Plan for Algebra 2 Unit 1: Sequences and Functions

Relevant Unit(s) to review: Algebra 1: Units 4 and 5

Essential prior concepts to engage with this unit	• Function notation. • Translating among table, graph, function, and description for both linear and exponential situations.
Brief narrative of approach	This unit is intentionally written to be an accessible way to start the year. There are opportunities to review linear and exponential functions embedded in the lessons so it is not necessary to pre-teach those topics. If the Check Your Readiness shows that students are not familiar with function notation it will be necessary to add activities from a previous course to teach that content.

Lessons to Add	**Lessons to Remove or Modify**
1. Alg 1 Unit 4 Activity 2.2 2. Alg 1 Unit 4 Lesson 4 3. Alg 1 Unit 5 Activity 3.1 (pay particular attention to the review in the launch)	1. Remove Alg2.1.4 - optional lesson on learning spreadsheets. 2. Remove Alg2.1.7 - a lesson comprised primarily of an info gap that reinforces all the parts of a recursive function.
Lessons added: 3	Lessons removed: 2

Modified Plan for Algebra 2 Unit 1

Day	IM lesson	Notes
	Alg2.1 Check Your Readiness Assessment	Note that the Check Your Readiness Assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in Alg2.1.
1	Alg2.1.1	
2	Alg2.1.2	
3	Alg2.1.3	
4	Alg1.4.2.2 Alg1.4.4	If the initial assessment shows that students are not familiar with function notation, include these activities before continuing with grade level content. Start with Algebra 1, Unit 4, Lesson 4, Activity 1. Then Algebra 1, Unit 4, Lesson 2, Activity 2. Followed by Algebra 1, Unit 4, Lesson 4, Activity 2.
5	Alg2.1.5	
6	Alg2.1.6	Only include this optional lesson if students demonstrate a need on the Check Your Readiness Assessment or during the previous lesson.
7	Alg1.5.3 Alg2.1.8	If the initial assessment shows that students are not familiar with exponent rules, consider replacing the warm-up from Lesson 8 with the warm-up from Algebra 1, Unit 5, Lesson 3 paying particular attention to the launch.
8	Alg2.1.9	
9	Alg2.1.10	
10	Alg2.1.11	
11	Alg2.1 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

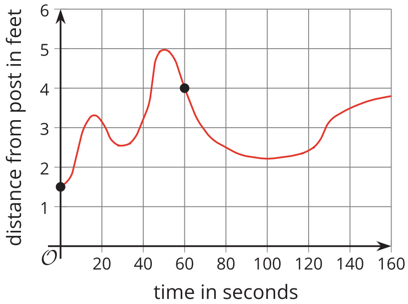
Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
Alg2.1.1	+	E	Introduces the vocabulary terms sequence and term.
Alg2.1.2	+	E	Introduces geometric sequences. Includes an opportunity to review exponential functions.
Alg2.1.3	+	E	Introduces arithmetic sequences. Includes an opportunity to review linear functions.
Alg2.1.4	-	E	This optional lesson focuses on learning how to use spreadsheets.
Alg2.1.5	+	D	Introduces recursive functions for sequences written using function notation.
Alg2.1.6	-	D	This optional lesson is available for students to review the topics introduced in lesson 5.
Alg2.1.7	-	A	This lesson is comprised primarily of an info gap that reinforces all the parts of a recursive function.
Alg2.1.8	0	E	Introduces closed form functions for sequences written using function notation. (Note that this is an embedded review of writing linear and exponential functions.)
Alg2.1.9	0	A	Applies sequences in situations to provide an opportunity to discuss domain.
Alg2.1.10	0	A	Applies sequences in situations to emphasize that different representations of functions are useful in different ways.
Alg2.1.11	0	A/E	Introduces summation of sequences which will be revisited in a subsequent unit.

Lesson 2: Function Notation

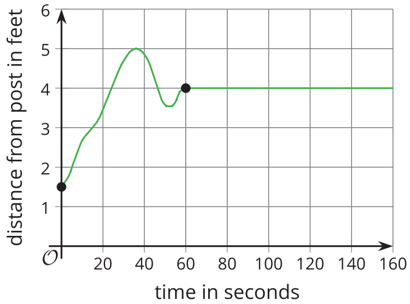
2.1: Back to the Post!

Here are the graphs of some situations you saw before. Each graph represents the distance of a dog from a post as a function of time since the dog owner left to purchase something from a store. Distance is measured in feet and time is measured in seconds.

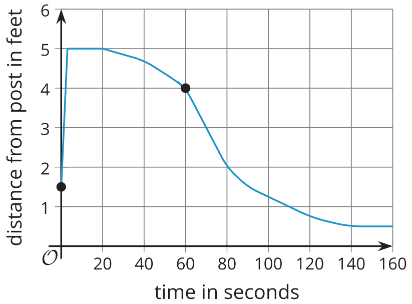
Day 1



Day 2



Day 3



1. Use the given graphs to answer these questions about each of the three days:

   a. How far away was the dog from the post 60 seconds after the owner left?

   Day 1: Day 2: Day 3:

   b. How far away was the dog from the post when the owner left?

   Day 1: Day 2: Day 3:

   c. The owner returned 160 seconds after he left. How far away was the dog from the post at that time?

   Day 1: Day 2: Day 3:

   d. How many seconds passed before the dog reached the farthest point it could reach from the post?

   Day 1: Day 2: Day 3:

2. Consider the statement, "The dog was 2 feet away from the post after 80 seconds." Do you agree with the statement?

3. What was the distance of the dog from the post 100 seconds after the owner left?

2.2: A Handy Notation

Let's name the functions that relate the dog's distance from the post and the time since its owner left: function f for Day 1, function g for Day 2, function h for Day 3. The input of each function is time in seconds, t.

1. Use function notation to complete the table.

	day 1	day 2	day 3
a. distance from post 60 seconds after the owner left			
b. distance from post when the owner left			
c. distance from post 150 seconds after the owner left			

2. Describe what each expression represents in this context:

   a. $f(15)$

   b. $g(48)$

   c. $h(t)$

3. The equation $g(120) = 4$ can be interpreted to mean: "On Day 2, 120 seconds after the dog owner left, the dog was 4 feet from the post."

   What does each equation mean in this situation?

   a. $h(40) = 4.6$

b. $f(t) = 5$

c. $g(t) = d$

2.3: Birthdays

Rule B takes a person's name as its input, and gives their birthday as the output.

input	output
Abraham Lincoln	February 12

Rule P takes a date as its input and gives a person with that birthday as the output.

input	output
August 26	Katherine Johnson

1. Complete each table with three more examples of input-output pairs.
2. If you use your name as the input to B, how many outputs are possible? Explain how you know.
3. If you use your birthday as the input to P, how many outputs are possible? Explain how you know.
4. Only one of the two relationships is a function. The other is not a function. Which one is which? Explain how you know.
5. For the relationship that is a function, write two input-output pairs from the table using function notation.

Are you ready for more?

1. Write a rule that describes these input-output pairs:

   $F(\text{ONE}) = 3$ $\quad$ $F(\text{TWO}) = 3$ $\quad$ $F(\text{THREE}) = 5$ $\quad$ $F(\text{FOUR}) = 4$

2. Here are some input-output pairs with the same inputs but different outputs:

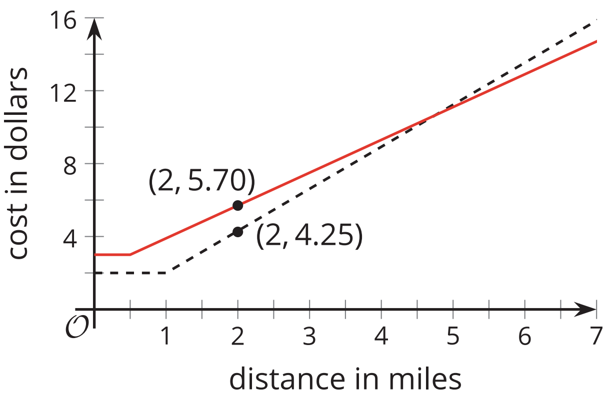
   $v(\text{ONE}) = 2$ $\quad$ $v(\text{TWO}) = 1$ $\quad$ $v(\text{THREE}) = 2$ $\quad$ $v(\text{FOUR}) = 2$

   What rule could define function v?

Lesson 2 Summary

Here are graphs of two functions, each representing the cost of riding in a taxi from two companies—Friendly Rides and Great Cabs.

For each taxi, the cost of a ride is a function of the distance traveled. The input is distance in miles, and the output is cost in dollars.

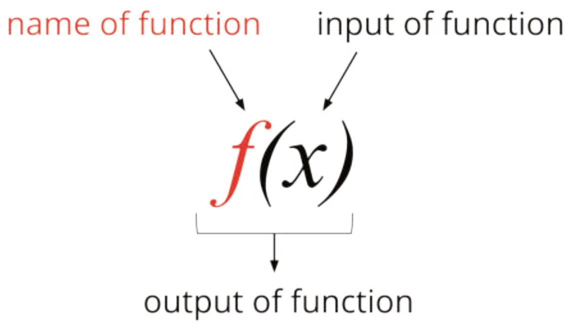


- The point $(2, 5.70)$ on one graph tells us the cost of riding a Friendly Rides taxi for 2 miles.
- The point $(2, 4.25)$ on the other graph tells us the cost of riding a Great Cabs taxi for 2 miles.

We can convey the same information much more efficiently by naming each function and using **function notation** to specify the input and the output.

- Let's name the function for Friendly Rides function f.
- Let's name the function for Great Cabs function g.
- To refer to the cost of riding each taxi for 2 miles, we can write: $f(2)$ and $g(2)$.
- To say that a 2-mile trip with Friendly Rides will cost \$5.70, we can write $f(2) = 5.70$.
- To say that a 2-mile trip with Great Cabs will cost \$4.25, we can write $g(2) = 4.25$.

In general, function notation has this form:



It is read "f of x" and can be interpreted to mean: $f(x)$ is the output of a function f when x is the input.

The function notation is a concise way to refer to a function and describe its input and output, which can be very useful. Throughout this unit and the course, we will use function notation to talk about functions.

Lesson 2: Function Notation

Cool Down: A Growing Puppy

Function Q gives a puppy's weight in pounds as a function of its age in months.

1. What does each expression or equation represent in this situation?

   a. $Q(18)$

   b. $Q(30) = 27.5$

2. Use function notation to represent each statement.

   a. When the puppy turned 12 months old, it weighed 19.6 pounds.

   b. When the puppy was a months old, it weighed w pounds.

Unit 4 Lesson 2 Cumulative Practice Problems

1. The height of water in a bathtub, w, is a function of time, t. Let P represent this function. Height is measured in inches and time in minutes.

   Match each statement in function notation with a description.

   A. $P(0) = 0$

   B. $P(4) = 10$

   C. $P(10) = 4$

   D. $P(20) = 0$

   1. After 20 minutes, the bathtub is empty.
   2. The bathtub starts out with no water.
   3. After 10 minutes, the height of the water is 4 inches.
   4. The height of the water is 10 inches after 4 minutes.

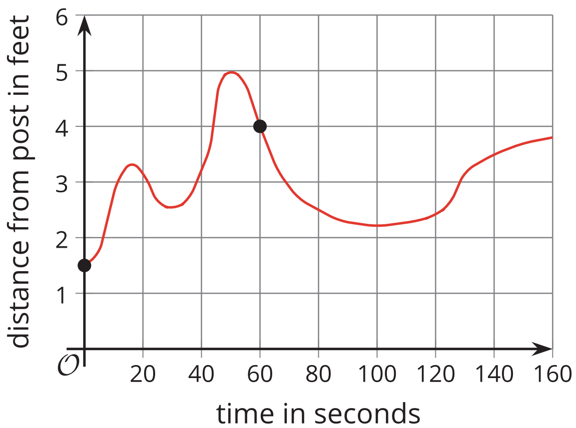
2. Function C takes time for its input and gives a student's Monday class for its output.

   a. Use function notation to represent: A student has English at 10:00.

   b. Write a statement to describe the meaning of $C(11:15) = \text{chemistry}$.

3. Function f gives the distance of a dog from a post, in feet, as a function of time, in seconds, since its owner left.

   Find the value of $f(20)$ and of $f(140)$.



4. Function C gives the cost, in dollars, of buying n apples. What does each expression or equation represent in this situation?

   a. $C(5) = 4.50$

   b. $C(2)$

5. A number of identical cups are stacked up. The number of cups in a stack and the height of the stack in centimeters are related.

   a. Can we say that the height of the stack is a function of the number of cups in the stack? Explain your reasoning.

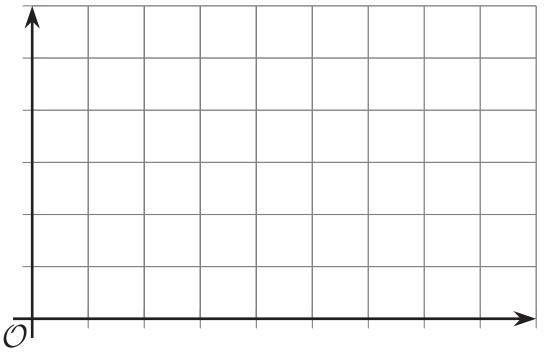
   b. Can we say that the number of cups in a stack is a function of the height of the stack? Explain your reasoning.

   (From Unit 4, Lesson 1.)

6. In a function, the number of cups in a stack is a function of the height of the stack in centimeters.

   a. Sketch a possible graph of the function on the coordinate plane. Be sure to label the axes.

   b. Identify one point on the graph and explain the meaning of the point in the situation.



   (From Unit 4, Lesson 1.)

7. Solve each system of equations without graphing. Show your reasoning.

a. $\begin{cases} -5x + 3y = -8 \\ 3x - 7y = -3 \end{cases}$

b. $\begin{cases} -8x - 2y = 24 \\ 5x - 3y = 2 \end{cases}$

(From Unit 2, Lesson 16.)

Lesson 4: Using Function Notation to Describe Rules (Part 1)

4.1: Notice and Wonder: Two Functions

What do you notice? What do you wonder?

x	$f(x) = 10 - 2x$
1	8
1.5	7
5	0
-2	14

x	$g(x) = x^3$
-2	-8
0	0
1	1
3	27

4.2: Four Functions

Here are descriptions and equations that represent four functions.

$f(x) = 3x - 7$

$g(x) = 3(x - 7)$

$h(x) = \frac{x}{3} - 7$

$k(x) = \frac{x - 7}{3}$

A. To get the output, subtract 7 from the input, then divide the result by 3.

B. To get the output, subtract 7 from the input, then multiply the result by 3.

C. To get the output, multiply the input by 3, then subtract 7 from the result.

D. To get the output, divide the input by 3, and then subtract 7 from the result.

1. Match each equation with a verbal description that represents the same function. Record your results.

2. For one of the functions, when the input is 6, the output is -3. Which is that function: f, g, h, or k? Explain how you know.

3. Which function value—$f(x)$, $g(x)$, $h(x)$, or $k(x)$—is the greatest when the input is 0? What about when the input is 10?

Are you ready for more?

Mai says $f(x)$ is always greater than $g(x)$ for the same value of x. Is this true? Explain how you know.

4.3: Rules for Area and Perimeter

1. A square that has a side length of 9 cm has an area of 81 cm^2. The relationship between the side length and the area of the square is a function.

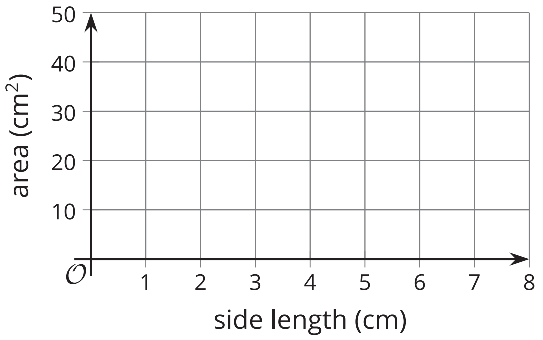
   a. Complete the table with the area for each given side length.

      Then, write a rule for a function, A, that gives the area of the square in cm^2 when the side length is s cm. Use function notation.

side length (cm)	area (cm^2)
1	
2	
4	
6	
s	

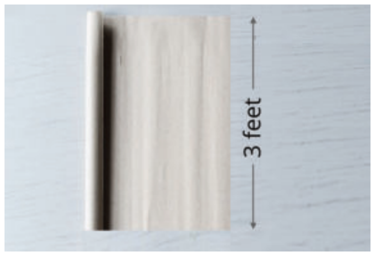
   b. What does $A(2)$ represent in this situation? What is its value?

c. On the coordinate plane, sketch a graph of this function.



2. A roll of paper that is 3 feet wide can be cut to any length.

a. If we cut a length of 2.5 feet, what is the perimeter of the paper?



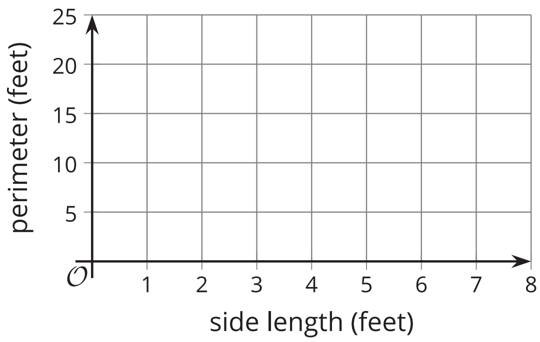
b. Complete the table with the perimeter for each given side length.

Then, write a rule for a function, P, that gives the perimeter of the paper in feet when the side length in feet is ℓ. Use function notation.

side length (feet)	perimeter (feet)
1	
2	
6.3	
11	
ℓ	

c. What does $P(11)$ represent in this situation? What is its value?

d. On the coordinate plane, sketch a graph of this function.



Lesson 4 Summary

Some functions are defined by rules that specify how to compute the output from the input. These rules can be verbal descriptions or expressions and equations. For example:

Rules in words:

- To get the output of function f, add 2 to the input, then multiply the result by 5.
- To get the output of function m, multiply the input by $\frac{1}{2}$ and subtract the result from 3.

Rules in function notation:

- $f(x) = (x + 2) \cdot 5$ or $5(x + 2)$
- $m(x) = 3 - \frac{1}{2}x$

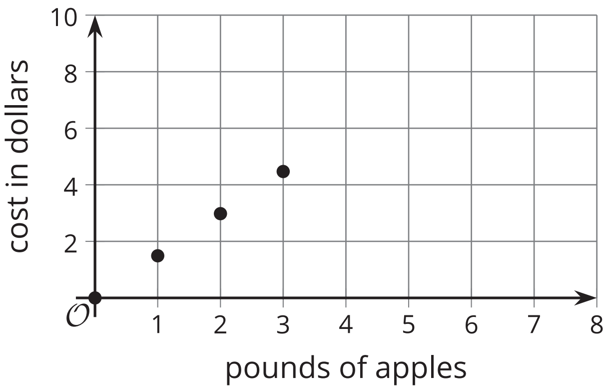
Some functions that relate two quantities in a situation can also be defined by rules and can therefore be expressed algebraically, using function notation.

Suppose function c gives the cost of buying n pounds of apples at \$1.49 per pound. We can write the rule $c(n) = 1.49n$ to define function c.

To see how the cost changes when n changes, we can create a table of values.

pounds of apples, n	cost in dollars, $c(n)$
0	0
1	1.49
2	2.98
3	4.47
n	$1.49n$

Plotting the pairs of values in the table gives us a graphical representation of c.



Lesson 4: Using Function Notation to Describe Rules (Part 1)

Cool Down: Perimeter of a Square

1. Complete the table with the perimeter of a square for each given side length.

side length (inches)	perimeter (inches)
0.5	
7	
20	

2. Write a rule for a function, P, that gives the perimeter of a square in inches when the side length is x inches.

3. What is the value of $P(9.1)$? What does it tell us about the side length and perimeter of the square?

Unit 4 Lesson 4 Cumulative Practice Problems

1. Match each equation with a description of the function it represents.

A. $f(x) = 2x + 4$

B. $g(x) = 2(x + 4)$

C. $h(x) = 4x + 2$

D. $k(x) = 4(x + 2)$

1. To get the output, add 4 to the input, then multiply the result by 2.
2. To get the output, add 2 to the input, then multiply the result by 4.
3. To get the output, multiply the input by 2, then add 4 to the result.
4. To get the output, multiply the input by 4, then add 2 to the result.

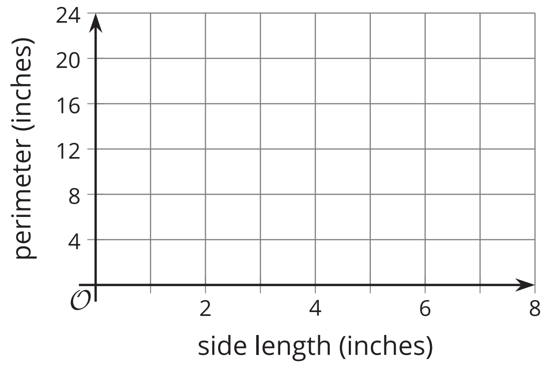
2. Function P represents the perimeter, in inches, of a square with side length x inches.

a. Complete the table.

x	0	1	2	3	4	5	6
$P(x)$							

b. Write an equation to represent function P.

c. Sketch a graph of function P.



3. Functions f and A are defined by these equations.

$$f(x) = 80 - 15x \qquad\qquad A(x) = 25 + 10x$$

Which function has a greater value when x is 2.5?

4. An equilateral triangle has three sides of equal length. Function P gives the perimeter of an equilateral triangle of side length s.

   a. Find $P(2)$

   b. Find $P(10)$

   c. Find $P(s)$

5. Imagine a situation where a person is using a garden hose to fill a child's pool. Think of two quantities that are related in this situation and that can be seen as a function.

   a. Define the function using a statement of the form "________ is a function of ________. Be sure to consider the units of measurement.

   b. Sketch a possible graph of the function. Be sure to label the axes.

   Then, identify the coordinates of one point on the graph and explain its meaning.

(From Unit 4, Lesson 1.)

6. Function C gives the cost, in dollars, of buying n apples.

   Which statement best represents the meaning of $C(10) = 9$?

A. The cost of buying 9 apples

B. The cost of 9 apples is $10.

C. The cost of 10 apples

D. Ten apples cost $9.

(From Unit 4, Lesson 2.)

7. Diego is baking cookies for a fundraiser. He opens a 5-pound bag of flour and uses 1.5 pounds of flour to bake the cookies.

   Which equation or inequality represents f, the amount of flour left in the bag after Diego bakes the cookies?

   A. $f = 1.5$

   B. $f < 1.5$

   C. $f = 3.5$

   D. $f > 3.5$

   (From Unit 2, Lesson 18.)

Lesson 3: Representing Exponential Growth

3.1: Math Talk: Exponent Rules

Rewrite each expression as a power of 2.

$2^3 \cdot 2^4$

$2^5 \cdot 2$

$2^{10} \div 2^7$

$2^9 \div 2$

3.2: What Does x^0 Mean?

1. Complete the table. Take advantage of any patterns you notice.

x	4	3	2	1	0
3^x	81	27			

2. Here are some equations. Find the solution to each equation using what you know about exponent rules. Be prepared to explain your reasoning.

    a. $9^? \cdot 9^7 = 9^7$

    b. $\frac{9^{12}}{9^?} = 9^{12}$

3. What is the value of 5^0? What about 2^0?

Are you ready for more?

We know, for example, that $(2+3)+5=2+(3+5)$ and $2\cdot(3\cdot5)=(2\cdot3)\cdot5$. The grouping with parentheses does not affect the value of the expression.

Is this true for exponents? That is, are the numbers $2^{(3^5)}$ and $(2^3)^5$ equal? If not, which is bigger? Which of the two would you choose as the meaning of the expression 2^{3^5} written without parentheses?

3.3: Multiplying Microbes

1. In a biology lab, 500 bacteria reproduce by splitting. Every hour, on the hour, each bacterium splits into two bacteria.

   a. Write an expression to show how to find the number of bacteria after each hour listed in the table.

   b. Write an equation relating n, the number of bacteria, to t, the number of hours.

   c. Use your equation to find n when t is 0. What does this value of n mean in this situation?

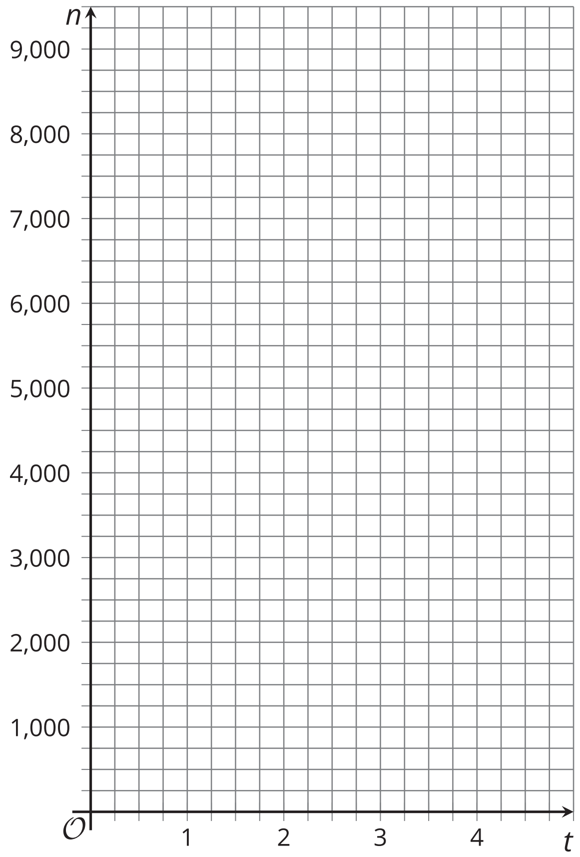
hour	number of bacteria
0	500
1	
2	
3	
6	
t	

2. In a different biology lab, a population of single-cell parasites also reproduces hourly. An equation which gives the number of parasites, p, after t hours is $p=100\cdot3^t$. Explain what the numbers 100 and 3 mean in this situation.

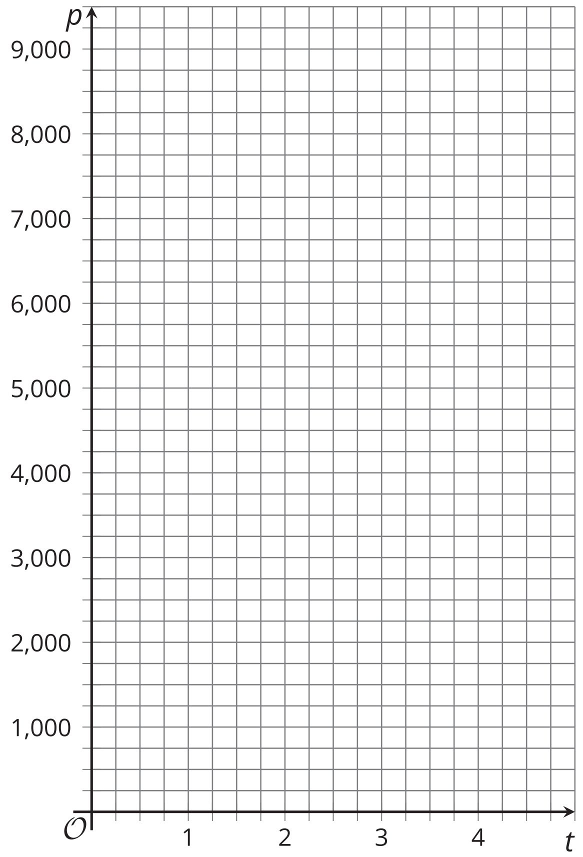
3.4: Graphing the Microbes

1. Refer back to your work in the table of the previous task. Use that information and the given coordinate planes to graph the following:

   a. Graph (t, n) when t is 0, 1, 2, 3, and 4.

   b. Graph (t, p) when t is 0, 1, 2, 3, and 4. (If you get stuck, you can create a table.)

2. On the graph of n, where can you see each number that appears in the equation?

3. On the graph of p, where can you see each number that appears in the equation?

Lesson 3 Summary

In relationships where the change is exponential, a quantity is repeatedly multiplied by the same amount. The multiplier is called the **growth factor**.

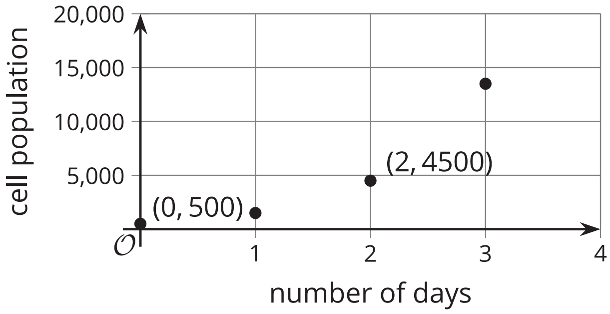
Suppose a population of cells starts at 500 and triples every day. The number of cells each day can be calculated as follows:

number of days	number of cells
0	500
1	1,500 (or $500 \cdot 3$)
2	4,500 (or $500 \cdot 3 \cdot 3$, or $500 \cdot 3^2$)
3	13,500 (or $500 \cdot 3 \cdot 3 \cdot 3$, or $500 \cdot 3^3$)
d	$500 \cdot 3^d$

We can see that the number of cells (p) is changing exponentially, and that p can be found by multiplying 500 by 3 as many times as the number of days (d) since the 500 cells were observed. The *growth factor* is 3. To model this situation, we can write this equation: $p = 500 \cdot 3^d$.

The equation can be used to find the population on any day, including day 0, when the population was first measured. On day 0, the population is $500 \cdot 3^0$. Since $3^0 = 1$, this is $500 \cdot 1$ or 500.

Here is a graph of the daily cell population. The point $(0, 500)$ on the graph means that on day 0, the population starts at 500.



Each point is 3 times higher on the graph than the previous point. $(1, 1500)$ is 3 times higher than $(0, 500)$, and $(2, 4500)$ is 3 times higher than $(1, 1500)$.

Lesson 3: Representing Exponential Growth

Cool Down: Mice in the Forest

A group of biologists is surveying the mice population in a forest. The equation $n = 75 \cdot 3^t$ gives the total number of mice, n, t years since the survey began. Explain what the numbers 75 and 3 mean in this situation.

Unit 5 Lesson 3 Cumulative Practice Problems

1. Which expression is equal to $4^0 \cdot 4^2$?

    A. 0

    B. 1

    C. 16

    D. 64

2. Select **all** expressions are equivalent to 3^8.

    A. 8^3

    B. $\frac{3^{10}}{3^2}$

    C. $3 \cdot 8$

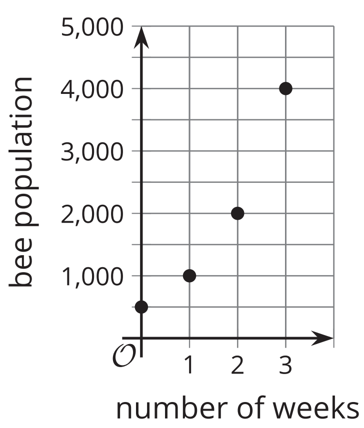
    D. $\left(3^4\right)^2$

    E. $(3 \cdot 3)^4$

    F. $\frac{1}{3^{-8}}$

    (From Unit 5, Lesson 1.)

3. A bee population is measured each week and the results are plotted on the graph.



a. What is the bee population when it is first measured?

b. Is the bee population growing by the same factor each week? Explain how you know.

c. What is an equation that models the bee population, b, w weeks after it is first measured?

4. A bond is initially bought for $250. It doubles in value every decade.

    a. Complete the table.

    b. How many decades does it take before the bond is worth more than $10,000?

    c. Write an equation relating v, the value of the bond, to d, the number of decades since the bond was bought.

decades since bond is bought	dollar value of bond
0	
1	
2	
3	
d	

5. A sea turtle population p is modeled by the equation $p = 400 \cdot \left(\frac{5}{4}\right)^y$ where y is the number of years since the population was first measured.

    a. How many turtles are in the population when it is first measured? Where do you see this in the equation?

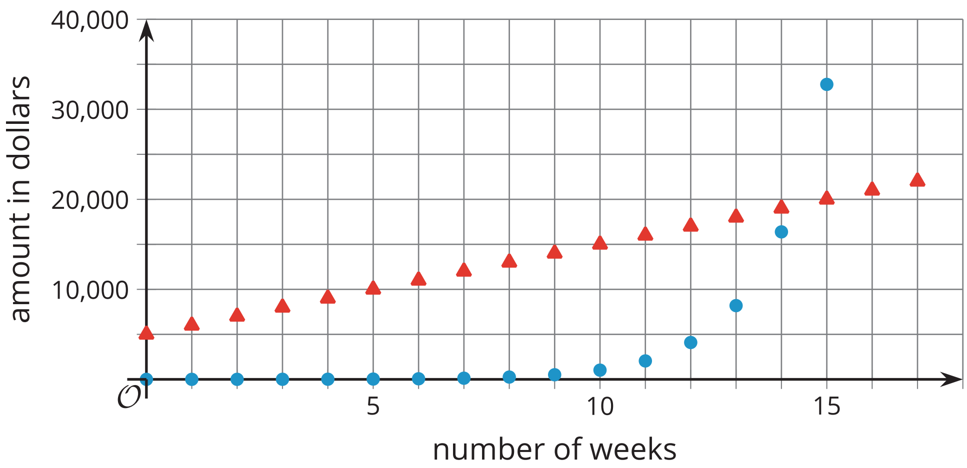
    b. Is the population increasing or decreasing? How can you tell from the equation?

    c. When will the turtle population reach 700? Explain how you know.

6. Bank account A starts with $5,000 and grows by $1,000 each week. Bank account B starts with $1 and doubles each week.

    a. Which account has more money after one week? After two weeks?

b. Here is a graph showing the two account balances. Which graph corresponds to which situation? Explain how you know.



c. Given a choice, which of the two accounts would you choose? Explain your reasoning.

(From Unit 5, Lesson 1.)

7. Match each equation in the first list to an equation in the second list that has the same solution.

A. $y = \frac{2}{5}x + 2$

B. $x = -5 - 2.5y$

C. $y = \frac{10}{5} - 0.4x$

D. $2x = 10 - 5y$

E. $-5y = 2x + 10$

F. $x = 5 - \frac{5}{2}y$

1. $2x + 5y = 10$
2. $-2x - 5y = 10$
3. $-2x + 5y = 10$

(From Unit 2, Lesson 9.)

8. Function F is defined so that its output $F(t)$ is the number of followers on a social media account t days after set up of the account.

   a. Explain the meaning of $F(30) = 8{,}950$ in this situation.

   b. Explain the meaning of $F(0) = 0$.

   c. Write a statement about function F that represents the fact that there were 28,800 followers 110 days after the set up of the account.

   d. Explain the meaning of t in the equation $F(t) = 100{,}000$.

(From Unit 4, Lesson 3.)