Plan for Algebra 2 Unit 5: Transformations of Functions

Relevant Unit(s) to review: Algebra 1 Unit 4: Functions,
Geometry Unit 3: Similarity,
Geometry Unit 6: Coordinate Geometry

Essential prior concepts to engage with this unit	• function notation • translations and reflections • scale factors
Brief narrative of approach	This unit connects geometric transformation language to graphs of functions. If the Check Your Readiness shows that students do not remember transformation vocabulary and concepts from geometry it will be necessary to add activities from a previous course to teach that content. If the Check Your Readiness shows that students are not fluent with function notation it will be necessary to add activities from a previous course to teach that content as well.

Lessons to Add	**Lessons to Remove or Modify**
1. Algebra 1 Unit 4 Lesson 3 2. Geometry Unit 3 Activities 1.1, 1.4 3. Geometry Unit 6 Activity 1.2 4. Geometry Unit 6 Lesson 2	Note: This unit is short and most of the lessons are important to include. The lessons listed here are possible to skip while still understanding the main ideas of the unit, but it may be best to find room for the additional needed lessons from other units. 1. Remove Alg2.5.10: a lesson on combining functions, a skill which will not be essential in the rest of the course 2. Remove Alg2.5.11: could be reformatted as a modeling prompt
Lessons added: 3	Lessons removed: 2

Modified Plan for Algebra 2 Unit 4

Day	IM lesson	Notes
	Alg2.5 Check Your Readiness assessment	Note that the Check Your Readiness assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in Alg2.5.
1	Alg1.4.3	If the initial assessment shows that students are not familiar with function notation, include this lesson before continuing with grade-level content.
2	Geo.3.1.1 Geo.3.1.4 Geo.6.1.2	If the initial assessment shows that students are not familiar with scale factors, include Geometry, Unit 3, Lesson 1, Activity 1 and Geometry, Unit 3, Lesson 1, Activity 4 before continuing with grade-level content. If the initial assessment also shows that students are not familiar with translation, rotation, and reflection, include Geometry, Unit 6, Lesson 1, Activity 2 before continuing with grade-level content.
3	Geo.6.2	If the initial assessment shows that students are not familiar with translation, rotation, and reflection, include this lesson before continuing with grade-level content.
4	Alg2.5.1	
5	Alg2.5.2	
6	Alg2.5.3	
7	Alg2.5.4	
8	Alg2.5.5	
9	Alg2.5.6	
10	Alg2.5.7	
11	Alg2.5.8	

12	Alg2.5.9	
13	Alg2.5 End Assessment	If Lesson 10 is removed as suggested, skip question 3 of this assessment.

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

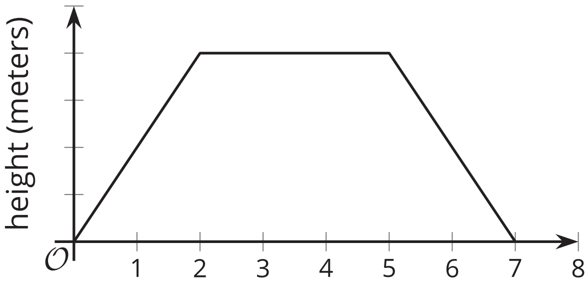
Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
Alg2.5.1	0	E	This lesson explores ways to transform functions to model sets of data.
Alg2.5.2	+	E	This lesson formalizes writing function transformations using function notation.
Alg2.5.3	0	D	This lesson expands on the previous lesson to include both horizontal and vertical translations of the same function.
Alg2.5.4	0	E	This lesson compares functions, where one is a reflection of the other across an axis, using tables, graphs, and equations.
Alg2.5.5	+	E	This lesson introduces the categories of even and odd functions.
Alg2.5.6	0	A	This lesson expands on the previous lesson to categorize equations as even, odd, or neither without using graphs.
Alg2.5.7	0	A	This lesson provides additional opportunities to apply ideas about transformations.
Alg2.5.8	0	E	This lesson explores the effect on a graph of multiplying the output of a function by a scale factor.
Alg2.5.9	0	D	This lesson expands on the previous lesson to include scale factors of the input of a

			function as well as the output.
Alg2.5.10	-	A	This lesson explores how to create new functions by combining two functions through adding, subtracting, multiplying, or dividing.
Alg2.5.11	-	A	This lesson offers the opportunity to practice applying transformations to different functions in order to model a set of data.

Lesson 3: Interpreting & Using Function Notation

3.1: Observing a Drone

Here is a graph that represents function f, which gives the height of a drone, in meters, t seconds after it leaves the ground.



Decide which function value is greater.

1. $f(0)$ or $f(4)$
2. $f(2)$ or $f(5)$
3. $f(3)$ or $f(7)$
4. $f(t)$ or $f(t+1)$

3.2: Smartphones

The function P gives the number of people, in millions, who own a smartphone, t years after year 2000.

1. What does each equation tell us about smartphone ownership?

   a. $P(17) = 2{,}320$

   b. $P(-10) = 0$

2. Use function notation to represent each statement.

   a. In 2010, the number of people who owned a smartphone was 296,600,000.

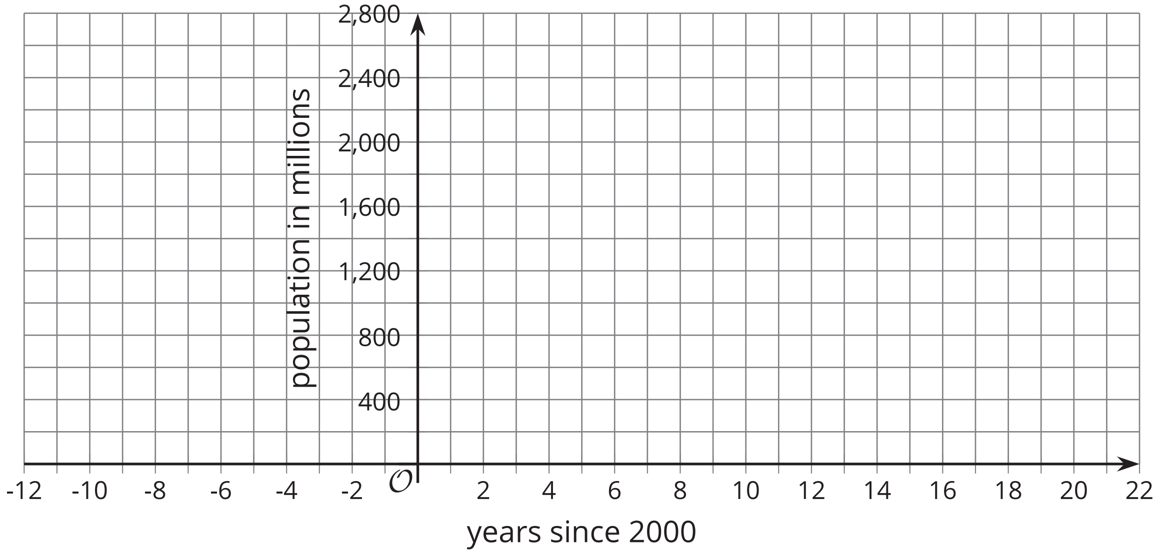
   b. In 2015, about 1.86 billion people owned a smartphone.

3. Mai is curious about the value of t in $P(t) = 1{,}000$.

   a. What would the value of t tell Mai about the situation?

   b. Is 4 a possible value of t here?

4. Use the information you have so far to sketch a graph of the function.



Are you ready for more?

What can you say about the value or values of t when $P(t) = 1{,}000$?

3.3: Boiling Water

The function W gives the temperature, in degrees Fahrenheit, of a pot of water on a stove, t minutes after the stove is turned on.

1. Take turns with your partner to explain the meaning of each statement in this situation. When it's your partner's turn, listen carefully to their interpretation. If you disagree, discuss your thinking and work to reach an agreement.

    a. $W(0) = 72$

    b. $W(5) > W(2)$

    c. $W(10) = 212$

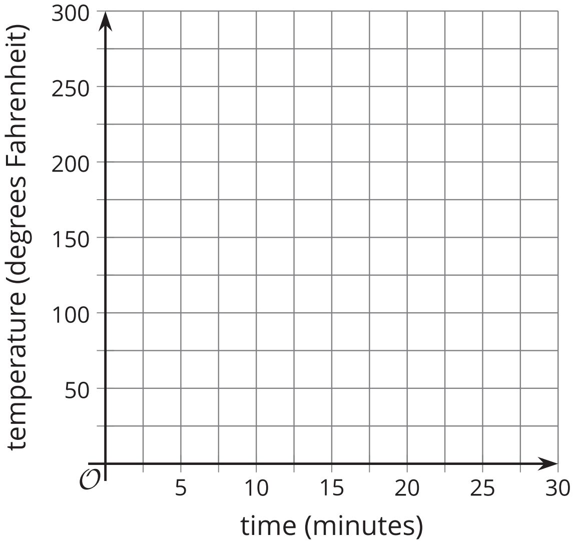
    d. $W(12) = W(10)$

    e. $W(15) > W(30)$

    f. $W(0) < W(30)$

2. If all statements in the previous question represent the situation, sketch a possible graph of function W.

    Be prepared to show where each statement can be seen on your graph.



Lesson 3 Summary

What does a statement like $p(3) = 12$ mean?

On its own, $p(3) = 12$ only tells us that when p takes 3 as its input, its output is 12.

If we know what quantities the input and output represent, however, we can learn much more about the situation that the function represents.

- If function p gives the perimeter of a square whose side length is x and both measurements are in inches, then we can interpret $p(3) = 12$ to mean "a square whose side length is 3 inches has a perimeter of 12 inches."

  We can also interpret statements like $p(x) = 32$ to mean "a square with side length x has a perimeter of 32 inches," which then allows us to reason that x must be 8 inches and to write $p(8) = 32$.

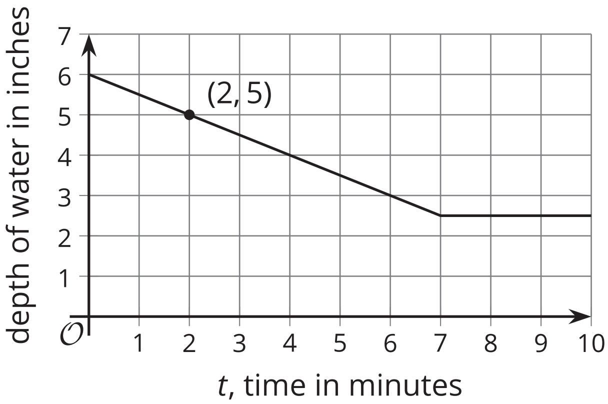
- If function p gives the number of blog subscribers, in thousands, x months after a blogger started publishing online, then $p(3) = 12$ means "3 months after a blogger started publishing online, the blog has 12,000 subscribers."

It is important to pay attention to the units of measurement when analyzing a function. Otherwise, we might mistake what is happening in the situation. If we miss that $p(x)$ is measured in thousands, we might misinterpret $p(x) = 36$ to mean "there are 36 blog subscribers after x months," while it actually means "there are 36,000 subscribers after x months."

A graph of a function can likewise help us interpret statements in function notation.

Function f gives the depth, in inches, of water in a tub as a function of time, t, in minutes, since the tub started being drained.

Here is a graph of f.



Each point on the graph has the coordinates $(t, f(t))$, where the first value is the input of the function and the second value is the output.

- $f(2)$ represents the depth of water 2 minutes after the tub started being drained. The graph passes through $(2, 5)$, so the depth of water is 5 inches when $t = 2$. The equation $f(2) = 5$ captures this information.

- $f(0)$ gives the depth of the water when the draining began, when $t = 0$. The graph shows the depth of water to be 6 inches at that time, so we can write $f(0) = 6$.

- $f(t) = 3$ tells us that t minutes after the tub started draining, the depth of the water is 3 inches. The graph shows that this happens when when t is 6.

Lesson 3: Interpreting & Using Function Notation

Cool Down: Visitors in a Museum

An art museum opens at 9 a.m. and closes at 5 p.m. The function V gives the number of visitors in a museum h hours after it opens.

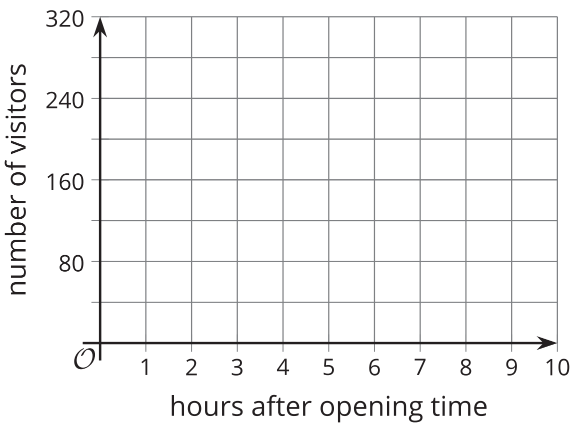
1. Explain what this statement tells us about the situation: $V(1.25) = 28$.

2. Use function notation to represent each statement:

    a. At 1 p.m., there were 257 visitors in the museum.

    b. At the time of closing, there were no visitors in the museum.

3. Use the previous statements about the visitors in the museum to sketch a graph that could represent the function.



Unit 4 Lesson 3 Cumulative Practice Problems

1. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Choose the equation that represents the statement: "At 1:30 p.m., the temperature was 20 degrees Celsius."

   A. $f(1:30) = 20$

   B. $f(1.5) = 20$

   C. $f(13:30) = 20$

   D. $f(13.5) = 20$

2. Tyler filled up his bathtub, took a bath, and then drained the tub. The function B gives the depth of the water, in inches, t minutes after Tyler began to fill the bathtub.

   Explain the meaning of each statement in this situation.

   a. $B(0) = 0$

   b. $B(1) < B(7)$

   c. $B(9) = 11$

   d. $B(10) = B(22)$

   e. $B(20) > B(40)$

3. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Use function notation to write an equation or expression for each statement.

   a. The temperature at 12 p.m.

   b. The temperature was the same at 9 a.m. and at 4 p.m.

c. It was warmer at 9 a.m. than at 6 a.m.

d. Some time after midnight, the temperature was 24 degrees Celsius.

4. Select **all** points that are on the graph of f if we know that $f(2) = \text{-}4$ and $f(5) = 3.4$.

A. $(\text{-}4, 2)$

B. $(2, \text{-}4)$

C. $(3.4, 5)$

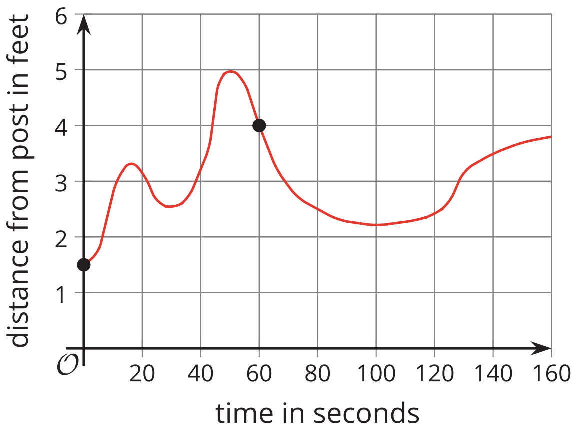
D. $(5, 3.4)$

E. $(2, 5)$

5. Write three statements that are true about this situation. Use function notation.

Function f gives the distance of a dog from a post, in feet, as a function of time, t, in seconds, since its owner left.

Use the $=$ sign in at least one statement and the $<$ sign in another statement.



6. Elena writes the equation $6x + 2y = 12$. Write a new equation that has:

a. exactly one solution in common with Elena's equation

b. no solutions in common with Elena's equation

c. infinitely many solutions in common with Elena's equation

(From Unit 2, Lesson 17.)

7. A restaurant owner wants to see if there is a relationship between the amount of sugar in some food items on her menu and how popular the items are.

   She creates a scatter plot to show the relationship between amount of sugar in menu items and the number of orders for those items. The correlation coefficient for the line of best fit is 0.58.

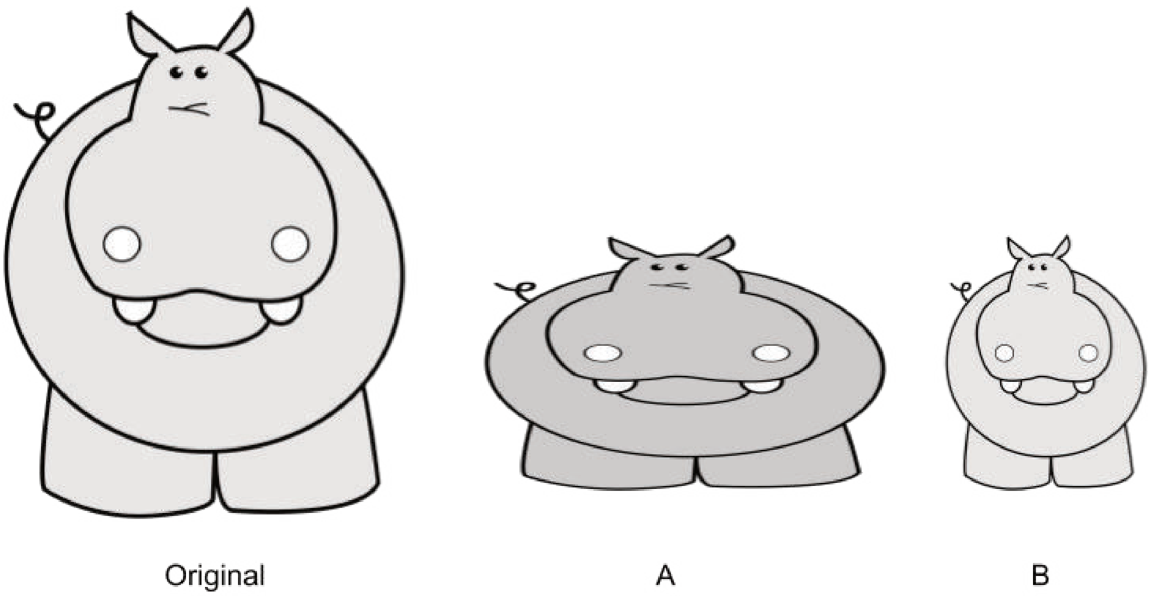
   a. Are the two variables correlated? Explain your reasoning.

   b. Does either of the variables cause the other to change? Explain your reasoning.

   (From Unit 3, Lesson 9.)

Lesson 1: Scale Drawings

1.1: Is That the Same Hippo?



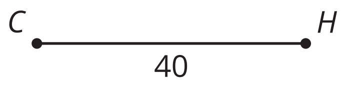
Diego took a picture of a hippo and then edited it. Which is the distorted image? How can you tell?

Is there anything about the pictures you could measure to test whether there's been a distortion?

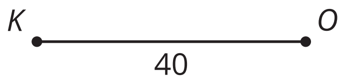
1.2: Sketching Stretching

A **dilation** with center O and positive **scale factor** r takes a point P along the ray OP to another point whose distance is r times farther away from O than P is. If r is less than 1 then the new point is really closer to O, not farther away.

1. Dilate H using C as the center and a scale factor of 3. H is 40 mm from C.

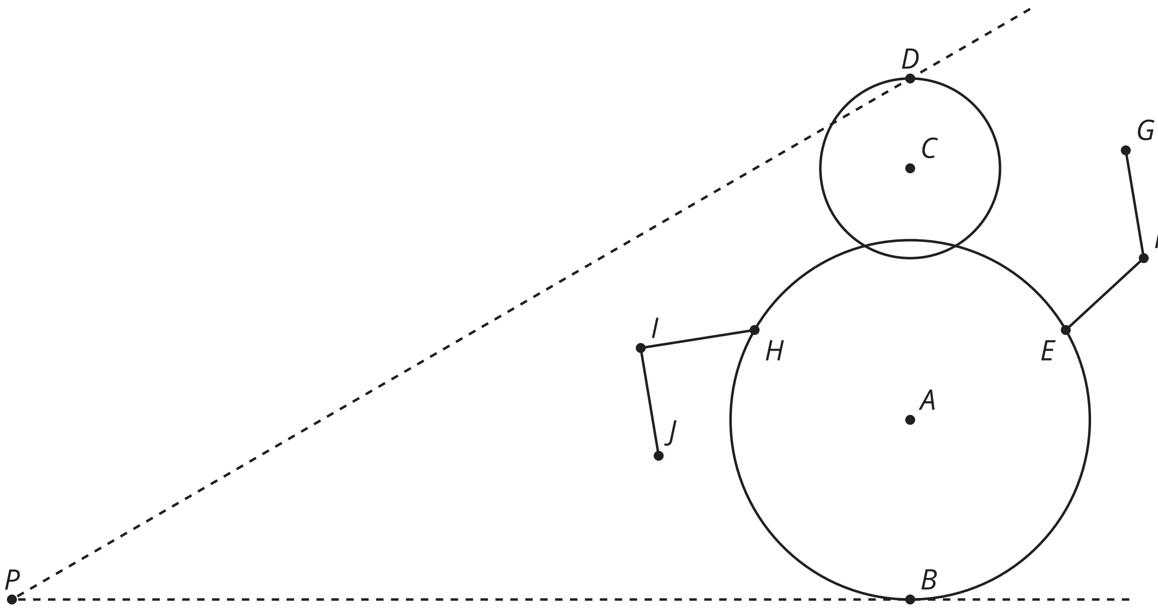


2. Dilate K using O as the center and a scale factor of $\frac{3}{4}$. K is 40 mm from O.



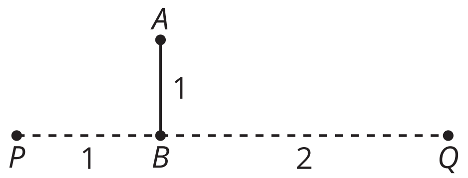
1.3: Mini Me

1. Dilate the figure using center P and scale factor $\frac{1}{2}$.



2. What do you notice? What do you wonder?

Are you ready for more?



1. Dilate segment AB using center P by scale factor $\frac{1}{2}$. Label the result $A' B'$.

2. Dilate the segment AB using center Q by scale factor $\frac{1}{2}$.

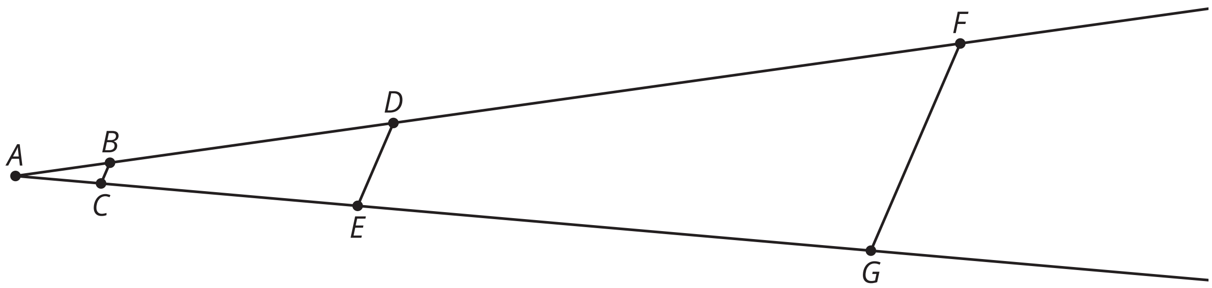
3. How does the length of $A'' B''$ compare to $A' B$? How would the length of $A'' B''$ change if Q was infinitely far away? Explain or show your answer.

Lesson 1 Summary

A scale drawing of an object is a drawing in which all lengths in the drawing correspond to lengths in the object by the same scale. When we scale a figure we need to be sure to scale all of the parts equally or else the image will become distorted.

Creating a scaled copy involves multiplying the lengths in the original figure by a **scale factor**. The scale factor is the factor by which every length in a original figure is multiplied when you make a scaled copy. A scale factor greater than 1 enlarges an object while a scale factor less than 1 shrinks an object. What would a scale factor equal to 1 do?

For example, segment BC is a scaled copy of segment DE with a scale factor of $\frac{1}{4}$. So $BC = \frac{1}{4}DE$. If $DE = 6$, then $BC = \frac{6}{4}$ or 1.5.



To perform a **dilation**, we need a center of dilation, a scale factor, and something to dilate. A dilation with center A and positive scale factor k takes a point D along the ray AD to another point whose distance is k times farther away from A than D is.

Segment FG is a dilation of segment DE using center A and a scale factor of 3. So $FA = 3 \cdot DA$. If $DA = 15$, then $FA = 45$.

Advice on Modeling

These are some steps that successful modelers often take, and questions that they ask themselves. You don’t necessarily have to do all of these steps, or do them in order. Only do the parts that you think will help you make progress.

	**Understand the Question** Think about what the question means before you start making a strategy to answer it. Are there words you want to look up? Does the scenario make sense? Is there anything you want to get clearer on before you start? Ask your classmates or teacher if you need to.
	**Refine the Question** If necessary, rewrite the question you are trying to answer so that it is more specific.
	**Estimate a Reasonable Answer** If you don’t have enough information to decide what’s reasonable, try to come up with an answer that would be too low, and an answer that would be too high.
	**Identify Unknowns** ● What are the meaningful quantities in this situation? Write them down. ● What information would be useful to know? In order to get that information, you could: look it up, take a measurement, or make an assumption.
	**Gather Information** Write down any of the unknown information that you find. As you work, organize your information in a way that makes sense to you.

Experiment!
Try different ideas to make progress toward answering your question. If you are stuck, think about:

- Helpful ways to organize the information you have or organize your work
- Questions you *can* answer using the information you have
- Ways to represent mathematical relationships or sets of data (tables, equations, scatter plots, graphs, statistical plots)
- Tools that are available for representing mathematics, both digital and analog

Check Your Reasoning
Do you have a first answer to your question? Great! See if it's reasonable.

- Make sure you can explain what the answer means in terms of the original problem.
- Check your precision: Is your answer overly precise (do you really need all those decimal places)? Not precise enough (were you overly aggressive with your rounding)?

Use and Improve Your Model

- Did you make assumptions or measurements? How can you express your model more generally, so that it would work for a range of numbers instead of the specific numbers you used?
- What are the limitations of your model? That is, what are some ways it is not realistic? Does it only work for certain inputs but not others? Are there any meaningful inputs affecting the outcome that are not accounted for? If possible, improve your model to take these into account.
- What are the implications of your model? That is, what should people or organizations do differently or smarter as a result of what your model shows? What would be effective ways to communicate with them?
- What are the areas for further research? That is, what new things are you wondering about that could be investigated, by you or someone else?

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">Decide What to Model</td><td>• Assumptions made are clearly identified and justified. Resulting limitations are stated when appropriate.
• Variables of interest are clearly identified and chosen wisely, and appropriate units of measure are used.</td><td>• Assumptions are noted but lacking in justification or difficult to find.
• Variables of interest are noted, but may lack justification, be difficult to find, or not be measured with appropriate units.</td><td>• No assumptions are stated.
• No variables are defined.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Ask questions about the situation to understand it better
• Check the assumptions you're making to see if they're reasonable (Try asking a friend, or imagining that you're a person involved in the scenario. Would those assumptions make sense to you?)
• Double-check the variables you've identified: Are there other quantities in the situation that could vary? Is there something you've identified as a variable that is actually fixed or determined? (Remember that more abstract things like time and speed are also quantities.)</td></tr>
<tr><td rowspan="2">Formulate a Mathematical Model</td><td>• An appropriate model is chosen and represented clearly.
• Diagrams, graphs, etc. are clear and appropriately labeled.</td><td>• Parts of the model are unclear, incomplete, or contain mistakes.</td><td>• No model is presented, or presentation contains significant errors.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
• Check your model more carefully to make sure it really fits well
• Consider a wider variety of possible models, to find one that fits the situation better
• Think about the situation more deeply before trying to find a model
• Convince a skeptic: Pretend that you think your model is inadequate, or ask a friend to pretend to be skeptical of it. What would a skeptic find wrong with your model? Try to fix those things, or explain why they're not actually problems.</td></tr>
</table>

<table>
<tr><th rowspan="2">Skill</th><th colspan="3">Score</th><th rowspan="2">Notes or Comments</th></tr>
<tr><th>Proficient</th><th>Developing</th><th>Needs Revisiting</th></tr>
<tr><td rowspan="2">Use Your Model to Reach a Conclusion</td><td>Solution is relevant to original problem.
Reader can easily understand the reasoning leading to the solution.
Relevant details are included like units of measure.</td><td>Solution is not well-aligned to original problem, or aspects of the solution are difficult to understand or incomplete.</td><td>No solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
Double-check your calculations: Show them to someone else to see if they agree, or take a break and look at your calculations again later
Make sure your calculations are justified by your model: Ask yourself how you decided what to calculate, and see if your reasoning matches up with your model
Think more deeply about what your conclusions mean in the original scenario: Imagine you're a person involved in the scenario, or explain your conclusions to someone else and see if they have questions</td></tr>
<tr><td rowspan="2">Refine and Share Your Model</td><td>The model's implications are clearly stated.
The limitations of the model and solution are addressed.</td><td>The limitations of the model and solution are addressed but lacking in depth or ignoring key components.</td><td>No interpretation of model and solution is provided.</td><td></td></tr>
<tr><td colspan="4">To improve at this skill, you could:
Think more creatively about what your conclusions mean: Ask yourself "If I was involved in this situation, what would I understand better because of these conclusions? What would I want to do next?"
Be skeptical of your model: What don't you like about it, and what can you do to fix those things?
Explain your model to someone else: Tell them how it works and why it's good. If you're not sure how it works or why it's good, you might need to change it.</td></tr>
</table>

Lesson 1: Scale Drawings

Cool Down: Match the Scale Factors

Match the image to the scale factor from FG to $F'G'$:

Image 1

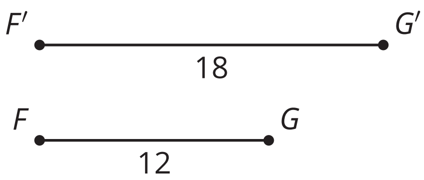


Image 2

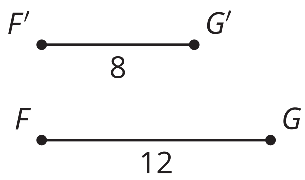


Image 3

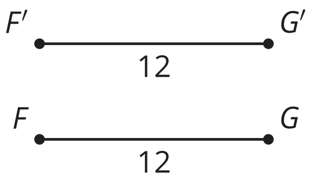


Image 4

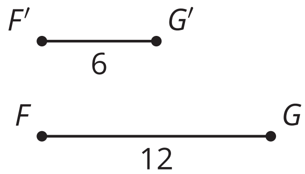
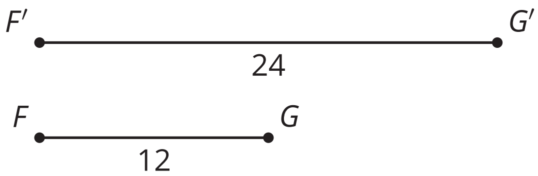


Image 5



A. 1

B. $\frac{3}{2}$

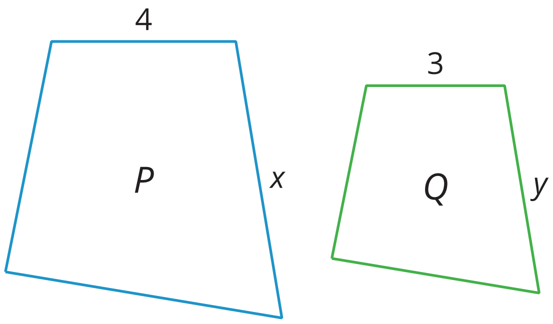
C. 2

D. $\frac{2}{3}$

E. $\frac{1}{2}$

Unit 3 Lesson 1 Cumulative Practice Problems

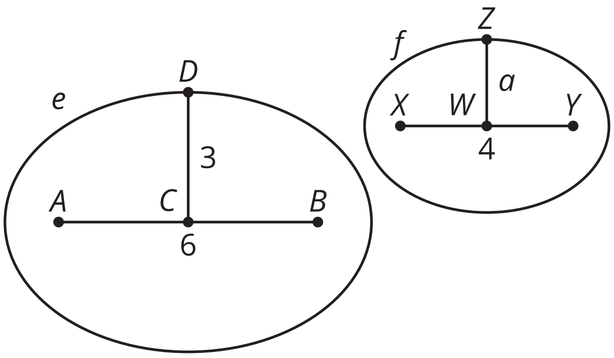
1. Polygon Q is a scaled copy of Polygon P.



   a. The value of x is 6, what is the value of y?

   b. What is the scale factor?

2. Figure f is a scaled copy of Figure e .



   We know:

   - $AB = 6$
   - $CD = 3$
   - $XY = 4$
   - $ZW = a$

   Select **all** true equations.

A. $\frac{6}{3} = \frac{4}{a}$

B. $\frac{6}{4} = \frac{3}{a}$

C. $\frac{3}{4} = \frac{6}{a}$

D. $\frac{6}{3} = \frac{a}{4}$

E. $\frac{6}{4} = \frac{a}{3}$

F. $\frac{3}{4} = \frac{a}{6}$

3. Solve each equation.

   a. $\frac{2}{5} = \frac{x}{15}$

   b. $\frac{4}{3} = \frac{x}{7}$

   c. $\frac{7}{5} = \frac{28}{x}$

   d. $\frac{11}{4} = \frac{5}{x}$

4. Select the shape that has 180 degree rotational symmetry.

   A. Rhombus

   B. Trapezoid

   C. Isosceles trapezoid

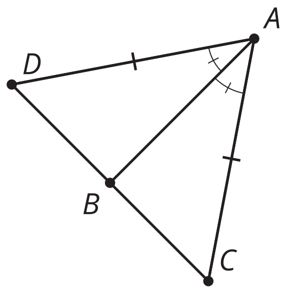
   D. Quadrilateral

   (From Unit 2, Lesson 14.)

5. Name a quadrilateral in which the diagonal is also a line of symmetry. Explain how you know the diagonal is a line of symmetry.

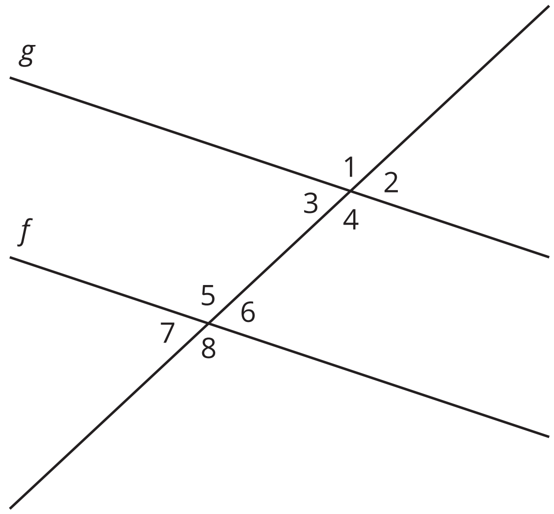
   (From Unit 2, Lesson 14.)

6. In isosceles triangle DAC, AD is congruent to AC and AB is an angle bisector of angle DAC. How does Kiran know that AB is a perpendicular bisector of segment CD?



(From Unit 2, Lesson 8.)

7. In the figure shown, lines f and g are parallel. Select **all** angles that are congruent to angle 1.



A. 1

B. 2

C. 3

D. 4

E. 5

F. 6

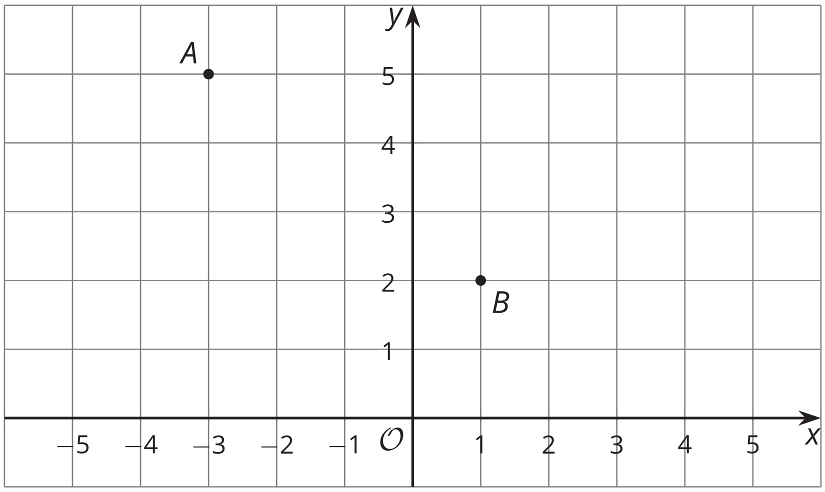
G. 7

H. 8

(From Unit 1, Lesson 20.)

Lesson 1: Rigid Transformations in the Plane

1.1: Traversing the Plane

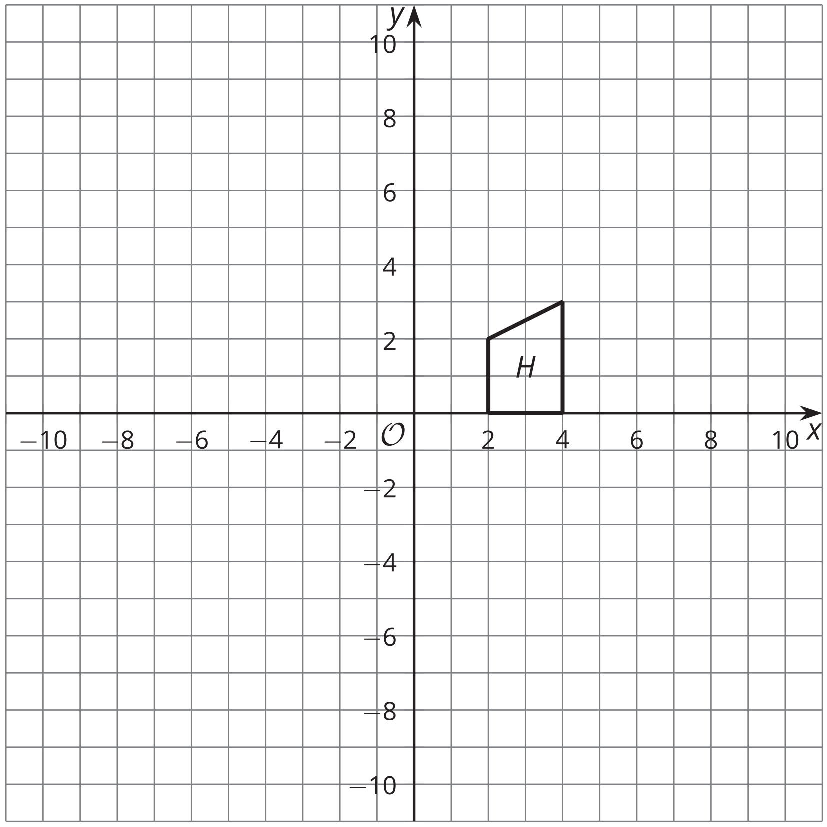


1. How far is point A from point B?

2. What transformations will take point A to point B?

1.2: Transforming with Coordinates

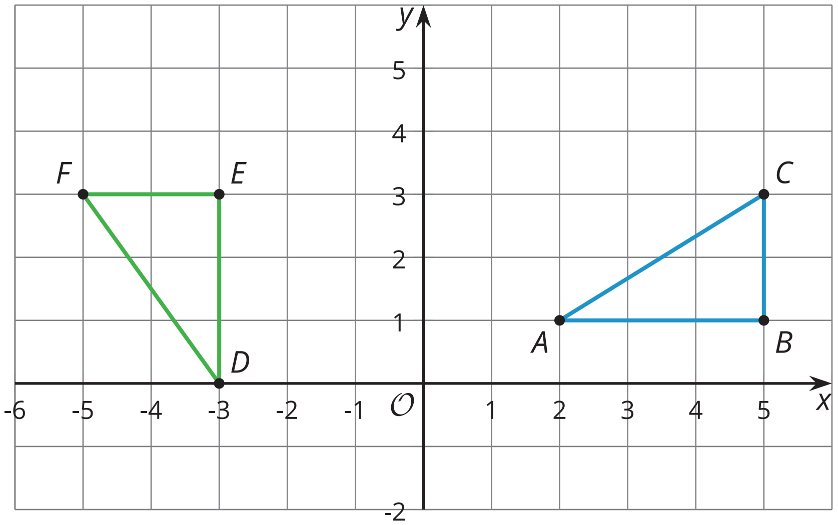
First, predict where each transformation will land. Next, carry out the transformation.



1. Rotate Figure H clockwise using center $(2, 0)$ by 90 degrees.
   Translate the image by the directed line segment from $(2, 0)$ to $(3, -4)$.
   Label the result R.

2. Reflect Figure H across the y-axis.
   Rotate the image counterclockwise using center $(0, 0)$ by 90 degrees.
   Label the result L.

1.3: Congruent by Coordinates



1. Calculate the length of each side in triangles ABC and DEF.

2. Calculate the measure of each angle in triangles ABC and DEF.

3. The triangles are congruent. How do you know this is true?

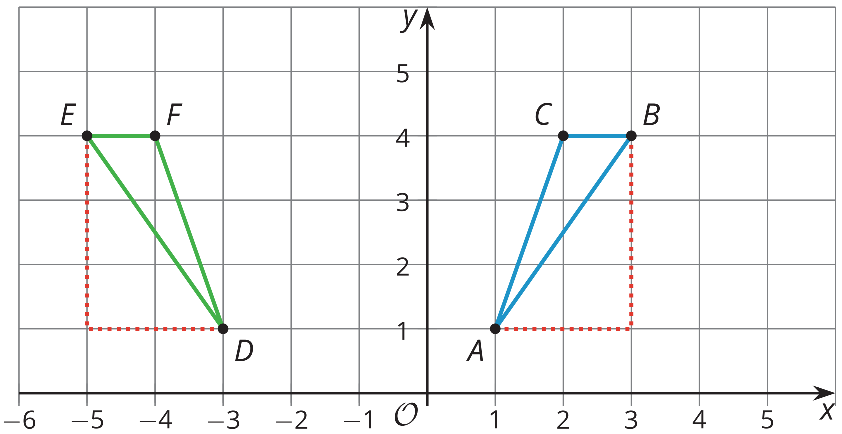
4. Because the triangles are congruent, there must be a sequence of rigid motions that takes one to the other. Find a sequence of rigid motions that takes triangle ABC to triangle DEF.

Are you ready for more?

What single transformation would take triangle ABC to triangle DEF?

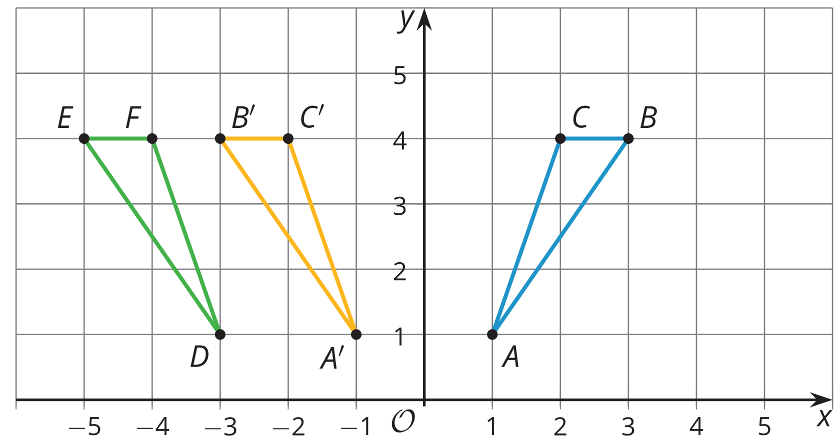
Lesson 1 Summary

The triangles shown here look like they might be congruent. Since we know the coordinates of all the vertices, we can compare lengths using the Pythagorean Theorem. The length of segment AB is $\sqrt{13}$ units because the segment is the hypotenuse of a right triangle with vertical side length 3 units and horizontal side length 2 units. The length of segment DE is $\sqrt{13}$ units as well, because this segment is also the hypotenuse of a right triangle with leg lengths 3 and 2 units.



The other sides of the triangles are congruent as well: The lengths of segments BC and FE are 1 unit each, and the lengths of segments AC and DF are each $\sqrt{10}$ units, because they are both hypotenuses of right triangles with leg lengths 1 and 3 units. So triangle ABC is congruent to triangle DEF by the Side-Side-Side Triangle Congruence Theorem.

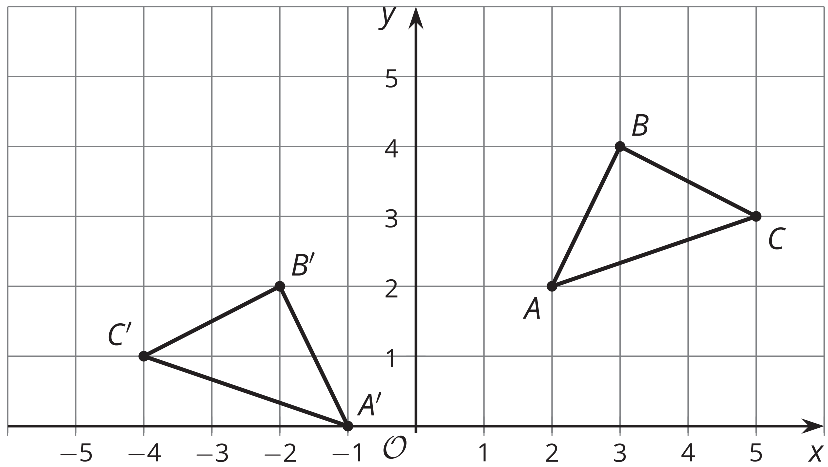
Since triangle ABC is congruent to triangle DEF, there is a sequence of rigid motions that takes triangle ABC to triangle DEF. Here is one possible sequence: First, reflect triangle ABC across the y-axis. Then, translate the image by the directed line segment from $(-1, 1)$ to $(-3, 1)$.



Lesson 1: Rigid Transformations in the Plane

Cool Down: A Transformed Triangle

Triangle $A'B'C'$ is the image of triangle ABC after a sequence of rigid motions.



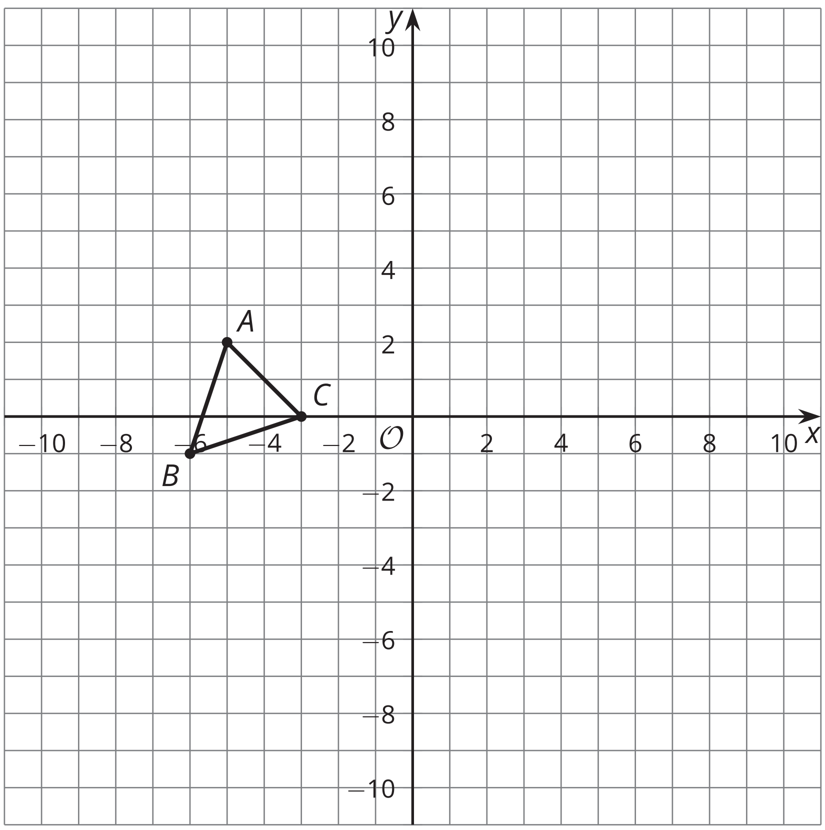
Find a sequence of transformations that takes triangle ABC to triangle $A'B'C'$.

Unit 6 Lesson 1 Cumulative Practice Problems

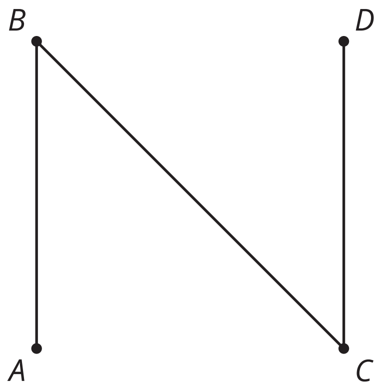
1. Reflect triangle ABC over the line $x = -3$.

   Translate the image by the directed line segment from $(0, 0)$ to $(4, 1)$.

   What are the coordinates of the vertices in the final image?



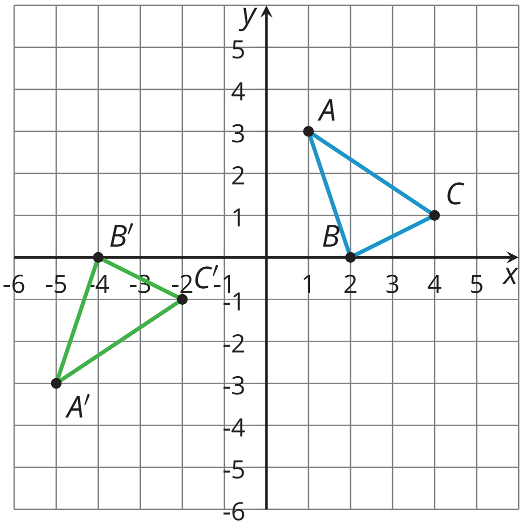
2. Three line segments form the letter N. Rotate the letter N counterclockwise around the midpoint of segment BC by 180 degrees. Describe the result.



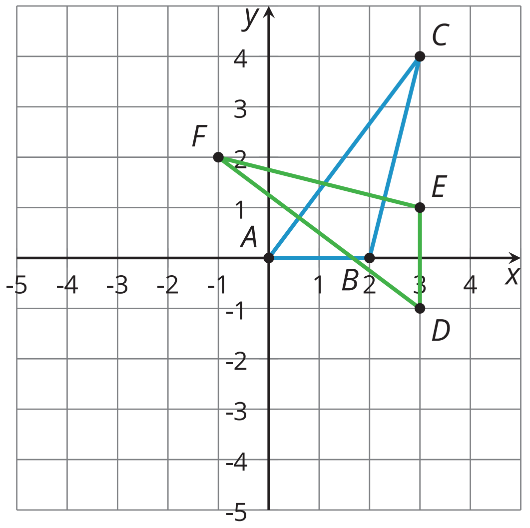
(From Unit 1, Lesson 14.)

3. Triangle ABC has coordinates $A = (1, 3)$, $B = (2, 0)$, and $C = (4, 1)$. The image of this triangle after a sequence of transformations is triangle $A'B'C'$ where $A' = (-5, -3)$, $B' = (-4, 0)$, and $C' = (-2, -1)$.

   Write a sequence of transformations that takes triangle ABC to triangle $A'B'C'$.



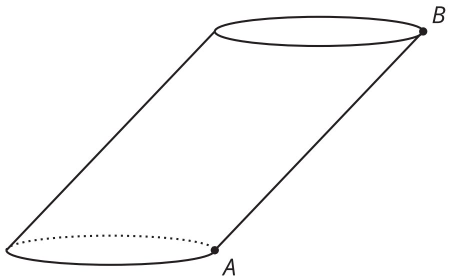
4. Prove triangle ABC is congruent to triangle DEF.



5. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a 1.17 gram diamond in the shape of a pyramid whose base has area 2 cm^2 and whose height is 0.5 centimeters sink or float? Explain your reasoning.

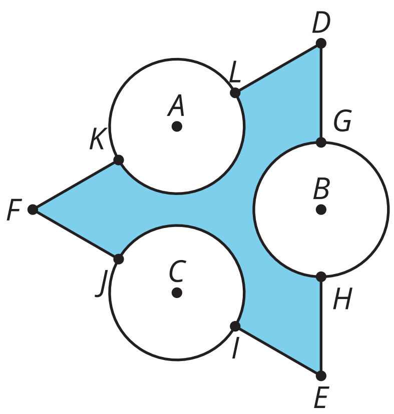
(From Unit 5, Lesson 17.)

6. *Technology required*. An oblique cylinder with a base of radius 2 units is shown. The top of the cylinder can be obtained by translating the base by the directed line segment AB which has length 16 units. The segment AB forms a 30° angle with the plane of the base. What is the volume of the cylinder?



(From Unit 5, Lesson 11.)

7. This design began from the construction of an equilateral triangle. Record at least 3 rigid transformations (rotation, reflection, translation) you see in the design.

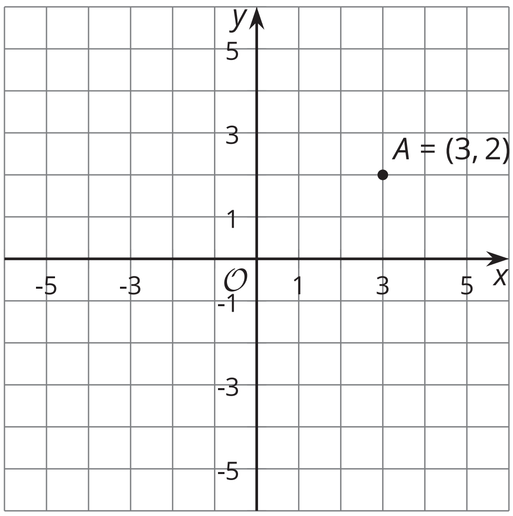


(From Unit 1, Lesson 22.)

Lesson 2: Transformations as Functions

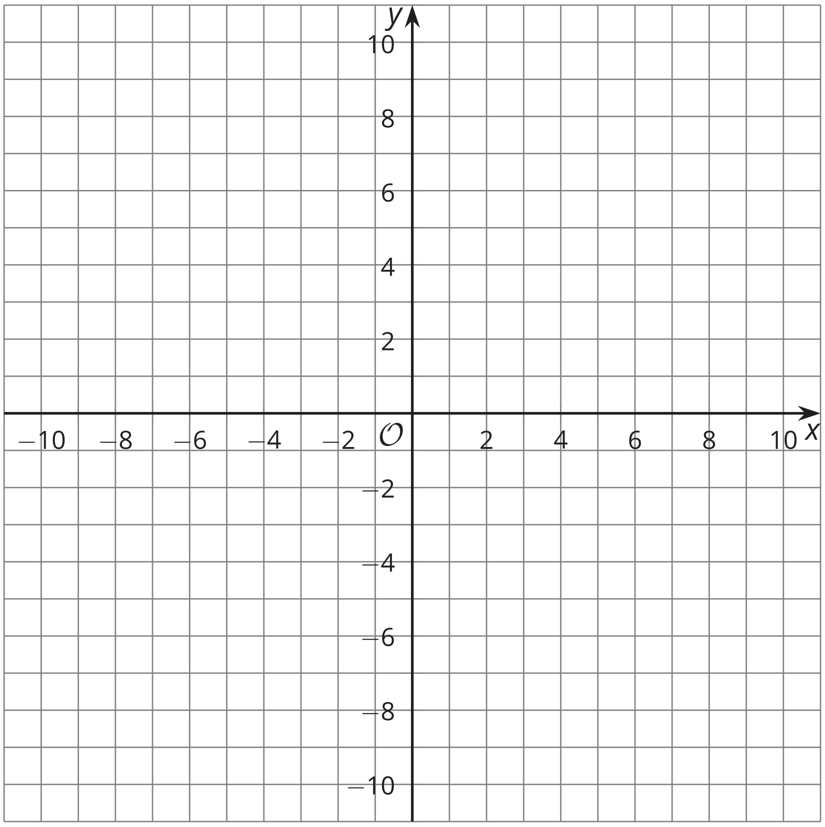
2.1: Math Talk: Transforming a Point

Mentally find the coordinates of the image of A under each transformation.



- Translate A by the directed line segment from $(0, 0)$ to $(0, 2)$.
- Translate A by the directed line segment from $(0, 0)$ to $(-4, 0)$.
- Reflect A across the x-axis.
- Rotate A 180 degrees clockwise using the origin as a center.

2.2: Inputs and Outputs



1. For each point (x, y), find its image under the transformation $(x + 12, y - 2)$.
   a. $A = (\text{-}10, 5)$

   b. $B = (\text{-}4, 9)$

   c. $C = (\text{-}2, 6)$

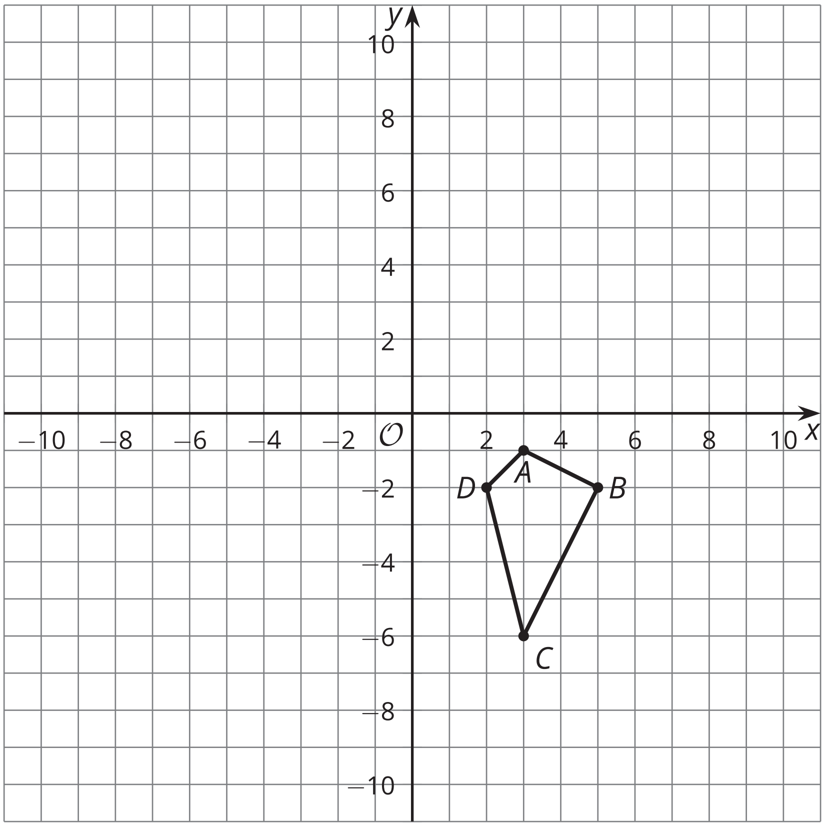
2. Next, sketch triangle ABC and its image on the grid. What transformation is $(x, y) \rightarrow (x + 12, y - 2)$?

3. For each point (x, y) in the table, find $(2x, 2y)$.

(x, y)	$(2x, 2y)$
(-1, -3)	
(-1, 1)	
(5, 1)	
(5, -3)	

4. Next, sketch the original figure (the (x, y) column) and image (the $(2x, 2y)$ column). What transformation is $(x, y) \rightarrow (2x, 2y)$?

2.3: What Does it Do?

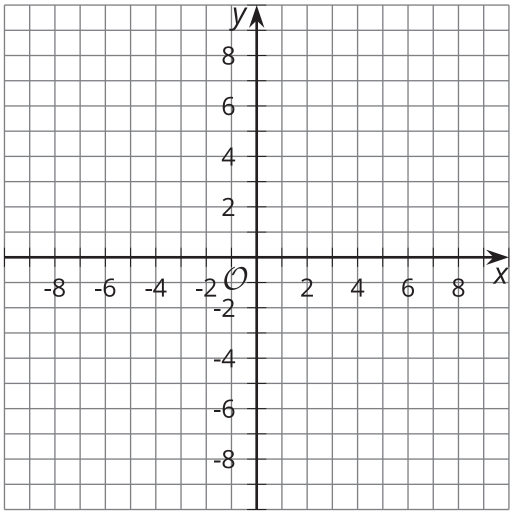


1. Here are some transformation rules. Apply each rule to quadrilateral $ABCD$ and graph the resulting image. Then describe the transformation.
    a. Label this transformation Q: $(x, y) \rightarrow (2x, y)$

    b. Label this transformation R: $(x, y) \rightarrow (x, -y)$

    c. Label this transformation S: $(x, y) \rightarrow (y, -x)$

Are you ready for more?



1. Plot the quadrilateral with vertices $(4, \text{-}2)$, $(8, 4)$, $(8, \text{-}6)$, and $(\text{-}6, \text{-}6)$. Label this quadrilateral A.

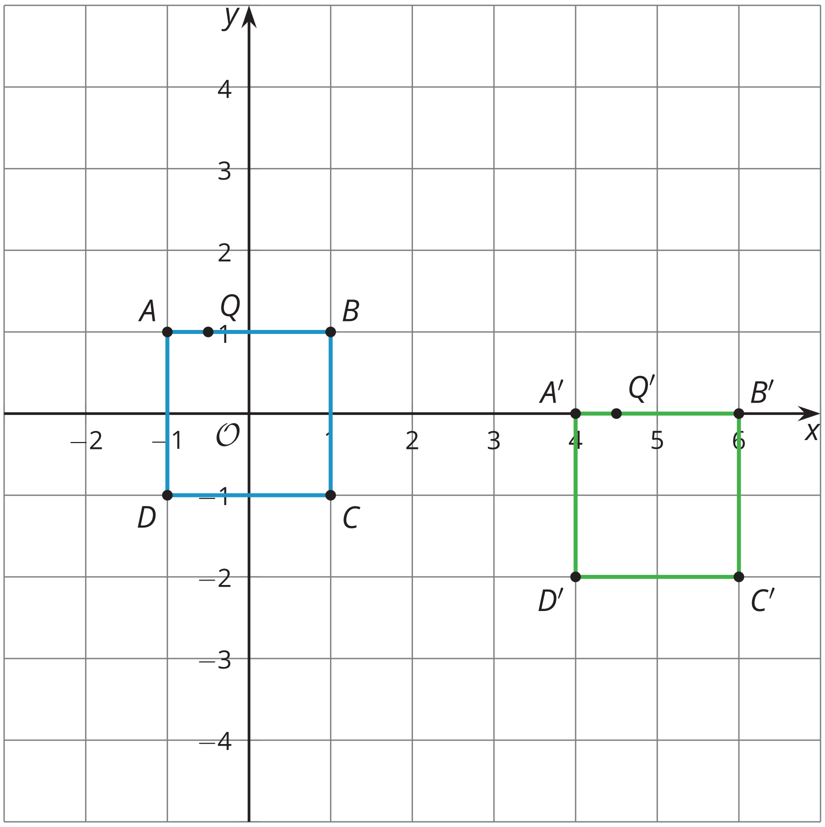
2. Plot the quadrilateral with vertices $(\text{-}2, 4)$, $(4, 8)$, $(\text{-}6, 8)$, and $(\text{-}6, \text{-}6)$. Label this quadrilateral A'.

3. How are the coordinates of quadrilateral A related to the coordinates of quadrilateral A'?

4. What single transformation takes quadrilateral A to quadrilateral A'?

Lesson 2 Summary

Square $ABCD$ has been translated by the directed line segment from $(-1, 1)$ to $(4, 0)$. The result is square $A'B'C'D'$.



Here is a list of coordinates in the original figure and corresponding coordinates in the image. Do you see the rule for taking points in the original figure to points in the image?

original figure	image
$A = (-1, 1)$	$A' = (4, 0)$
$B = (1, 1)$	$B' = (6, 0)$
$C = (1, -1)$	$C' = (6, -2)$
$D = (-1, -1)$	$D' = (4, -2)$
$Q = (-0.5, 1)$	$Q' = (4.5, 0)$

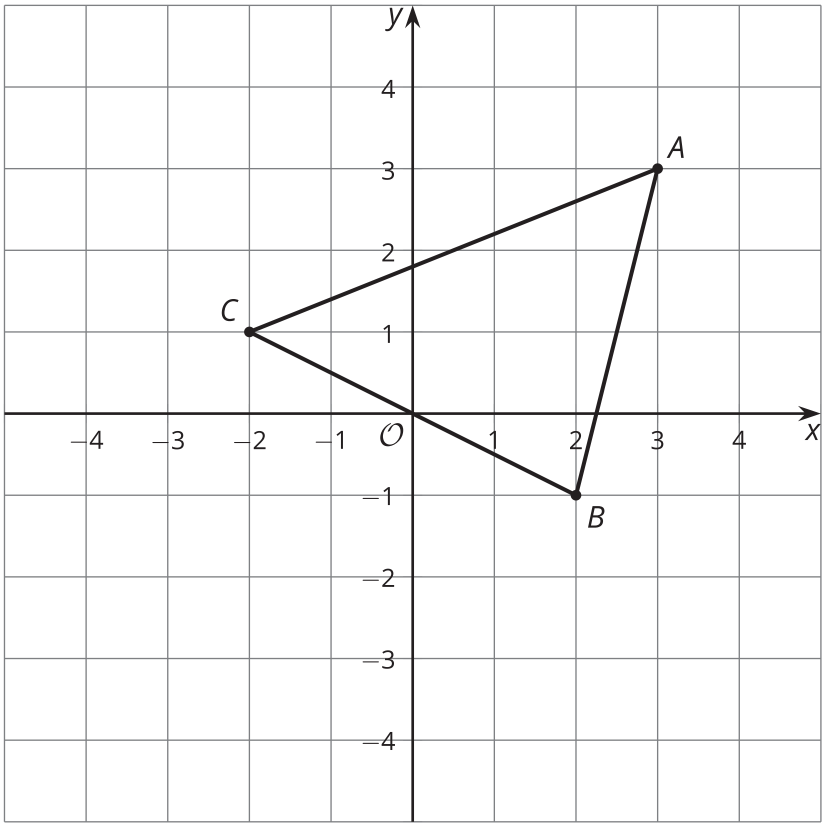
This table looks like a table that shows corresponding inputs and outputs of a function. A transformation is a special type of function that takes points in the plane as inputs and gives other points as outputs. In this case, the function's rule is to add 5 to the x-coordinate and subtract 1 from the y-coordinate.

We write the rule this way: $(x, y) \rightarrow (x + 5, y - 1)$.

Lesson 2: Transformations as Functions

Cool Down: Ready? Transform!

Triangle ABC is graphed.



1. Transform triangle ABC using the rule $(x, y) \rightarrow (-x, y)$.

2. Describe the transformation precisely.

Unit 6 Lesson 2 Cumulative Practice Problems

1. Match each coordinate rule to a description of its resulting transformation.

   A. $(x, y) \rightarrow (x + 3, y)$

   B. $(x, y) \rightarrow (2x, 2y)$

   C. $(x, y) \rightarrow (x, y + 4)$

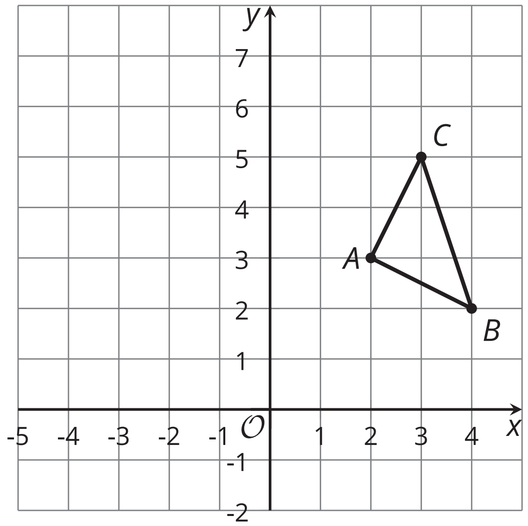
   D. $(x, y) \rightarrow (x, y - 4)$

   E. $(x, y) \rightarrow (x - 3, y + 4)$

   1. Translate by the directed line segment from $(0, 0)$ to $(0, 4)$.
   2. Translate by the directed line segment from $(0, 0)$ to $(3, 0)$.
   3. Dilate using the origin as the center and a scale factor of 2.
   4. Translate by the directed line segment from $(0, 0)$ to $(0, \text{-}4)$.
   5. Translate by the directed line segment from $(0, 0)$ to $(\text{-}3, 4)$.

2. a. Draw the image of triangle ABC under the transformation $(x, y) \rightarrow (x - 4, y + 1)$. Label the result T.

   b. Draw the image of triangle ABC under the transformation $(x, y) \rightarrow (\text{-}x, y)$. Label the result R.



3. Here are some transformation rules. For each rule, describe whether the transformation is a rigid motion, a dilation, or neither.

   a. $(x, y) \rightarrow (x - 2, y - 3)$

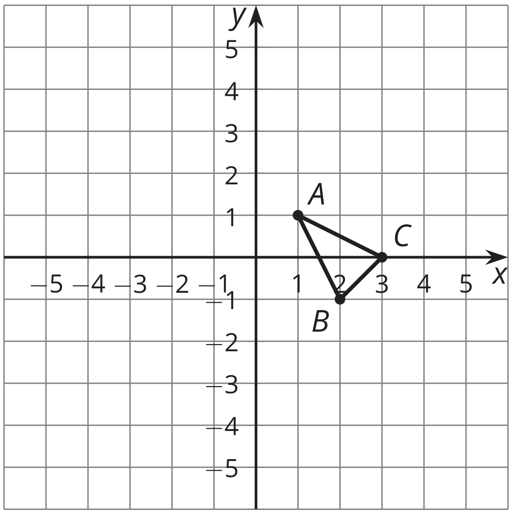
   b. $(x, y) \rightarrow (2x, 3y)$

   c. $(x, y) \rightarrow (3x, 3y)$

   d. $(x, y) \rightarrow (2 - x, y)$

4. Reflect triangle ABC over the line $x = 0$. Call this new triangle $A'B'C'$. Then reflect triangle $A'B'C'$ over the line $y = 0$. Call the resulting triangle $A''B''C''$.

   Which single transformation takes ABC to $A''B''C''$?



   A. Translate triangle ABC by the directed line segment from $(1, 1)$ to $(-2, 1)$.

   B. Reflect triangle ABC across the line $y = -x$.

   C. Rotate triangle ABC counterclockwise using the origin as the center by 180 degrees.

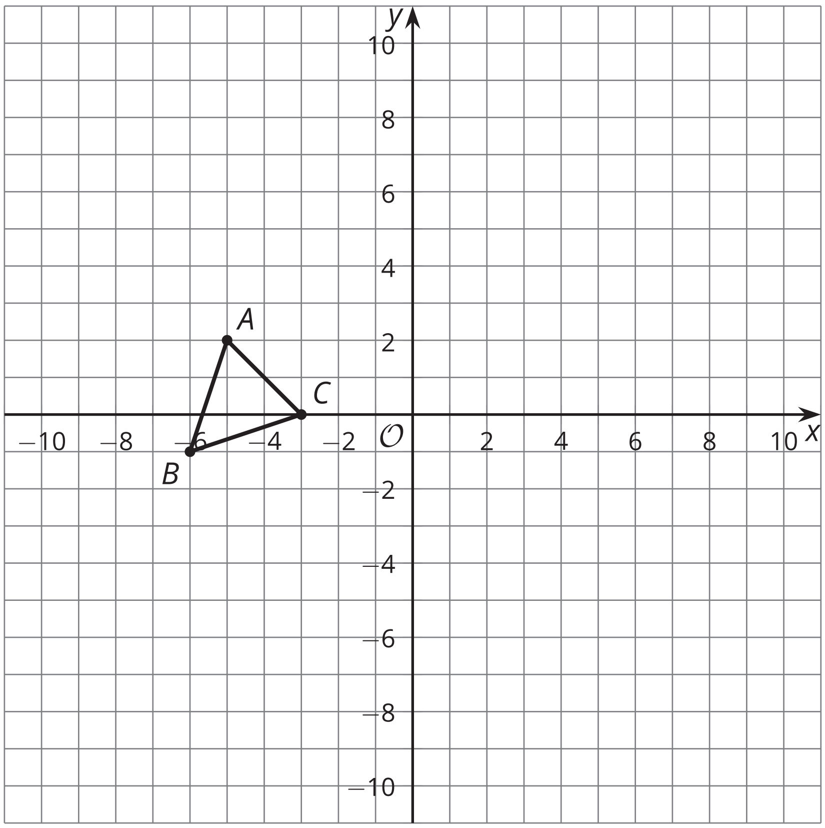
   D. Dilate triangle ABC using the origin as the center and a scale factor of 2.

(From Unit 6, Lesson 1.)

5. Reflect triangle ABC over the line $y = 2$.

   Translate the image by the directed line segment from $(0, 0)$ to $(3, 2)$.

   What are the coordinates of the vertices in the final image?

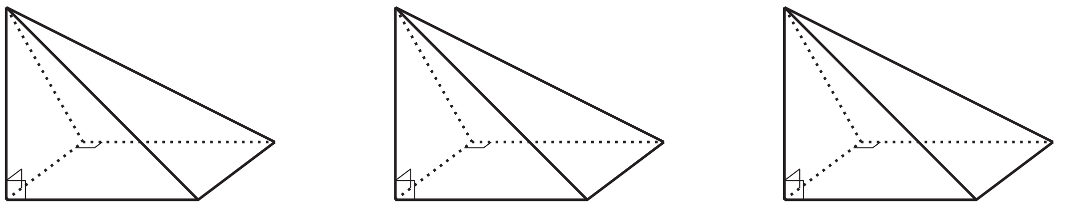


(From Unit 6, Lesson 1.)

6. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a cylindrical log with radius 0.4 meters, height 5 meters, and mass 1,950 kilograms sink or float? Explain your reasoning.

(From Unit 5, Lesson 17.)

7. These 3 congruent square pyramids can be assembled into a cube with side length 3 feet. What is the volume of each pyramid?



A. 1 cubic foot

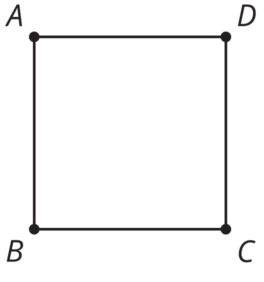
B. 3 cubic feet

C. 9 cubic feet

D. 27 cubic feet

(From Unit 5, Lesson 12.)

8. Reflect square $ABCD$ across line CD. What is the ratio of the length of segment AA' to the length of segment AD? Explain or show your reasoning.



(From Unit 2, Lesson 1.)