Plan for Algebra 1 Unit 5: Exponential Functions

Relevant Unit(s) to review: Grade 8 Unit 7: Exponents and Scientific Notation,
Grade 7 Unit 4: Proportional Relationships and Percentages

Essential prior concepts to engage with this unit	• Percent change • Exponents
Brief narrative of approach	Students learn the laws for operating on exponents at the end of grade 8. While some review of this content is included in the Algebra 1 curriculum, it may not be sufficient for some students because of interruptions to instruction last spring. Students explored percent change in grade 7, and review is included already in the Algebra 1 curriculum, so no additional lessons were included here. If the Check Your Readiness assessment reveals substantial struggles with percentages, consider spending additional time on Lesson 14.

Lessons to Add	**Lessons to Remove or Modify**
1. 8.7 Lessons 2–4: Emphasize Activities 2.3, 3.2, and 4.2. Students can complete practice problems or other activities outside of class. Note that Algebra 1 Lesson 3 provides additional opportunities to review these concepts. 2. 8.7 Lessons 5–6: Emphasize the meaning of negative exponents, and the idea that exponent laws for 10 apply to all numbers. Assign practice problems for outside of class work.	1. Omit Lesson 20. This lesson explores how exponential functions grow over equal intervals. It is not essential for the successful completion of this course and will be addressed in more detail in Algebra 2. 2. Omit Lesson 21. In this lesson students apply their learning from the unit to model different populations with linear and exponential functions. While it provides an opportunity to consolidate learning from the unit, it does not introduce new topics.
Lessons added: 2	Lessons removed: 2

Modified Plan for Algebra 1 Unit 5

Day	IM lesson	Notes
	assessment	A1.5 Check Your Readiness assessment Note that the Check Your Readiness assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in A1.5
1	A1.5.1	Invites students into the big ideas of the unit.
2	A1.5.2	Invites students into the big ideas of the unit.
3	8.7.2 8.7.3 8.7.4	If the initial assessment shows that students are not familiar with exponent laws for exponential expressions with base 10, include activities from these lessons before continuing with grade-level content.
4	8.7.5 8.7.6	If the initial assessment shows that students are not familiar with exponent laws for exponential expressions with other bases or negative exponents, include activities from these lessons before continuing with grade-level content.
5	A1.5.3	Reviews grade 8 content and bridges to Algebra 1 topics on exponential relationships.
6	A1.5.4	Explores exponential decay
7	A1.5.5	Explores exponential expressions in context.
8	A1.5.6	
9	A1.5.7	
10	A1.5.8	
11	A1.5.9	
12	A1.5.10	
13	A1.5.11	

14	A1.5.12	
15	A1.5.13	
16	MUA	
17	A1.5.14	
18	A1.5.15	
19	A1.5.16	
20	A1.5.17	
21	A1.5.18	
22	A1.5.19	
23	EOU Assessment	If lesson 20 is skipped, omit question 6.

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

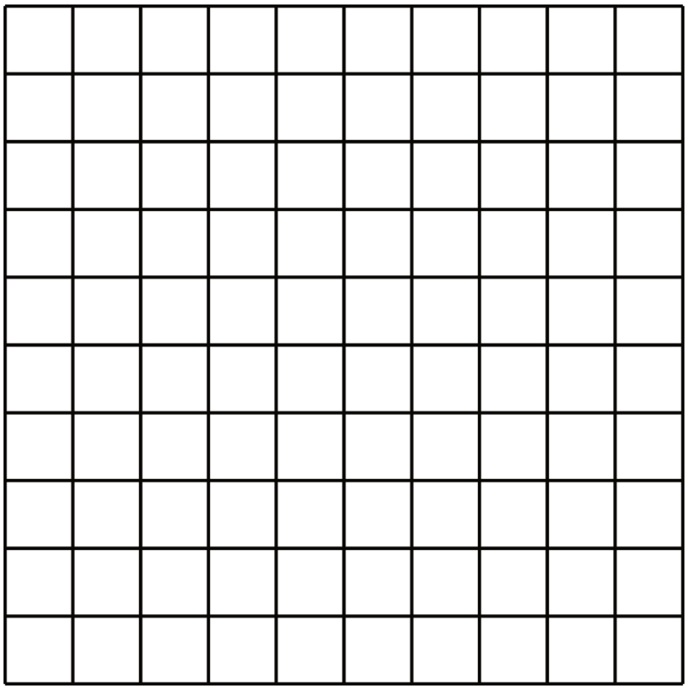
E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	**Priority (+, 0, -)**	**Category (E, D, A)**	**Notes**
A1.5.1	+	E	Introduces concept of exponential growth.
A1.5.2	0	D	
A1.5.3	+	E	Introduces growth factor and the meaning of the terms *a*, *b*, and *x* in the equation $y = a \cdot b^x$.
A1.5.4	+	E	Introduces exponential decay.
A1.5.5	+	D	Formalizes expressing exponential decay in an equation.
A1.5.6	0	D	Analyzes graphs representing exponential decay.
A1.5.7	0	A	Introduces the meaning of negative exponents in the context of exponential expressions.
A1.5.8	+	E	Incorporates function notation for the first time in this unit.
A1.5.9	0	D	Explores exponential functions and graphs in context.
A1.5.10	+	A	Revisits average rate of change, which will be used to compare graphs in future lessons.
A1.5.11	0	E	Prompts student thinking about equations to represent exponential functions that will be explored further in Lessons 12 and 13.

A1.5.12	+	D	Students closely examine different graphs of exponential functions and consider the meaning of the *a* and *b* terms.
A1.5.13	0	A	Students consolidate learning about graphs and equations of exponential functions in context.
A1.5.14	0	E	Reviews percent change concepts from grade 7.
A1.5.15	+	E	Explores percent change in the context of exponential functions and repeated multiplication.
A1.5.16	+	D	Formalizes learning from the previous lesson and connects the standard form of exponential functions to percent change.
A1.5.17	+	D	Students calculate percent growth and decay in context.
A1.5.18	0	A	Students solve problems on percent growth and decay in context.
A1.5.19	+	E	Compares growth rate between linear and exponential functions.
A1.5.20	-	D	Explores change over equal intervals.
A1.5.21	-	A	Provides an opportunity to model linear and exponential situations.

Lesson 2: Multiplying Powers of Ten

2.1: 100, 1, or $\frac{1}{100}$?



Clare said she sees 100.

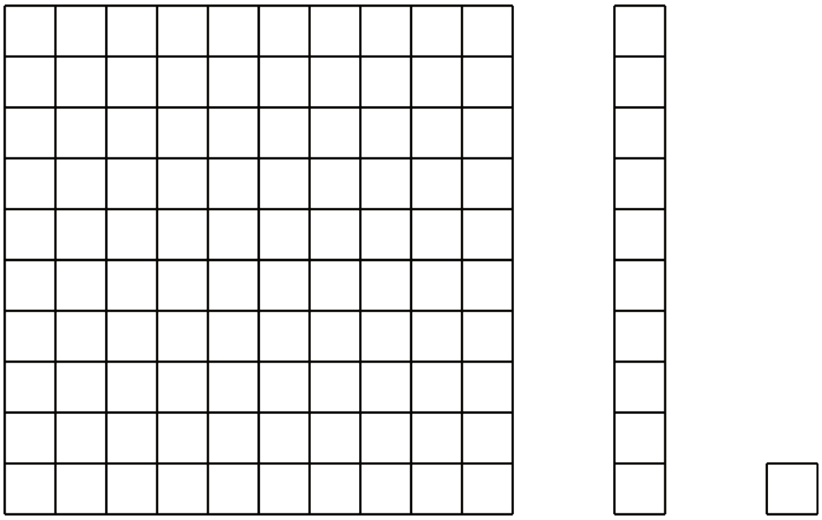
Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Who do you agree with?

2.2: Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could you represent the large square as a power of 10?

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?

4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

2.3: Multiplying Powers of Ten

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Are you ready for more?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?

2. How about 10^7? 10^8?

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: multiplying powers of 10 corresponds to adding the exponents together. To see this, multiply 10^5 and 10^2. We know that 10^5 has five factors that are 10 and 10^2 has two factors that are 10. That means that $10^5 \cdot 10^2$ has 7 factors that are 10.

$$10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7.$$

This will work for other powers of 10 too. So $10^{14} \cdot 10^{47} = 10^{61}$.

This rule makes it easier to understand and work with expressions that have exponents.

Lesson 2: Multiplying Powers of Ten

Cool Down: That's a Lot of Dough, Though!

1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.

2. Each year, roughly 10^6 computer programmers each make about \$$10^5$. How much money is this all together? Express your answer both as a power of 10 and as a dollar amount.

Unit 7 Lesson 2 Cumulative Practice Problems

1. Write each expression with a single exponent:

    a. $10^3 \cdot 10^9$

    b. $10 \cdot 10^4$

    c. $10^{10} \cdot 10^7$

    d. $10^3 \cdot 10^3$

    e. $10^5 \cdot 10^{12}$

    f. $10^6 \cdot 10^6 \cdot 10^6$

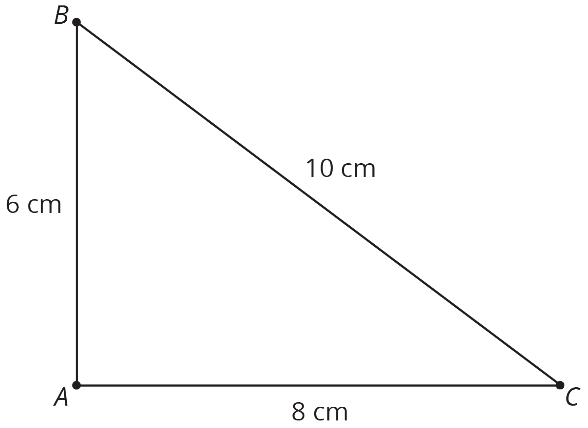
2. A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water.

    a. What is the area of the surface of the water in the pool?

    b. How much water does the pool hold?

    c. Express your answers to the previous two questions as powers of 10.

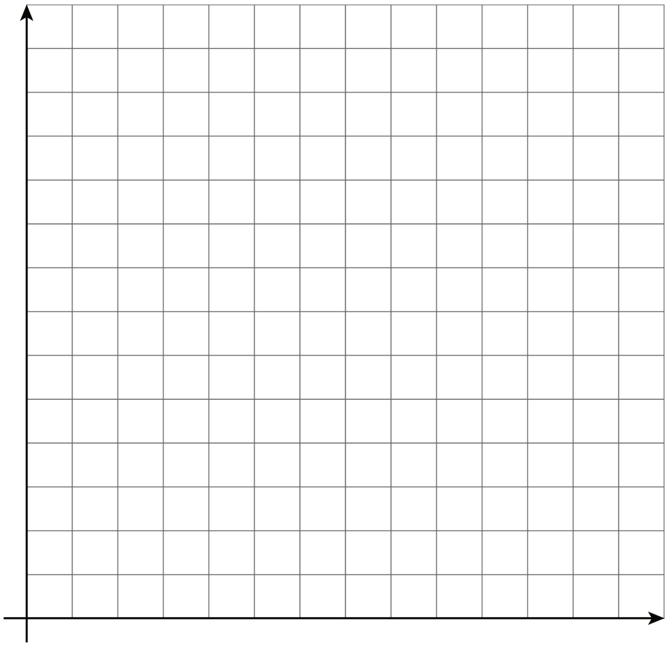
3. Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



(From Unit 2, Lesson 7.)

4. Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

   Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



(From Unit 3, Lesson 3.)

Lesson 3: Powers of Powers of 10

3.1: Big Cube

What is the volume of a giant cube that measures 10,000 km on each side?

3.2: Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

3.3: How Do the Rules Work?

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, “When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$.”
- Elena says, “10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$.”

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Lesson 3 Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$\left(10^4\right)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $\left(10^6\right)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Lesson 3: Powers of Powers of 10

Cool Down: Making a Million

Here are some equivalent ways of writing 10^4:

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents and multiplication.

Unit 7 Lesson 3 Cumulative Practice Problems

1. Write each expression with a single exponent:

    a. $(10^7)^2$

    b. $(10^9)^3$

    c. $(10^6)^3$

    d. $(10^2)^3$

    e. $(10^3)^2$

    f. $(10^5)^7$

2. You have 1,000,000 number cubes, each measuring one inch on a side.

    a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

    b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

    c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

3. An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

   a. How many amoebas are there after 1 hour?

   b. How many amoebas are there after 2 hours?

   c. Write an expression for the number of amoebas after 6 hours.

   d. Write an expression for the number of amoebas after 24 hours.

   e. Why might exponential notation be preferable to answer these questions?

   (From Unit 7, Lesson 1.)

4. Elena noticed that, nine years ago, her cousin Katie was twice as old as Elena was then. Then Elena said, “In four years, I’ll be as old as Katie is now!” If Elena is currently e years old and Katie is k years old, which system of equations matches the story?

   A. $\begin{cases} k-9=2e \\ e+4=k \end{cases}$

   B. $\begin{cases} 2k=e-9 \\ e=k+4 \end{cases}$

   C. $\begin{cases} k=2e-9 \\ e+4=k+4 \end{cases}$

   D. $\begin{cases} k-9=2(e-9) \\ e+4=k \end{cases}$

   (From Unit 4, Lesson 15.)

Illustrative Mathematics

Lesson 4: Dividing Powers of 10

4.1: A Surprising One

What is the value of the expression?

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

4.2: Dividing Powers of Ten

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the “expanded” column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression of the form $10^{\square}$.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Are you ready for more?

expression	expanded	single power
$10^4 \div 10^6$		

4.3: Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. What would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \boldsymbol{\cdot} 10^0$ with a power of 10 with a single exponent using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you multiply 10^{12} by to get this same answer?

2. a. Write $\frac{10^8}{10^0}$ with a single power of 10 using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you divide 10^8 by to get this same answer?

3. If we want the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?
4. Noah says, “If I try to write 10^0 expanded, it should have zero factors that are 10, so it must be equal to 0.” Do you agree? Discuss with your partner.

4.4: Making Millions

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents, multiplication, and division. What patterns do you notice with the exponents?

Lesson 4 Summary

In an earlier lesson, we learned that when multiplying powers of 10, the exponents add together. For example, $10^6 \cdot 10^3 = 10^9$ because 6 factors that are 10 multiplied by 3 factors that are 10 makes 9 factors that are 10 all together. We can also think of this multiplication equation as division:

$$10^6 = \frac{10^9}{10^3}$$

So when dividing powers of 10, the exponent in the denominator is subtracted from the exponent in the numerator. This makes sense because

$$\frac{10^9}{10^3} = \frac{10^3 \cdot 10^6}{10^3} = \frac{10^3}{10^3} \cdot 10^6 = 1 \cdot 10^6 = 10^6$$

This rule works for other powers of 10 too. For example, $\frac{10^{56}}{10^{23}} = 10^{33}$ because 23 factors that are 10 in the numerator and in the denominator are used to make 1, leaving 33 factors remaining.

This gives us a new exponent rule:

$$\frac{10^n}{10^m} = 10^{n-m}.$$

So far, this only makes sense when n and m are positive exponents and $n > m$, but we can extend this rule to include a new power of 10, 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means that if we want the rule to work when the exponent is 0, then it must be that

$$10^0 = 1$$

Lesson 4: Dividing Powers of 10

Cool Down: Why Subtract?

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11}? Explain or show your thinking.

Unit 7 Lesson 4 Cumulative Practice Problems

1. Evaluate:

   a. 10^0

   b. $\frac{10^3}{10^3}$

   c. $10^2 + 10^1 + 10^0$

2. Write each expression as a single power of 10.

   a. $\frac{10^3 \cdot 10^4}{10^5}$

   b. $(10^4) \cdot \frac{10^{12}}{10^7}$

   c. $\left(\frac{10^5}{10^3}\right)^4$

   d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$

   e. $\frac{(10^5)^2}{(10^2)^3}$

3. The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

4. Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

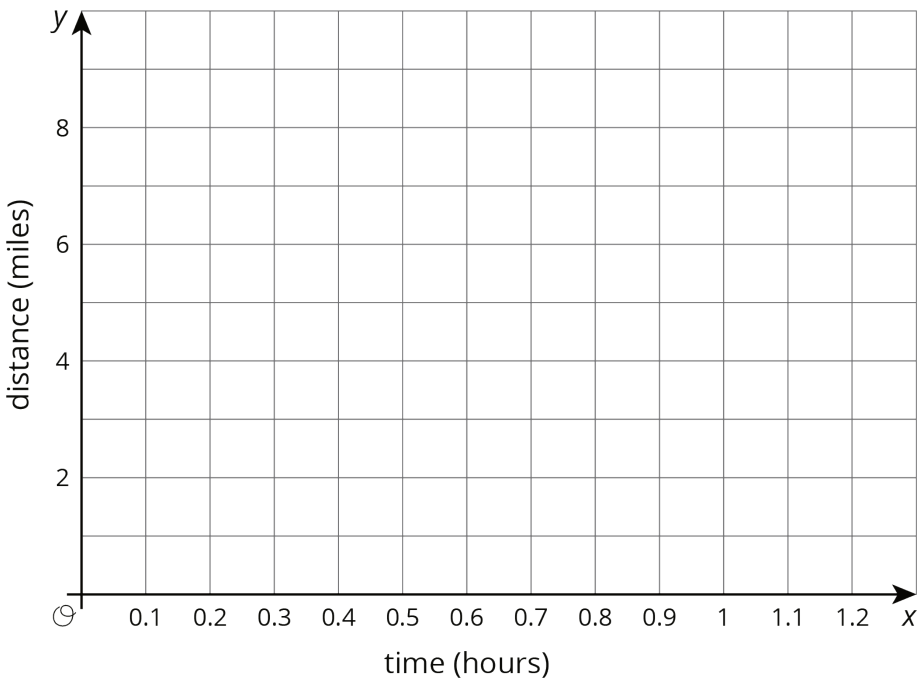
   a. Write an equation relating the two variables.

   b. Rearrange the equation so b is the independent variable.

   c. Rearrange the equation so g is the independent variable.

(From Unit 5, Lesson 3.)

5. Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



(From Unit 3, Lesson 1.)

Lesson 5: Negative Exponents with Powers of 10

5.1: Number Talk: What's That Exponent?

Solve each equation mentally.

$\frac{100}{1} = 10^x$

$\frac{100}{x} = 10^1$

$\frac{x}{100} = 10^0$

$\frac{100}{1{,}000} = 10^x$

5.2: Negative Exponent Table

Complete the table to explore what negative exponents mean.

	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	
using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$
	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	

1. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?

2. How does a multiplier of 10 affect the placement of the decimal in the product? How does the other multiplier affect the placement of the decimal in the product?

3. Use the patterns you found in the table to write 10^{-7} as a fraction.

4. Use the patterns you found in the table to write 10^{-5} as a decimal.

5. Write $\frac{1}{100{,}000{,}000}$ using a single exponent.

6. Use the patterns in the table to write 10^{-n} as a fraction.

5.3: Follow the Exponent Rules

1. a. Match each exponential expression with an equivalent multiplication expression:

$\left(10^2\right)^3$

$\left(10^2\right)^{-3}$

$\left(10^{-2}\right)^3$

$\left(10^{-2}\right)^{-3}$

$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
$\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)$
$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$

b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$

$\frac{10^2}{10^{-5}}$

$\frac{10^{-2}}{10^5}$

$\frac{10^{-2}}{10^{-5}}$

$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$
$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$10^4 \cdot 10^{-3}$

$10^{-4} \cdot 10^3$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10)$
$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Are you ready for more?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, “I knew I’d be the only one left, since I went first.”

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?

2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Lesson 5 Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1:

$$10^8 \cdot \frac{1}{10} = 10^7$$

This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2:

$$\left(\frac{1}{10}\right)^2 \cdot 10^8 = 10^6$$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make

$$10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents:

$$\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

To see why, notice that

$$\frac{10^3}{10^5} = \frac{10^3}{10^3 \cdot 10^2} = \frac{10^3}{10^3} \cdot \frac{1}{10^2} = \frac{1}{10^2}$$

which is equal to 10^{-2}.

Here is an example of extending the rule $(10^m)^n = 10^{m \cdot n}$ to use negative exponents:

$$\left(10^{-2}\right)^3 = 10^{(-2)(3)} = 10^{-6}$$

To see why, notice that $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. This means that

$$\left(10^{-2}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$$

Lesson 5: Negative Exponents with Powers of 10

Cool Down: Negative Exponent True or False

Mark each of the following equations as true or false. Explain or show your reasoning.

1. $10^{-5} = -10^{5}$

2. $(10^{2})^{-3} = (10^{-2})^{3}$

3. $\frac{10^{3}}{10^{14}} = 10^{-11}$

Unit 7 Lesson 5 Cumulative Practice Problems

1. Write with a single exponent: (ex: $\frac{1}{10} \cdot \frac{1}{10} = 10^{-2}$)

   a. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   b. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   c. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^2$

   d. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^3$

   e. $(10 \cdot 10 \cdot 10)^{-2}$

2. Write each expression as a single power of 10.

   a. $10^{-3} \cdot 10^{-2}$

   b. $10^{4} \cdot 10^{-1}$

   c. $\frac{10^5}{10^7}$

   d. $(10^{-4})^5$

   e. $10^{-3} \cdot 10^{2}$

   f. $\frac{10^{-9}}{10^5}$

3. Select **all** of the following that are equivalent to $\frac{1}{10,000}$:

   A. $(10,000)^{-1}$

   B. $(-10,000)$

   C. $(100)^{-2}$

   D. $(10)^{-4}$

   E. $(-10)^{2}$

4. Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

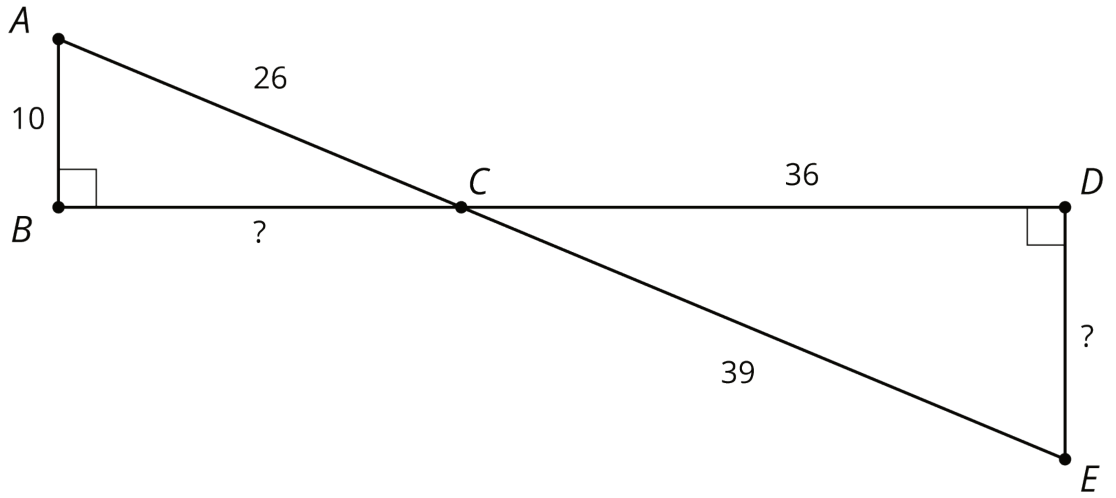
d. $y = \frac{5}{2}x$

Situations:

- A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- A store has a “4 for $10” sale on hats.
- For every 48 cookies I bake, my students get 24.

(From Unit 3, Lesson 2.)

5. a. Explain why triangle ABC is similar to EDC.



b. Find the missing side lengths.

(From Unit 2, Lesson 8.)

Lesson 6: What about Other Bases?

6.1: True or False: Comparing Expressions with Exponents

Is each statement true or false? Be prepared to explain your reasoning.

1. $3^5 < 4^6$
2. $(-3)^2 < 3^2$
3. $(-3)^3 = 3^3$
4. $(-5)^2 > -5^2$

6.2: What Happens with Zero and Negative Exponents?

Complete the table to show what it means to have an exponent of zero or a negative exponent.

$\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$

value	16					$\frac{1}{2}$			
exponent form	2^4								

$\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?
2. Use the patterns you found in the table to write 2^{-6} as a fraction.
3. Write $\frac{1}{32}$ as a power of 2 with a single exponent.
4. What is the value of 2^0?
5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?
6. How would you write 3^{-4} as a fraction?

Are you ready for more?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

6.3: Exponent Rules with Bases Other than 10

Lin, Noah, Diego, and Elena decide to test each other's knowledge of exponents with bases other than 10. They each chose an expression to start with and then came up with a new list of expressions; some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:
   $(5^3)^{-3}$ $\quad$ -5^9 $\quad$ $\frac{5^{-6}}{5^3}$ $\quad$ $(5^3)^{-2}$ $\quad$ $\frac{5^{-4}}{5^{-5}}$ $\quad$ $5^{-4} \cdot 5^{-5}$

2. Noah's original expression is 3^{10} and his list is:
   $3^5 \cdot 3^2$ $\quad$ $(3^5)^2$ $\quad$ $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$

   $\left(\frac{1}{3}\right)^{-10}$ $\quad$ $3^7 \cdot 3^3$ $\quad$ $\frac{3^{20}}{3^{10}}$ $\quad$ $\frac{3^{20}}{3^2}$

3. Diego's original expression is x^4 and his list is:

$\frac{x^8}{x^4}$	$x \cdot x \cdot x \cdot x$	$\frac{x^{-4}}{x^{-8}}$	$\frac{x^{-4}}{x^8}$
$(x^2)^2$	$4 \cdot x$	$x \cdot x^3$	

4. Elena's original expression is 8^0 and her list is:

1	0	$8^3 \cdot 8^{-3}$
$\frac{8^2}{8^2}$	10^0	11^0

Lesson 6 Summary

Earlier we focused on powers of 10 because 10 plays a special role in the decimal number system. But the exponent rules that we developed for 10 also work for other bases. For example, if $2^0 = 1$ and $2^{-n} = \frac{1}{2^n}$, then

$$\begin{aligned} 2^m \cdot 2^n &= 2^{m+n} \\ (2^m)^n &= 2^{m \cdot n} \\ \frac{2^m}{2^n} &= 2^{m-n}. \end{aligned}$$

These rules also work for powers of numbers less than 1. For example, $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3}$ and $\left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$. We can also check that $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^{2+4}$.

Using a variable x helps to see this structure. Since $x^2 \cdot x^5 = x^7$ (both sides have 7 factors that are x), if we let $x = 4$, we can see that $4^2 \cdot 4^5 = 4^7$. Similarly, we could let $x = \frac{2}{3}$ or $x = 11$ or any other positive value and show that these relationships still hold.

Lesson 6: What about Other Bases?

Cool Down: Spot the Mistake

1. Diego was trying to write $2^3 \cdot 2^2$ with a single exponent and wrote $2^3 \cdot 2^2 = 2^{3 \cdot 2} = 2^6$. Explain to Diego what his mistake was and what the answer should be.

2. Andre was trying to write $\frac{7^4}{7^{-3}}$ with a single exponent and wrote $\frac{7^4}{7^{-3}} = 7^{4-3} = 7^1$. Explain to Andre what his mistake was and what the answer should be.

Unit 7 Lesson 6 Cumulative Practice Problems

1. Priya says “I can figure out 5^0 by looking at other powers of 5. 5^3 is 125, 5^2 is 25, then 5^1 is 5.”

   a. What pattern do you notice?

   b. If this pattern continues, what should be the value of 5^0? Explain how you know.

   c. If this pattern continues, what should be the value of 5^{-1}? Explain how you know.

2. Select **all** the expressions that are equivalent to 4^{-3}.

   A. -12

   B. 2^{-6}

   C. $\frac{1}{4^3}$

   D. $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

   E. 12

   F. $(-4) \cdot (-4) \cdot (-4)$

   G. $\frac{8^{-1}}{2^2}$

3. Write each expression using a single exponent.

   a. $\frac{5^3}{5^6}$

   b. $(14^3)^6$

   c. $8^3 \cdot 8^6$

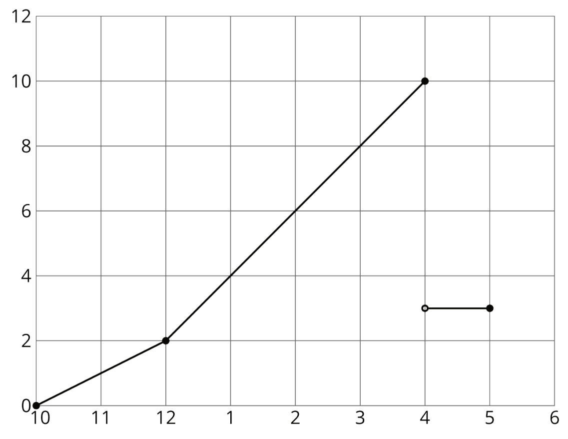
   d. $\frac{16^6}{16^3}$

   e. $(21^3)^{-6}$

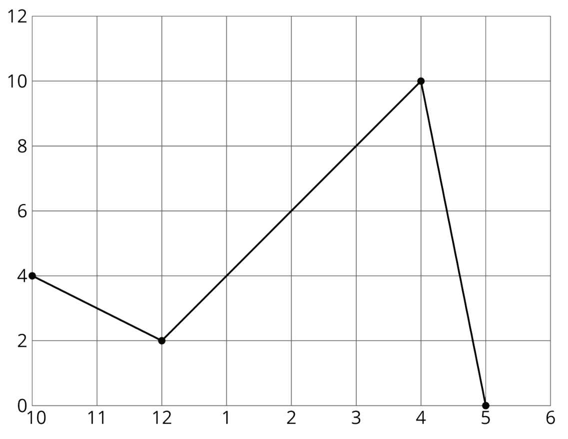
4. Andre sets up a rain gauge to measure rainfall in his back yard. On Tuesday, it rains off and on all day.

    - He starts at 10 a.m. with an empty gauge when it starts to rain.
    - Two hours later, he checks, and the gauge has 2 cm of water in it.
    - It starts raining even harder, and at 4 p.m., the rain stops, so Andre checks the rain gauge and finds it has 10 cm of water in it.
    - While checking it, he accidentally knocks the rain gauge over and spills most of the water, leaving only 3 cm of water in the rain gauge.
    - When he checks for the last time at 5 p.m., there is no change.

Graph A



Graph B



a. Which of the two graphs could represent Andre's story? Explain your reasoning.

b. Label the axes of the correct graph with appropriate units.

c. Use the graph to determine how much total rain fell on Tuesday.

(From Unit 5, Lesson 6.)