Plan for Algebra 2 Unit 2: Polynomials and Rational Functions

Relevant Unit(s) to review: Algebra 1, Unit 2 and Unit 6

Essential prior concepts to engage with this unit	• Multiplying binomials. • Solving systems of linear equations by substitution.
Brief narrative of approach	This unit builds on the previous unit as well as the foundation students established in Algebra 1. In order to successfully investigate whether polynomials are closed under addition, subtraction, and multiplication, students need to be familiar with those three operations for polynomials. While polynomial identities are interesting and provide a good opportunity for practice manipulating expressions, lessons from this section are reasonable to skip given time constraints.

Lessons to Add	Lessons to Remove or Modify
1. Algebra 1, Unit 6, Lesson 8	1. Remove Alg2.2.24. Students practice determining when an equation is an identity, a lesson that is nice but not necessary.
Lessons added: 1	Lessons removed: 1

Modified Plan for Algebra 2 Unit 2

Day	IM lesson	Notes
	Alg2.2 Check Your Readiness Assessment	Note that the Check Your Readiness assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in Alg2.2
1	Alg2.2.1	
2	Alg2.2.2	
3	Alg2.2.3	
4	Alg1.6.8	If the initial assessment shows that students are not familiar with multiplying binomials or factoring quadratics, include this lesson before continuing with grade-level content.
5	Alg2.2.4	
6	Alg2.2.5	
7	Alg2.2.6	
8	Alg2.2.7	
9	Alg2.2.8	
10	Alg2.2.9	
11	Alg2.2.10	
12	Alg2.2.11	
13	Alg2.2.12	
14	Alg2.2.13	
15	Alg2.2.14	

16	Alg2.2.15	
17	Alg2.2 Mid-Unit Assessment	
18	Alg2.2.16	
19	Alg2.2.17	
20	Alg2.2.18	
21	Alg2.2.19	
22	Alg2.2.20	
23	Alg2.2.21	
24	Alg2.2.22	
25	Alg2.2.23	
26	Alg2.2.25	
27	Alg2.2.26	
28	Alg2.2 End of Unit Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

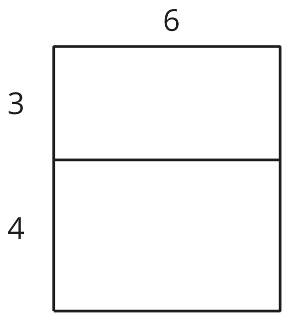
Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
Alg2.2.1	+	E	This lesson introduces the vocabulary term polynomial. The situation of a folded box is one students will return to in a later lesson.
Alg2.2.2	+	E	This lesson offers an opportunity to evaluate, write, and interpret polynomials.
Alg2.2.3	+	D	This lesson introduces the vocabulary terms degree, relative minimum, and relative maximum.
Alg2.2.4	0	E	This lesson offers an opportunity to add, subtract, and multiply polynomials. Students conclude that the result of these operations will always be a polynomial.
Alg2.2.5	+	D	This lesson offers an opportunity to connect the factors of a polynomial, its zeros, and the horizontal intercepts of its graph.
Alg2.2.6	+	E	This lesson offers an opportunity to compare factored form with standard form. Students identify information about the graph of the polynomial from each.
Alg2.2.7	+	D	This lesson leads to the conclusion that a polynomial with a factor of $x - a$ has a zero when $x = a$.
Alg2.2.8	0	E	This lesson introduces the idea of end behavior.
Alg2.2.9	0	D	This lesson continues to study end behavior, now considering polynomials with leading coefficients.
Alg2.2.10	+	A	This lesson introduces the vocabulary term multiplicity. Students make solid connections

			between the shape of a graph and the structure of the factored equation for polynomial functions.
Alg2.2.11	+	D	This lesson expands on systems of equations to include systems of equations involving quadratics.
Alg2.2.12	0	E	This lesson introduces polynomial division using a diagram.
Alg2.2.13	+	D	This lesson introduces polynomial division using long division.
Alg2.2.14	-	A	This lesson consists primarily of an info gap that reinforces the characteristics of a polynomial needed to draw a sketch.
Alg2.2.15	0	D	This lesson leads to students understanding the Remainder Theorem.
Alg2.2.16	+	E	This lesson introduces rational functions.
Alg2.2.17	+	D	This lesson introduces vertical asymptotes.
Alg2.2.18	+	D	This lesson introduces horizontal asymptotes.
Alg2.2.19	0	D	This lesson introduces rational functions with new types of end behavior.
Alg2.2.20	0	E	This lesson begins the study of rational equations
Alg2.2.21	0	A	This lesson offers an opportunity to write and solve rational equations. Students interpret the equations and solutions in context.
Alg2.2.22	0	D	This lesson introduces extraneous solutions
Alg2.2.23	-	E	This lesson introduces polynomial identities. It offers an opportunity to practice manipulating polynomial expressions if needed.

Alg2.2.24	-	D	This lesson continues to study polynomial identities.
Alg2.2.25	-	D	This lesson leads to students deriving the formula for the sum of the first *n* terms in a geometric sequence.
Alg2.2.26	-	A	This lesson offers an opportunity to apply the formula from the previous lesson.

Lesson 8: Equivalent Quadratic Expressions

8.1: Diagrams of Products

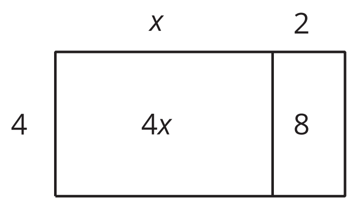


1. Explain why the diagram shows that $6(3+4) = 6 \cdot 3 + 6 \cdot 4$.

2. Draw a diagram to show that $5(x+2) = 5x + 10$.

8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.

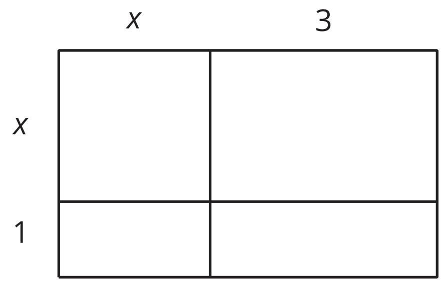


1. Draw a diagram to show that $n(2n+5)$ and $2n^2 + 5n$ are equivalent expressions.

2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
   a. $6\left(\frac{1}{3}n+2\right)$ b. $p(4p+9)$ c. $5r\left(r+\frac{3}{5}\right)$ d. $(0.5w+7)w$

8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x + 1$ and $x + 3$. Use this diagram to show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:
    a. $(x + 5)^2$

    b. $2x(x + 4)$

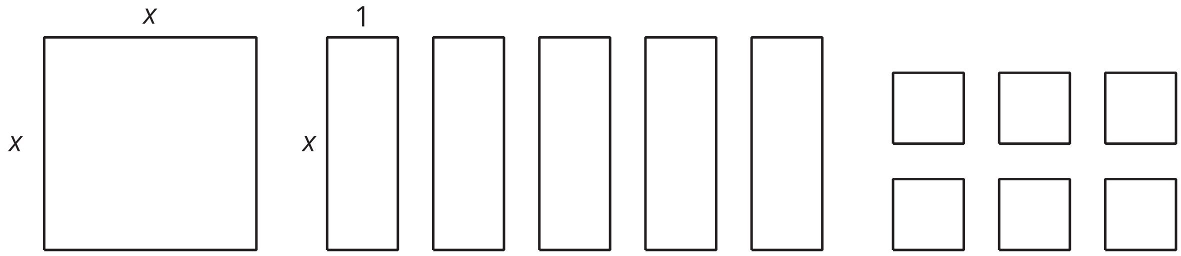
    c. $(2x + 1)(x + 3)$

    d. $(x + m)(x + n)$

3. Write an equivalent expression for each expression without drawing a diagram:
    a. $(x + 2)(x + 6)$

    b. $(x + 5)(2x + 10)$

Are you ready for more?



1. Is it possible to arrange an x by x square, five x by 1 rectangles and six 1 by 1 squares into a single large rectangle? Explain or show your reasoning.

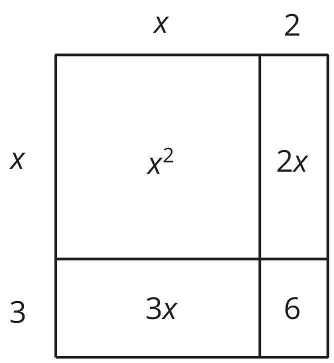
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?

3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed with $x(18 - x)$, which can also be written as $18x - x^2$. The former is a product of x and $18 - x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x + 2)(x + 3)$. We can write an equivalent expression by thinking about each factor, the $(x + 2)$ and $(x + 3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x + 2)$ and $(x + 3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x + 2)(x + 3)$ is equivalent to $x^2 + 2x + 3x + 6$, or to $x^2 + 5x + 6$.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x + 2$) is multiplied by every term in the other factor (the x and the 3 in $x + 3$).

$$
\begin{aligned}
&(x + 2)(x + 3) \\
&= x(x + 3) + 2(x + 3) \\
&= x^2 + 3x + 2x + (2)(3) \\
&= x^2 + (3+2)x + (2)(3)
\end{aligned}
$$

In general, when a quadratic expression is written in the form of $(x + p)(x + q)$, we can apply the distributive property to rewrite it as $x^2 + px + qx + pq$ or $x^2 + (p + q)x + pq$.

Lesson 8: Equivalent Quadratic Expressions

Cool Down: Writing Equivalent Expressions

1. Use a diagram to show that $(3x+1)(x+2)$ is equivalent to $3x^2+7x+2$.

2. Is $(x+4)^2$ equivalent to $2x^2+8x+8$? Explain or show your reasoning.

Unit 6 Lesson 8 Cumulative Practice Problems

1. Draw a diagram to show that $(2x + 5)(x + 3)$ is equivalent to $2x^2 + 11x + 15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   A. $(x + 2)(x + 6)$
   B. $(2x + 8)(x + 2)$
   C. $(x + 8)(x + 4)$
   D. $(x + 2)(2x + 6)$

   1. $x^2 + 12x + 32$
   2. $2x^2 + 10x + 12$
   3. $2x^2 + 12x + 16$
   4. $x^2 + 8x + 12$

3. Select **all** expressions that are equivalent to $x^2 + 4x$.

   A. $x(x + 4)$
   B. $(x + 2)^2$
   C. $(x + x)(x + 4)$
   D. $(x + 2)^2 - 4$
   E. $(x + 4)x$

4. Tyler drew a diagram to expand $(x + 5)(2x + 3)$.

   a. Explain Tyler's mistake.

   b. What is the correct expanded form of $(x + 5)(2x + 3)$?

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

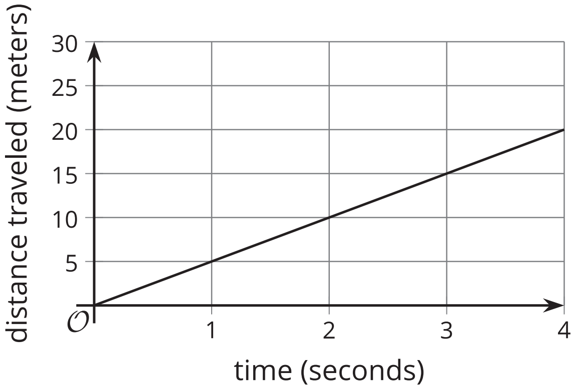
5. Explain why the values of the exponential expression 3^x will eventually overtake the values of the quadratic expression $10x^2$.

(From Unit 6, Lesson 4.)

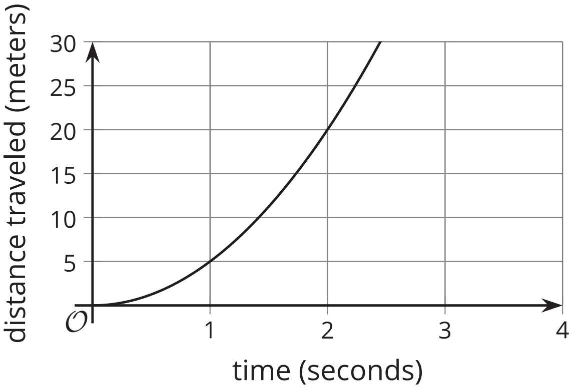
6. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   Which graph could represent this situation? Explain how you know.

Graph A



Graph B



(From Unit 6, Lesson 5.)

7. Consider a function q defined by $q(x) = x^2$. Explain why negative values are not included in the range of q.

(From Unit 4, Lesson 10.)

8. Based on past concerts, a band predicts selling $600 - 10p$ concert tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many concert tickets the band expects to sell and what revenues it expects to receive at the given ticket prices.

ticket price (dollars)	number of tickets	revenue (dollars)
10		
15		
20		
30		
35		
45		
50		
60		
p		

   b. In this model, at what ticket prices will the band earn no revenue at all?

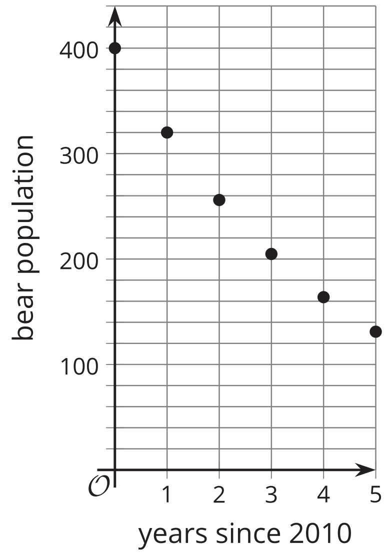
   c. At what ticket prices should the band sell the tickets if it must earn at least 8,000 dollars in revenue to break even (to not lose money) on a given concert. Explain how you know.

(From Unit 6, Lesson 7.)

9. A population of bears decreases exponentially.

    a. What is the annual factor of decrease for the bear population? Explain how you know.

    b. Using function notation, represent the relationship between the bear population, b, and the number of years since the population was first measured, t. That is, find a function, f, so that $b = f(t)$.



(From Unit 5, Lesson 8.)

10. Equations defining functions a, b, c, d, and f are shown here.

    Select **all** the equations that represent exponential functions.

    A. $a(x) = 2^3 \cdot x$

    B. $b(t) = \left(\frac{2}{3}\right)^t$

    C. $c(m) = \frac{1}{5} \cdot 2^m$

    D. $d(x) = 3x^2$

    E. $f(t) = 3 \cdot 2^t$

(From Unit 5, Lesson 8.)