Summary

All right triangles that contain the same acute angles are similar to each other. This means that the ratios of corresponding side lengths are equal for all right triangles with the same acute angles.



These triangles are all similar by the Angle-Angle Triangle Similarity Theorem. Focusing on the 25 degree angles, we see that all 3 triangles have adjacent leg to hypotenuse ratios of approximately 0.91.

Because all right triangles with the same acute angle measures have the same ratios, we can look for patterns that will help us solve problems. The right triangle table comes from measuring and finding ratios in several right triangles with different angle measures.

angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
25°	0.906	0.423	0.466
35°	0.819	0.574	0.700
45°	0.707	0.707	1.000
55°	0.574	0.819	1.428
65°	0.423	0.906	2.145

Some ratios in this table are repeated. Notice that the rows for 25 degrees and 65 degrees have 2 of the same ratios. What is special about 25 and 65? They are **complementary** angles, that is, the 2 angles sum to 90 degrees. This seems to be true for other complementary angles. Notice that $35 + 55 = 90$ and those rows both have 0.819 as a ratio.

	angle A	**adjacent leg**	**opposite leg**	**hypotenuse**	**adjacent leg ÷ hypotenuse**	**opposite leg ÷ hypotenuse**	**opposite leg ÷ adjacent leg**
Trial 1							
Trial 2							
Trial 3							
Trial 4							
Trial 5							
Mean							

	angle B	**adjacent leg**	**opposite leg**	**hypotenuse**	**adjacent leg ÷ hypotenuse**	**opposite leg ÷ hypotenuse**	**opposite leg ÷ adjacent leg**
Trial 1							
Trial 2							
Trial 3							
Trial 4							
Trial 5							
Mean							

angle:	**adjacent leg ÷ hypotenuse**	**opposite leg ÷ hypotenuse**	**opposite leg ÷ adjacent leg**
10°	0.985	0.174	0.176
20°	0.940	0.342	0.364
30°	0.866	0.500	0.577
40°	0.766	0.643	0.839
50°	0.643	0.766	1.192
60°	0.500	0.866	1.732
70°	0.342	0.940	2.747
80°	0.174	0.985	5.671

angle:	**adjacent leg ÷ hypotenuse**	**opposite leg ÷ hypotenuse**	**opposite leg ÷ adjacent leg**
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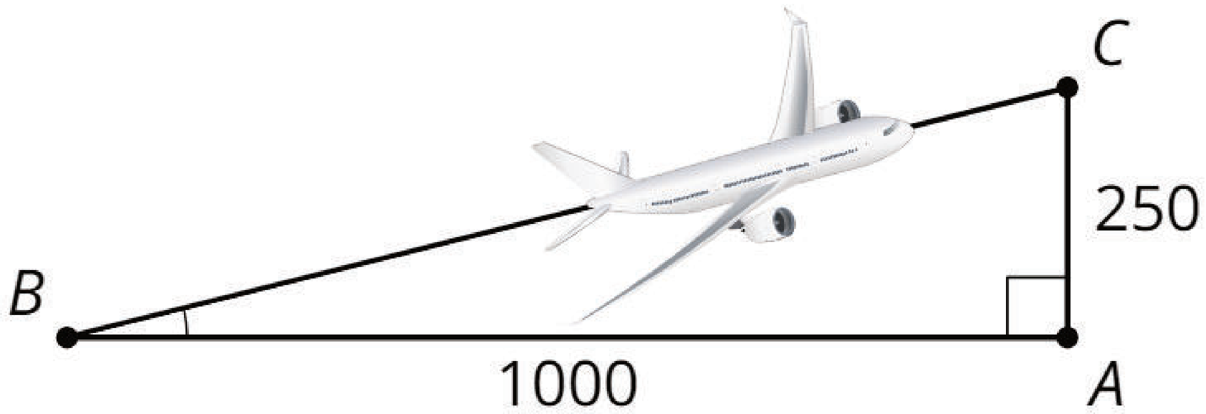
Lesson 4: Ratios in Right Triangles

Cool Down: Lift Off

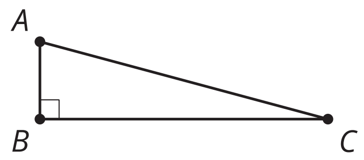
Shortly after takeoff, an airplane is climbing 250 feet for every 1,000 feet it travels. Estimate the airplane's climb angle while this is happening.



Unit 4 Lesson 4 Cumulative Practice Problems

1. Angle B is an acute angle in a right triangle. What is a reasonable approximation for angle B if the ratio for the opposite leg divided by the hypotenuse is 0.67?

2. Estimate the values to complete the table.



angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
A			
C	0.97	0.26	0.27

3. Priya says, "I know everything about a right triangle with a 30 degree angle and a hypotenuse with length 1 cm. Here, look."

    - The other angle is 60 degrees.
    - The leg adjacent to the 30 degree angle is 0.866 cm long.
    - The side opposite the 30 degree angle is 0.5 cm long.

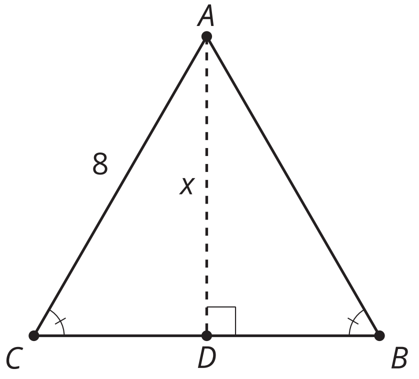
    Han asks, "What would happen if a right triangle with a 30 degree angle has a hypotenuse that is 2 cm instead?"

    Help them find the missing angles and side lengths in the new triangle. Explain or show your reasoning.

4. Triangle ABC is equilateral.

   a. What is the value of x?

   b. What is the measure of angle B?



(From Unit 4, Lesson 3.)

5. An equilateral triangle has side length 8 units. What is the area?

   A. $16\sqrt{3}$ square units

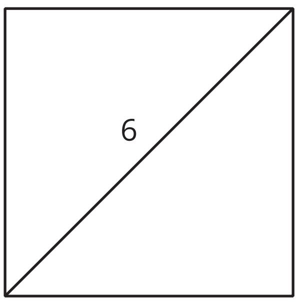
   B. 24 square units

   C. $24\sqrt{3}$ square units

   D. 32 square units

(From Unit 4, Lesson 3.)

6. What is the length of the square's side?



   A. 3 units

   B. $\frac{6}{\sqrt{2}}$ units

   C. $6\sqrt{2}$ units

   D. 12 units

7. A step has a height of 6 inches. A ramp starts 5 feet away from the base of the step, making a 5.9° angle with the ground. What can you say about the angle the ramp would make with the ground if the ramp starts closer to the step?

   A. The angle would decrease.

   B. The angle would increase.

   C. The angle would stay the same.

   D. We cannot determine anything about the angle.

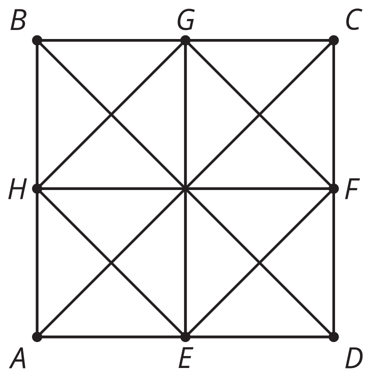
   (From Unit 4, Lesson 1.)

8. The quilt is made of squares with diagonals.

   The length of BD is 4.

   a. Find the length of AE.

   b. Find the area of square $ABCD$.

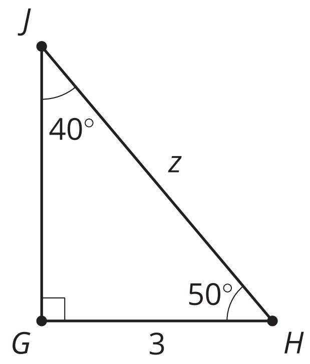


   (From Unit 3, Lesson 12.)

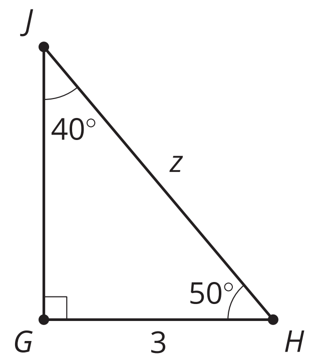
Lesson 6: Working with Trigonometric Ratios

6.1: This Time with Strategies

Estimate the value of z.



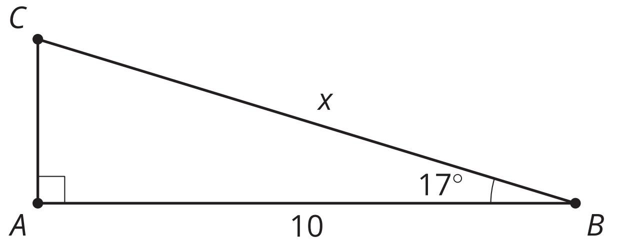
6.2: New Names, Same Ratios



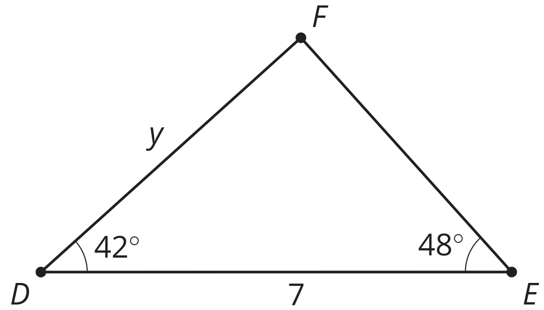
1. Use your calculator to determine the values of $\cos(50)$, $\sin(50)$, and $\tan(50)$.
2. Use your calculator to determine the values of $\cos(40)$, $\sin(40)$, and $\tan(40)$.
3. How do these values compare to your chart?
4. Find the value of z.

6.3: Solve These Triangles

1. Solve for x.

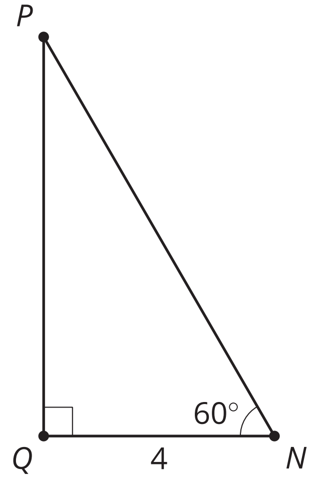


2. Solve for y.



3. Find all the missing sides and angle measures.

   a. The measure of angle X is 90 degrees and angle Y is 12 degrees. Side XZ has length 2 cm.

b. 

c. The measure of angle K is 90 degrees and angle L is 71 degrees. Side LM has length 20 cm.

Are you ready for more?

Complete the table.

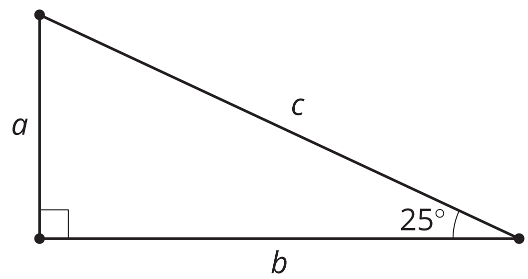
angle	cosine	sine	tangent
80°			
85°			
89°			

Based on this information, what do you think are the cosine, sine, and tangent of 90 degrees? Explain or show your reasoning.

Lesson 6 Summary

We have a column in the right triangle table for "adjacent leg ÷ hypotenuse." We use this ratio so frequently it has a name. It is called the **cosine** of the angle. We write $\cos(25)$ to say the cosine of 25 degrees. A scientific calculator can display the cosine of any angle. This means we can more precisely calculate unknown side lengths rather than estimating using the table. The right triangle table is sometimes called a trigonometry table since cosine, **sine**, and **tangent** are **trigonometric ratios**. Here is what the table looks like with the ratios labeled with their special names:

	cosine	**sine**	**tangent**
angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
25°	$\cos(25) = 0.906$	$\sin(25) = 0.423$	$\tan(25) = 0.466$

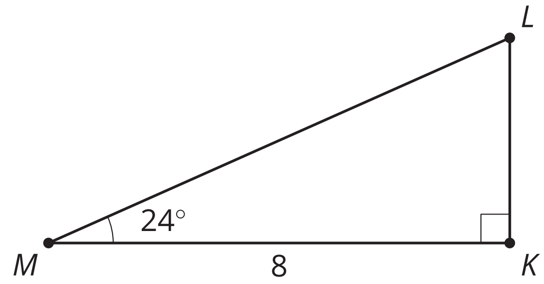


If the length b is 7, we can find c by solving the equation $\cos(25) = \frac{7}{c}$. So c is about 7.7 units. To solve for a we have 3 choices: the Pythagorean Theorem, sine, and tangent. Let's use tangent by solving the equation $\tan(25) = \frac{a}{7}$. So a is about 3.3 units. We can check our answers using the Pythagorean Theorem. It should be true that $3.3^2 + 7^2 = 7.7^2$. The two expressions are almost equal, which makes sense because we expect some error due to rounding.

Lesson 6: Working with Trigonometric Ratios

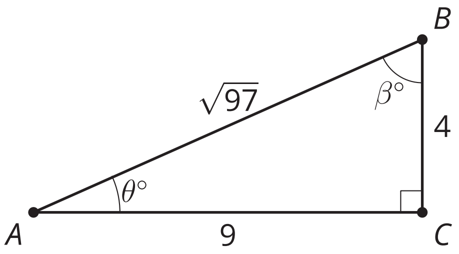
Cool Down: Solve That Triangle

Find all the missing side and angle measures.



Unit 4 Lesson 6 Cumulative Practice Problems

1. Select **all** true statements:



A. $\sin(\theta) = \frac{4}{\sqrt{97}}$

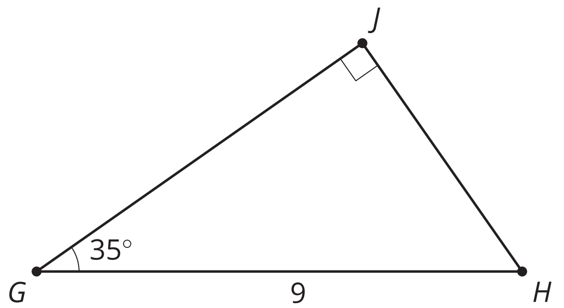
B. $\tan(\beta) = \frac{9}{4}$

C. $\tan(\beta) = \frac{4}{9}$

D. $\cos(\beta) = \frac{4}{\sqrt{97}}$

E. $4^2 + 9^2 = 97$

2. Write an expression that can be used to find the length of JH and an expression that can be used to find the length of GJ.



3. Andre and Clare are discussing triangle ABC that has a right angle at C and a hypotenuse of length 15 units. Andre thinks the triangle could have legs that are 9 and 12 units long. Clare thinks angle B could be 20 degrees and then side BC would be 14.1 units long. Do you agree with either of them? Explain or show your reasoning.

(From Unit 4, Lesson 5.)

4. A triangle has sides with lengths 5, 12, and 13.

   a. Verify this is a Pythagorean triple.

   b. Approximate the acute angles in this triangle.

(From Unit 4, Lesson 5.)

5. Approximate the angles that have the following quotients:

   a. adjacent leg ÷ hypotenuse = 0.966

   b. opposite leg ÷ hypotenuse = 0.469

   c. adjacent leg ÷ hypotenuse = 0.309

   d. opposite leg ÷ adjacent leg = 1.036

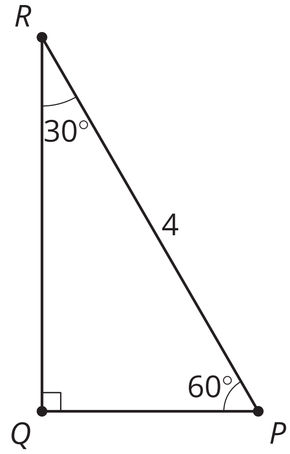
(From Unit 4, Lesson 4.)

6. Lin missed class and Tyler is helping her use the table to approximate the angle measures that have the ratios listed. Tyler says, "You can use the right triangle table to figure this out." Lin notices that some of the ratios are the same in each row. Estimate the angles and explain why some of the values are repeated.

angle	adjacent leg ÷ hypotenuse	opposite leg ÷ hypotenuse	opposite leg ÷ adjacent leg
	0.573	0.819	1.428
	0.819	0.573	0.700

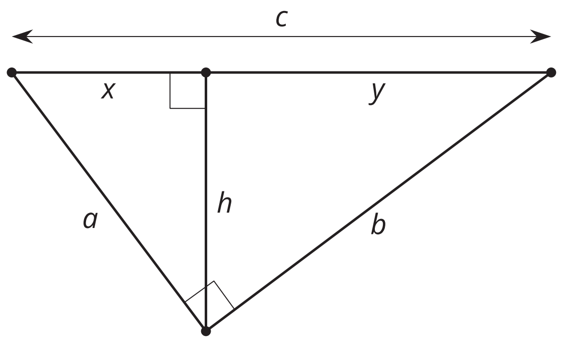
(From Unit 4, Lesson 4.)

7. Find the length of each leg.



(From Unit 4, Lesson 3.)

8. Elena is proving the Pythagorean Theorem. She knows the goal is to prove that $a^2 + b^2 = c^2$ in a right triangle. So far she has:



In a right triangle the altitude that intersects the hypotenuse decomposes the triangle into 2 smaller right triangles. These triangles are similar to the large triangle by the Angle-Angle Triangle Similarity Theorem since each smaller triangle shares one angle with the larger triangle and has a right angle. Similar triangles have proportional side lengths, so _____1_____ and _____2_____. I can rewrite those equations to get $a^2 = xc$ and $b^2 = yc$. Therefore $a^2 + b^2 = \ldots$

Fill in the blanks and finish the proof Elena started.

(From Unit 3, Lesson 14.)

Lesson 11: A New Way to Measure Angles

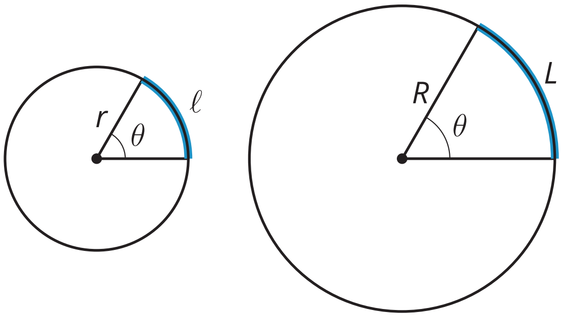
11.1: A One-Unit Radius

A circle has radius 1 unit. Find the length of the arc defined by each of these central angles. Give your answers in terms of π.

1. 180 degrees
2. 45 degrees
3. 270 degrees
4. 225 degrees
5. 360 degrees

11.2: A Constant Ratio

Diego and Lin are looking at 2 circles.



Diego says, "It seems like for a given central angle, the arc length is proportional to the radius. That is, the ratio $\frac{\ell}{r}$ has the same value as the ratio $\frac{L}{R}$ because they have the same central angle measure. Can we prove that this is true?"

Lin says, "The big circle is a dilation of the small circle. If k is the scale factor, then $R = kr$."

Diego says, "The arc length in the small circle is $\ell = \frac{\theta}{360} \cdot 2\pi r$. In the large circle, it's $L = \frac{\theta}{360} \cdot 2\pi R$. We can rewrite that as $L = \frac{\theta}{360} \cdot 2\pi rk$. So $L = k\ell$."

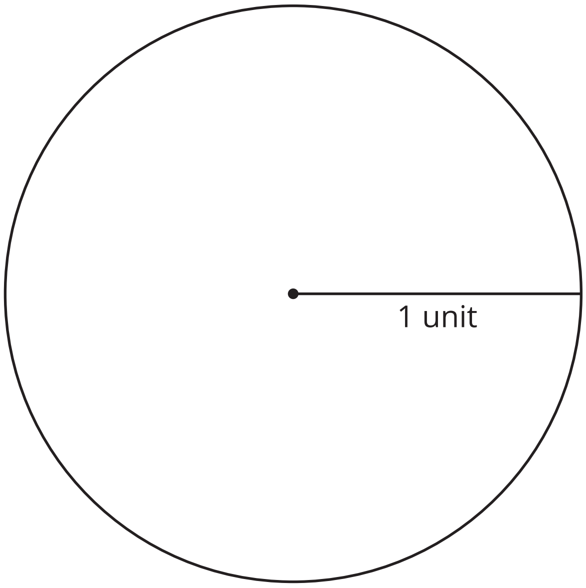
Lin says, "Okay, from here I can show that $\frac{\ell}{r}$ and $\frac{L}{R}$ are equivalent."

1. How does Lin know that the big circle is a dilation of the small circle?
2. How does Lin know that $R = kr$?
3. Why could Diego write $\ell = \frac{\theta}{360} \cdot 2\pi r$?
4. When Diego says that $L = k\ell$, what does that mean in words?
5. Why could Diego say that $L = k\ell$?
6. How can Lin show that $\frac{\ell}{r} = \frac{L}{R}$?

11.3: Defining Radians

Suppose we have a circle that has a central angle. The **radian** measure of the angle is the ratio of the length of the arc defined by the angle to the circle's radius. That is,
$\theta = \frac{\text{arc length}}{\text{radius}}$.

1. The image shows a circle with radius 1 unit.



   a. Cut a piece of string that is the length of the radius of this circle.

   b. Use the string to mark an arc on the circle that is the same length as the radius.

   c. Draw the central angle defined by the arc.

   d. Use the definition of radian to calculate the radian measure of the central angle you drew.

2. Draw a 180 degree central angle (a diameter) in the circle. Use your 1-unit piece of string to measure the approximate length of the arc defined by this angle.

3. Calculate the radian measure of the 180 degree angle. Give your answer both in terms of π and as a decimal rounded to the nearest hundredth.

4. Calculate the radian measure of a 360 degree angle.

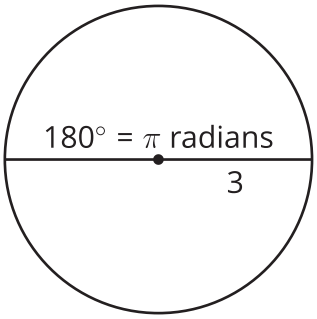
Are you ready for more?

Research where the “360” in 360 degrees comes from. Why did people choose to define a degree as $\frac{1}{360}$ of the circumference of a circle?

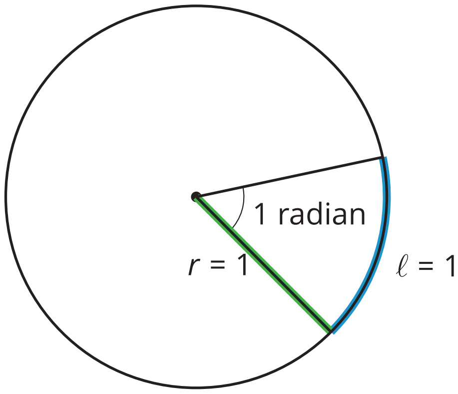
Lesson 11 Summary

Degrees are one way to measure the size of an angle. **Radians** are another way to measure angles. Assume an angle's vertex is the center of a circle. The radian measure of the angle is the ratio between the length of the arc defined by the angle and the radius of the circle. We can write this as $\theta = \frac{\text{arc length}}{\text{radius}}$. This ratio is constant for a given angle, no matter the size of the circle.

Consider a 180 degree central angle in a circle with radius 3 units. The arc length defined by the angle is 3π units. The radian measure of the angle is the ratio of the arc length to the radius, which is π radians because $\frac{3\pi}{3} = \pi$.



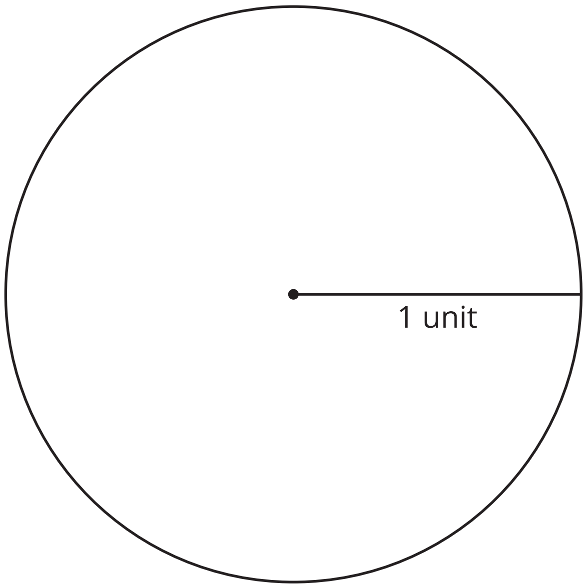
Another way to think of the radian measure of the angle is that it measures the number of radii that would make up the length of the arc defined by the angle. For example, if we draw an arc that is the same length as the radius, both the arc length and the radius are 1 unit. The radian measure of the central angle that defines this arc is the quotient of those values, or 1 radian.



Lesson 11: A New Way to Measure Angles

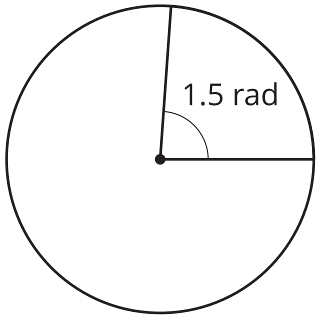
Cool Down: Find a Radian Measure

Calculate the radian measure of a 60 degree angle. Use any method you like, including sketching in the circle diagram provided. Explain or show your reasoning.



Unit 7 Lesson 11 Cumulative Practice Problems

1. Here is a central angle that measures 1.5 radians. Select **all** true statements.



   A. The radius is 1.5 times longer than the length of the arc defined by the angle.

   B. The length of the arc defined by the angle is 1.5 times longer than the radius.

   C. The ratio of arc length to radius is 1.5.

   D. The ratio of radius to arc length is 1.5.

   E. The area of the whole circle is 1.5 times the area of the slice.

   F. The circumference of the whole circle is 1.5 times the length of the arc formed by the angle.

2. Match each arc length ℓ and radius r with the measure of the central angle of the arc in radians.

   A. $r = 2, \ell = \frac{\pi}{2}$

   B. $r = 3, \ell = 2$

   C. $r = 3.5, \ell = 2.8$

   D. $r = 4, \ell = 3$

   E. $r = 5, \ell = \pi$

   F. $r = 6, \ell = 4$

   1. $\frac{\pi}{5}$ radians

   2. $\frac{2}{3}$ radians

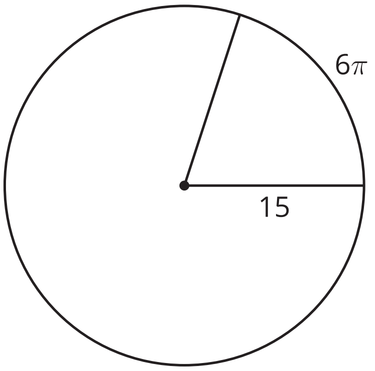
   3. 0.75 radians

   4. $\frac{\pi}{4}$ radians

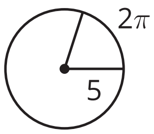
   5. 0.8 radians

3. Han thinks that since the arc length in circle A is longer, its central angle is larger. Do you agree with Han? Show or explain your reasoning.

circle A

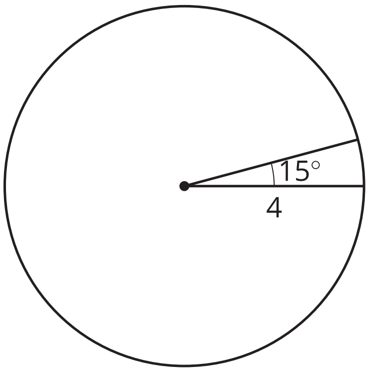


circle B

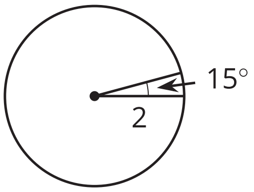


4. Circle B is a dilation of circle A.

circle A



circle B



a. What is the scale factor?

b. What is the area of the 15 degree sector in circle A?

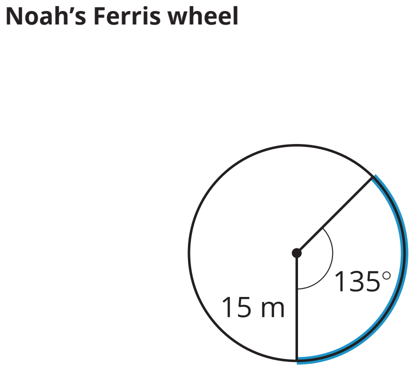
c. What is the area of the 15 degree sector in circle B?

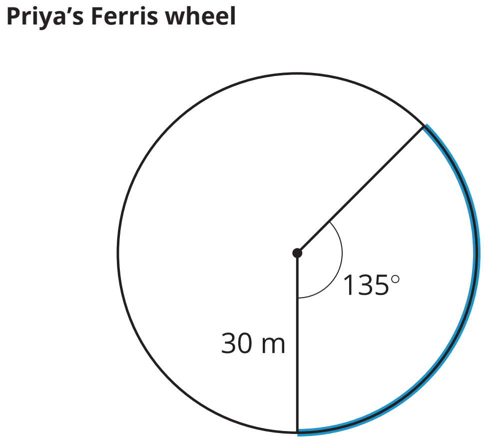
d. What is the ratio of the areas of the sectors?

e. How does the ratio of areas of the sectors compare to the scale factor?

(From Unit 7, Lesson 10.)

5. Priya and Noah are riding different size Ferris wheels at a carnival. They started at the same time. The highlighted arcs show how far they have traveled.





a. How far has Noah traveled?

b. How far has Priya traveled?

c. If the Ferris wheels will each complete 1 revolution, who do you think will finish first?

(From Unit 7, Lesson 10.)

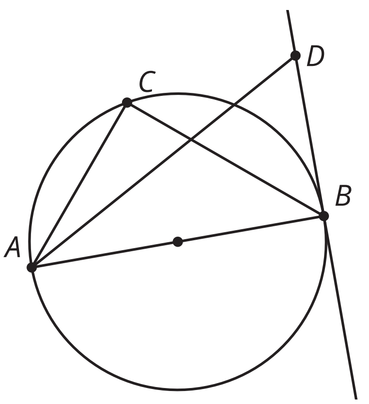
6. A circle has radius 8 units, and a central angle is drawn in. The length of the arc defined by the central angle is 4π units. Find the area of the sector outlined by this arc.

(From Unit 7, Lesson 9.)

7. Clare is trying to explain how to find the area of a sector of a circle. She says, “First, you find the area of the whole circle. Then, you divide by the radius.“ Do you agree with Clare? Explain or show your reasoning.

(From Unit 7, Lesson 8.)

8. Line BD is tangent to a circle with diameter AB. List 2 right angles.



(From Unit 7, Lesson 3.)