Plan for Algebra 1 Unit 2: Linear Equations, Inequalities, and Systems

Relevant Unit(s) to review: Grade 8 Units 3 and 4,
Grade 7 Unit 6 (Inequalities Lessons 13–17)

Essential prior concepts to engage with this unit	• Solving equations by rewriting them using operations that do not change the equivalence, combining like terms, and applying the distributive property. • Graphing equations in slope-intercept form and interpreting the slope and vertical intercept in context • Solving systems of equations by graphing
Brief narrative of approach	In this unit, students expand and deepen their prior understanding of expressions, equations, and inequalities. This unit builds on understanding students bring from grade 8 about graphing lines in slope-intercept form and interpreting slope in context, as well as solving equations and solving systems of equations by graphing. Because of the variety of prerequisite understandings and skills, the grade 8 supplemental lessons should be taught at the beginning of the section where they are most relevant, rather than as a series of lessons taught before the grade-level work begins. The first section introduces equations in contexts, and can be taught without alteration (unless you need to compress the first two lessons for time). The second section includes two big ideas: solving equations and connecting equations and their graphs. Lessons to support solving equations have been included at the beginning of the section, and lessons to review or re-teach slope-intercept form have been included for that section as well. It may be helpful to think of section 2 in two parts: one focused on solving equations, and one focused on graphing. Section 3 builds on previous understandings from grade 8 of solving systems of equations by graphing, and extends that understanding to solving systems by substitution and elimination. A lesson on solving systems by graphing has been included for students who need more time with that idea.

	The rest of the unit is an opportunity for students to apply all of these understandings to solving linear inequalities and linear systems of inequalities. Supplemental work on inequalities has not been included, but teachers may consider looking to Grade 7 Unit 6 Lessons 13–17 to get a sense of student prior learning.

Lessons to Add	Lessons to Remove or Modify
1. Combine 8.4.3 and 4, particularly Activity 3 in Lesson 3 and Activities 2 and 3 in Lesson 4. Focus on the idea of using the same operation with the expressions on each side of an equation or changing the form of one of the expressions through combining like terms, applying the distributive property, and similar operations 2. Combine 8.3.8 and 8.3.9. Introduce $y = mx + b$, and include an activity from Lesson 9 to introduce negative slope. 3. 8.3.10: Calculating slope 4. 8.3.11: Equations for horizontal and vertical lines 5. 8.4.12: Solving systems by graphing	1. Combine Lessons 1 and 2: Use an activity from Lesson 1 to introduce the idea of constraint. Remove Activities 1 and 4 from Lesson 2. 2. Combine Lessons 8 and 9. (Remove 8.2 and the cool down for Lesson 8, and 9.2). 3. Combine Lessons 15 and 16. 4. Remove Lesson 26.
Lessons added: 5	Lessons removed: 4

Modified Plan for Algebra 1 Unit 2

Day	IM lesson	Notes
	assessment	Check Your Readiness for Algebra Unit 2
1	A1.2.1 A1.2.2	Use equations to represent situations, introduces constraint. Consolidate Lessons 1 and 2. The Pizza Party activity can be shortened to get at this idea, and the cool-down can be safely skipped. Activities 2 and 3 from Lesson 2 should be emphasized. The percent work from the warm-up and Activity 4 are good activities to think about percentages in context, but omitting them will not get in the way of learning in future lessons.
2	A1.2.3	Write equations to model relationships.
3	A1.2.4	Equations and their solutions
4	A1.2.5	Equations and their graphs
5	8.4.3 8.4.4	Solving equations. Choose activities from these lessons to activate prior knowledge of solving equations and focus on the concept of “balancing equations.” This will support student thinking in the Algebra 1 lessons where they move to more formal understandings of solving equations.
6	A1.2.6	Solve equations.
7	A1.2.7	Solve equations and explain the steps for solving.
8	A1.2.8 A1.2.9	Rearrange equations to solve for a specific variable.
9	8.3.8 8.3.9	Focus on introducing slope-intercept form and understanding negative slope.
10	8.3.10	Calculate slope.
11	8.3.11	Write equations of horizontal and vertical lines.
12	A1.2.10	Connect equations to graphs (emphasis on standard form of equations).

13	A1.2.11	Connect equations to graphs (emphasis on standard form of equations and rearranging the equation to move between standard and slope-intercept form).
14	8.4.12	Solve systems of equations by graphing.
15	A1.2.12	Write and solve systems of equations by graphing.
16	A1.2.13	Solve systems of equations by substitution.
17	A1.2.14	Solve systems of equations by elimination.
18	A1.2.15 A1.2.16	Solve systems of equations by elimination. (Lesson 15 emphasizes multiplying one equation by a constant to eliminate. Lesson 16 emphasizes multiplying both equations by constants to eliminate.)
19	A1.2.17	Systems of linear equations and their solutions
20	MUA	
21	A1.2.18	Represent situations with inequalities.
22	A1.2.19	Solutions to inequalities in one variable
23	A1.2.20	Write and solve inequalities in one variable.
24	A1.2.21	Graph linear inequalities in two variables.
25	A1.2.22	Graph linear inequalities in two variables.
26	A1.2.23	Solve problems with inequalities in two variables.
27	A1.2.24	Solutions to systems of linear inequalities
28	A1.2.25	Solve problems with systems of linear inequalities.
29	EOU	End-of-Unit Assessment

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
A1.2.1	0	E	This lesson introduces the unit and activates students' prior knowledge of writing equations to represent situations. It also introduces the term constraint, which will be used throughout the unit.
A1.2.2	0	E	This lesson allows students to explore further writing equations to represent situations that are likely to be familiar to them.
A1.2.3	+	D	This lesson dives deeper into modeling with equations, including unfamiliar situations and through looking for patterns in tables to write an equation.
A1.2.4	+	D	This lesson is designed to support thinking around the meaning of solution.
A1.2.5	0	A	This lesson connects equations to graphs.
A1.2.6	+	E	This lesson introduces the idea that one way to solve equations is to generate equivalent equations (or equivalent expressions on one side of the equation).
A1.2.7	+	D	In this lesson, students further develop their understanding of solving equations by describing the steps they take to solve them.
A1.2.8	-	D	This lesson introduces the concept of solving for a variable. It is marked as low priority, based on the assumption that students may need additional support with the equation-solving process in Lessons 6 and 7, and that they will learn in Lesson 11 to transform equations from standard form to slope-intercept form.
A1.2.9	-	D	This lesson is further work in solving for a variable and understanding it as an efficient

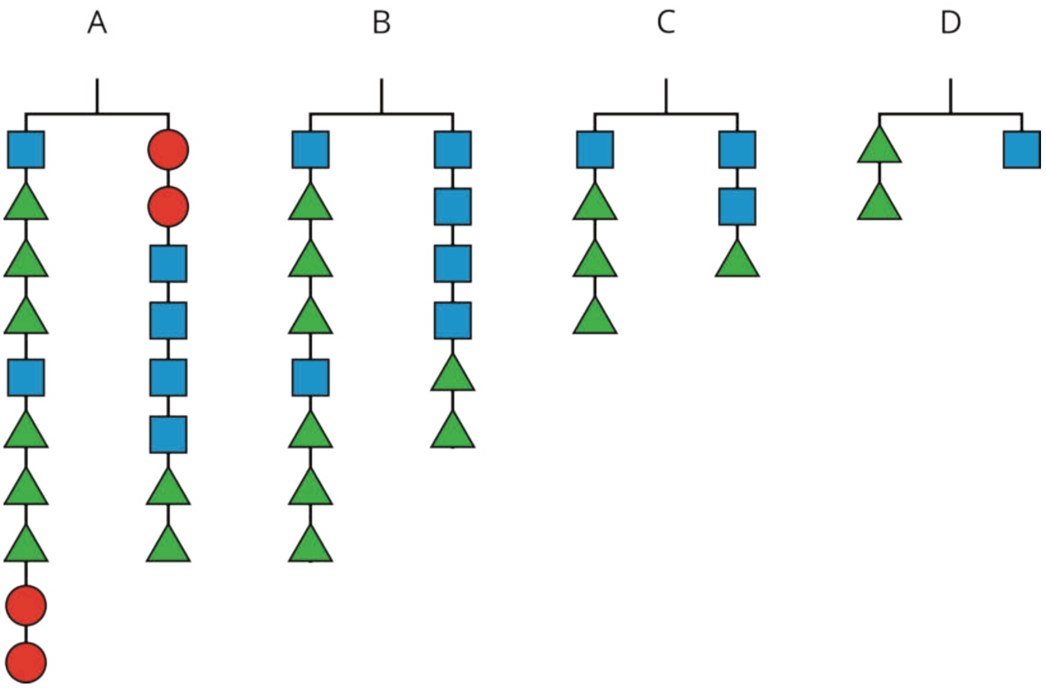
			strategy.
A1.2.10	+	A	This lesson connects equations in standard form to their graphs.
A1.2.11	0	A	This lesson connects equations in standard form and emphasizes transforming those equations to slope-intercept form.
A1.2.12	0	E	This lesson introduces systems of equations by graphing (which students should have been exposed to in grade 8).
A1.2.13	0	D	This lesson introduces systems of equations by substitution.
A1.2.14	0	E	This is the first of three lessons on solving systems by elimination.
A1.2.15	+	D	This lesson explores systems by elimination more deeply, including considering solving by multiplying one of the equations by a factor.
A1.2.16	-	D	This lesson explores solving systems by elimination and introduces the strategy of multiplying both equations by a factor. If time is an issue, the concept of multiplying by a factor in Lesson 15 will extend easily to Lesson 16, and some activities can be deprioritized.
A1.2.17	+	A	This lesson applies understandings of systems of equations and their graphs and extends that reasoning to systems with no solutions and infinitely many solutions.
A1.2.18	0	E	The focus of this lesson is on interpreting and writing inequalities that represent the constraints in various situations.
A1.2.19	0	D	This lesson continues work on interpreting and writing inequalities, and introduces the idea of a solution set.
A1.2.20	0	A	This lesson prompts students to write and solve inequalities to answer a question about a situation, and to reason about solution sets.

A1.2.21	0	E	This lesson introduces graphs of linear inequalities in two variables, and invites students to consider solutions and non-solutions.
A1.2.22	0	D	Students deepen their understanding of linear inequalities in two variables by studying them in context.
A1.2.23	+	A	This lesson puts all of the skills from this section together: Students write inequalities, to represent constraints and use those representations to answer questions about the situation. Graphing technology is used to support student understanding.
A1.2.24	0	E	Students explore systems of inequalities and learn that the solutions to the situations represent solutions that satisfy both constraints. They learn that graphically those solutions are represented by the region where the graphs of the inequalities overlap.
A1.2.25	+	D	This lesson digs deeper into writing, graphing, and interpreting solutions of systems of inequalities in context.
A1.2.26	0	A	This culminating lesson allows students to integrate ideas from the unit and explore modeling activities related to modeling.

Lesson 3: Balanced Moves

3.1: Matching Hangers

Figures A, B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.



Here are some equations. Each equation represents one of the hanger diagrams.

$$2(x+3y)=4x+2y$$
$$2y=x$$
$$2(x+3y)+2z=2z+4x+2y$$
$$x+3y=2x+y$$

1. Write the equation that goes with each figure:
   A:
   B:
   C:
   D:

2. Each variable (x, y, and z) represents the weight of one shape. Which goes with which?

3. Explain what was done to each equation to create the next equation. If you get stuck, think about how the hangers changed.

3.2: Matching Equation Moves

Your teacher will give you some cards. Each of the cards 1 through 6 show two equations. Each of the cards A through E describe a move that turns one equation into another.

1. Match each number card with a letter card.

2. One of the letter cards will not have a match. For this card, write two equations showing the described move.

3.3: Keeping Equality

1. Noah and Lin both solved the equation $14a = 2(a - 3)$.
   Do you agree with either of them? Why?

   Noah's solution:

   $$14a = 2(a - 3)$$
   $$14a = 2a - 6$$
   $$12a = -6$$
   $$a = -\frac{1}{2}$$

   Lin's solution:

   $$14a = 2(a - 3)$$
   $$7a = a - 3$$
   $$6a = -3$$
   $$a = -\frac{1}{2}$$

2. Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?

3. Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

Are you ready for more?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

HANGER + HANGER + HANGER = ALGEBRA

Lesson 3 Summary

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $\text{-}2x - 3$ have equal value, we can write the equation

$$4x + 9 = \text{-}2x - 3$$

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add *negative numbers*! For example, we can add -9 to each side of the equation.

$$\begin{aligned} 4x + 9 + \text{-}9 &= \text{-}2x - 3 + \text{-}9 && \text{add -9 to each side} \\ 4x &= \text{-}2x - 12 && \text{combine like terms} \end{aligned}$$

Since expressions represent numbers, we can also add *expressions* to each side of an equation. For example, we can add $2x$ to each side and still maintain equality.

$$\begin{aligned} 4x + 2x &= \text{-}2x - 12 + 2x && \text{add } 2x \text{ to each side} \\ 6x &= \text{-}12 && \text{combine like terms} \end{aligned}$$

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (so long as we do not divide by zero).

$$6x \cdot \frac{1}{6} = \text{-}12 \cdot \frac{1}{6} \qquad \text{multiply each side by } \frac{1}{6}$$

or

$$6x \div 6 = \text{-}12 \div 6 \qquad \text{divide each side by } 6$$

Now we can see that $x = \text{-}2$ is the solution to our equation.

We will use these moves in systematic ways to solve equations in future lessons.

Lesson 3: Balanced Moves

Cool Down: More Matching Moves

Match these equation balancing steps with the description of what was done in each step.

Step 1:

$$\begin{aligned} 12x - 6 &= 10 \\ 6x - 3 &= 5 \end{aligned}$$

Step 2:

$$\begin{aligned} 6x - 3 &= 5 \\ 6x &= 8 \end{aligned}$$

Step 3:

$$\begin{aligned} 6x &= 8 \\ x &= \frac{4}{3} \end{aligned}$$

Descriptions to match with each step:

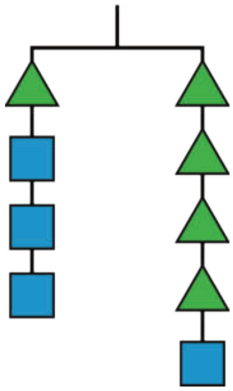
A: Add 3 to both sides

B: Multiply both sides by $\frac{1}{6}$

C: Divide both sides by 2

Unit 4 Lesson 3 Cumulative Practice Problems

1. In this hanger, the weight of the triangle is x and the weight of the square is y.



   a. Write an equation using x and y to represent the hanger.

   b. If x is 6, what is y?

2. Andre and Diego were each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each make to the equation.

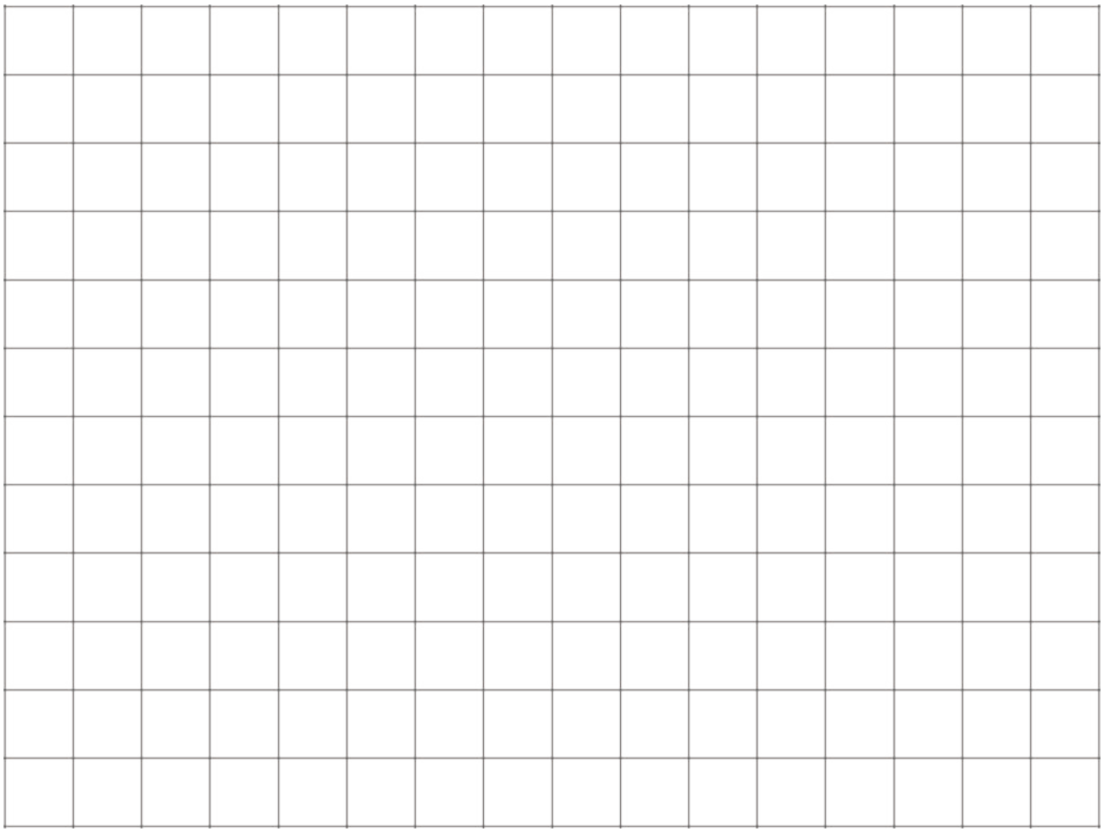
   a. The result of Andre's first step was $-x + 6 = -8$.

   b. The result of Diego's first step was $6 = x - 8$.

3. a. Complete the table with values for x or y that make this equation true: $3x + y = 15$.

x	2		6	0	3		
y		3				0	8

b. Create a graph, plot these points, and find the slope of the line that goes through them.



(From Unit 3, Lesson 11.)

4. Match each set of equations with the move that turned the first equation into the second.

   A. $6x + 9 = 4x - 3$
      $2x + 9 = -3$

   B. $-4(5x - 7) = -18$
      $5x - 7 = 4.5$

   C. $8 - 10x = 7 + 5x$
      $4 - 10x = 3 + 5x$

   D. $\frac{-5x}{4} = 4$
      $5x = -16$

   E. $12x + 4 = 20x + 24$
      $3x + 1 = 5x + 6$

   1. Multiply both sides by $\frac{-1}{4}$
   2. Multiply both sides by -4
   3. Multiply both sides by $\frac{1}{4}$
   4. Add $-4x$ to both sides
   5. Add -4 to both sides

5. Select **all** the situations for which only zero or positive solutions make sense.

   A. Measuring temperature in degrees Celsius at an Arctic outpost each day in January.

   B. The height of a candle as it burns over an hour.

   C. The elevation above sea level of a hiker descending into a canyon.

   D. The number of students remaining in school after 6:00 p.m.

   E. A bank account balance over a year.

   F. The temperature in degrees Fahrenheit of an oven used on a hot summer day.

   (From Unit 3, Lesson 14.)

Lesson 4: More Balanced Moves

4.1: Different Equations?

Equation 1

$$x - 3 = 2 - 4x$$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

Equation A	Equation B	Equation C	Equation D
$2x - 6 = 4 - 8x$	$x - 5 = -4x$	$2(1 - 2x) = x - 3$	$-3 = 2 - 5x$

4.2: Step by Step by Step by Step

Here is an equation, and then all the steps Clare wrote to solve it:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 3(4x + 1) &= 3(5x + 9) \\ 4x + 1 &= 5x + 9 \\ 1 &= x + 9 \\ -8 &= x \end{aligned}$$

Here is the same equation, and the steps Lin wrote to solve it:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 12x + 3 &= 15x + 27 \\ 12x &= 15x + 24 \\ -3x &= 24 \\ x &= -8 \end{aligned}$$

1. Are both of their *solutions* correct? Explain your reasoning.

2. Describe some ways the steps they took are alike and different.

3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.

Mai:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 7x + 3 &= 3(9) \\ 7x + 3 &= 27 \\ 7x &= 24 \\ x &= \frac{24}{7} \end{aligned}$$

Noah:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 15x + 27 \\ 27x + 3 &= 27 \\ 27x &= 24 \\ x &= \frac{24}{27} \end{aligned}$$

4.3: Make Your Own Steps

Solve these equations for x.

1. $\frac{12+6x}{3} = \frac{5-9}{2}$
2. $x - 4 = \frac{1}{3}(6x - 54)$
3. $-(3x - 12) = 9x - 4$

Are you ready for more?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

Lesson 4 Summary

How do we make sure the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an x from one line to the next–there are many possible mistakes to watch out for!

Fortunately, each step we take solving an equation results in a new equation with the same solution as the original. This means we can check our work by substituting the value of the solution into the original equation. For example, say we solve the following equation:

$$\begin{aligned} 2x &= \text{-}3(x+5) \\ 2x &= \text{-}3x+15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Substituting 3 in place of x into the original equation,

$$\begin{aligned} 2(3) &= \text{-}3(3+5) \\ 6 &= \text{-}3(8) \\ 6 &= \text{-}24 \end{aligned}$$

we get a statement that isn't true! This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3. Fixing it, we now have

$$\begin{aligned} 2x &= \text{-}3(x+5) \\ 2x &= \text{-}3x-15 \\ 5x &= \text{-}15 \\ x &= \text{-}3 \end{aligned}$$

Substituting -3 in place of x into the original equation to make sure we didn't make another mistake:

$$\begin{aligned} 2(\text{-}3) &= \text{-}3(\text{-}3+5) \\ \text{-}6 &= \text{-}3(2) \\ \text{-}6 &= \text{-}6 \end{aligned}$$

This equation is true, so $x = \text{-}3$ is the solution.

Lesson 4: More Balanced Moves

Cool Down: Mis-Steps

Lin solved the equation $8(x-3)+7=2x(4-17)$ incorrectly. Find the errors in her solution. What should her answer have been?

Lin's solution:

$$\begin{aligned} 8(x-3)+7 &= 2x(4-17) \\ 8(x-3)+7 &= 2x(13) \\ 8x-24+7 &= 26x \\ 8x-17 &= 26x \\ -17 &= 34x \\ -\frac{1}{2} &= x \end{aligned}$$

Unit 4 Lesson 4 Cumulative Practice Problems

1. Mai and Tyler work on the equation $\frac{2}{5}b + 1 = -11$ together. Mai's solution is $b = -25$ and Tyler's is $b = -28$. Here is their work. Do you agree with their solutions? Explain or show your reasoning.

   Mai:
   $\frac{2}{5}b + 1 = -11$
   $\frac{2}{5}b = -10$
   $b = -10 \cdot \frac{5}{2}$
   $b = -25$

   Tyler:
   $\frac{2}{5}b + 1 = -11$
   $2b + 1 = -55$
   $2b = -56$
   $b = -28$

2. Solve $3(x - 4) = 12x$

3. Describe what is being done in each step while solving the equation.

   a. $2(-3x + 4) = 5x + 2$

   b. $-6x + 8 = 5x + 2$

   c. $8 = 11x + 2$

   d. $6 = 11x$

   e. $x = \frac{6}{11}$

4. Andre solved an equation, but when he checked his answer he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Andre's mistake and what is the solution to the equation?

$$\begin{aligned} -2(3x - 5) &= 4(x + 3) + 8 \\ -6x + 10 &= 4x + 12 + 8 \\ -6x + 10 &= 4x + 20 \\ 10 &= -2x + 20 \\ -10 &= -2x \\ 5 &= x \end{aligned}$$

5. Choose the equation that has solutions $(5, 7)$ and $(8, 13)$.

   A. $3x - y = 8$

   B. $y = x + 2$

   C. $y - x = 5$

   D. $y = 2x - 3$

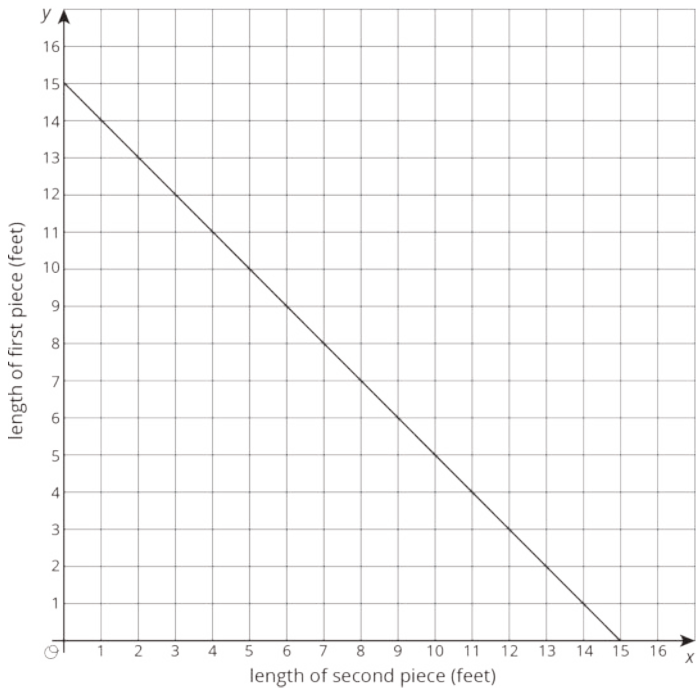
   (From Unit 3, Lesson 12.)

6. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the second piece, x, for each length of the first piece, y.

   a. How long is the ribbon? Explain how you know.

   b. What is the slope of the line?

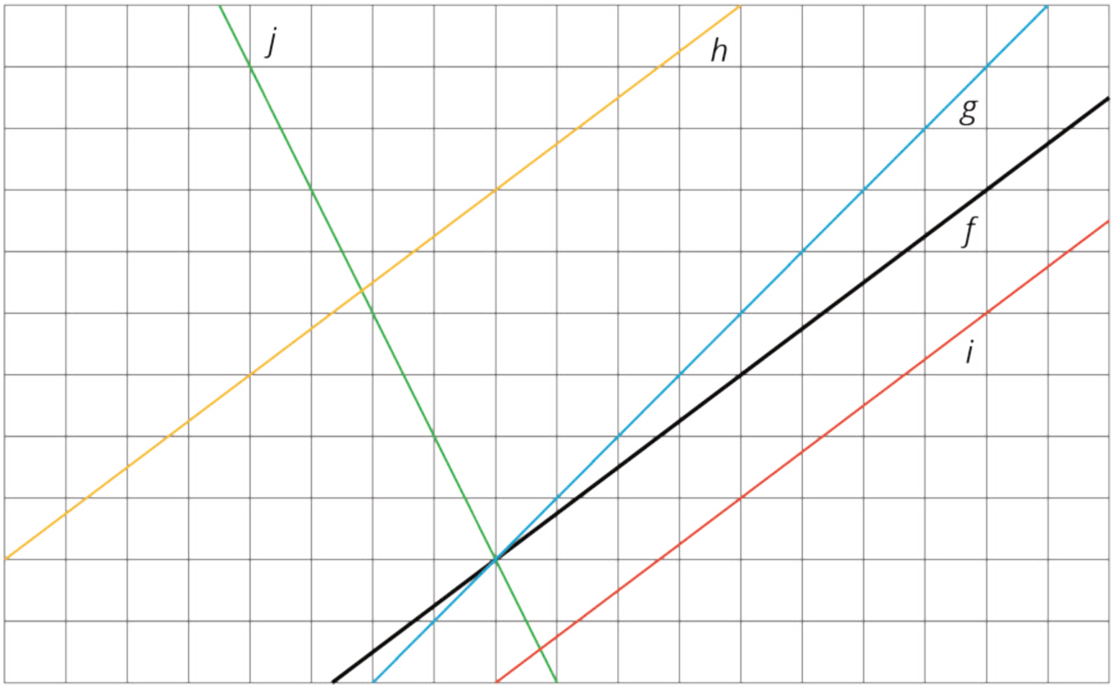
   c. Explain what the slope of the line represents and why it fits the story.



   (From Unit 3, Lesson 9.)

Lesson 8: Translating to $y = mx + b$

8.1: Lines that Are Translations



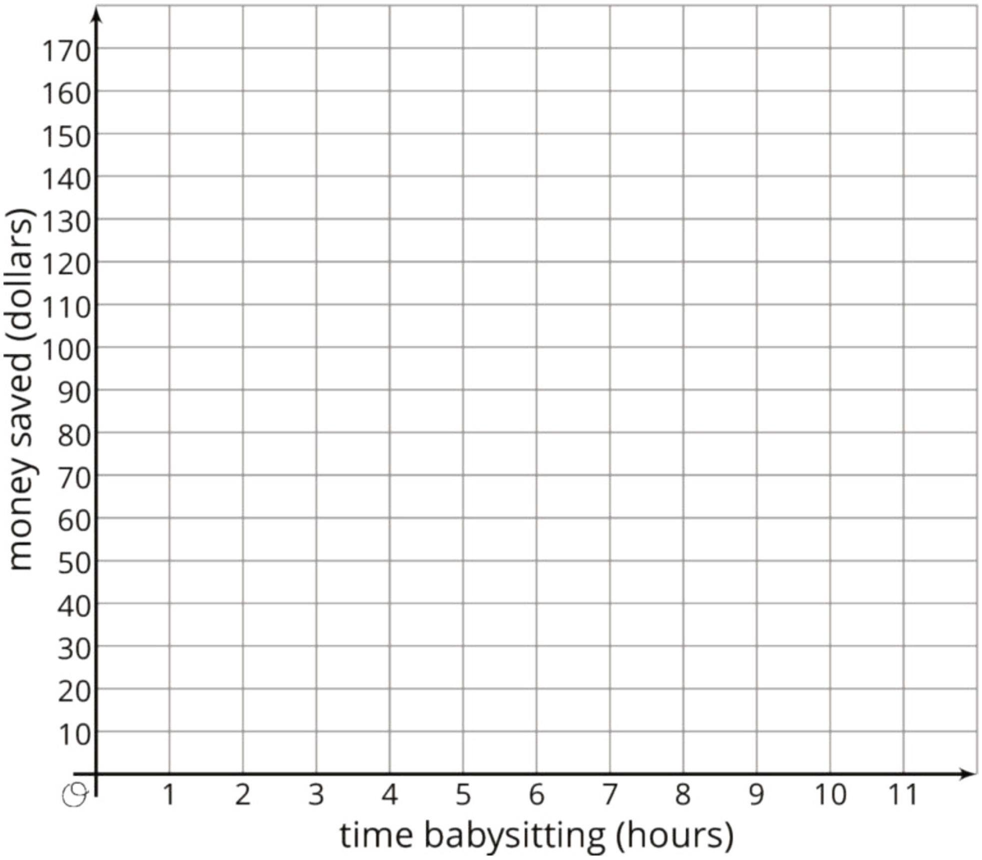
The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.

1. Which lines are images of line f under a translation?

2. For each line that is a translation of f, draw an arrow on the grid that shows the vertical translation distance.

8.2: Increased Savings

1. Diego earns $10 per hour babysitting. Assume that he has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.



2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.

3. Compare the second line with the first line. How much *more* money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

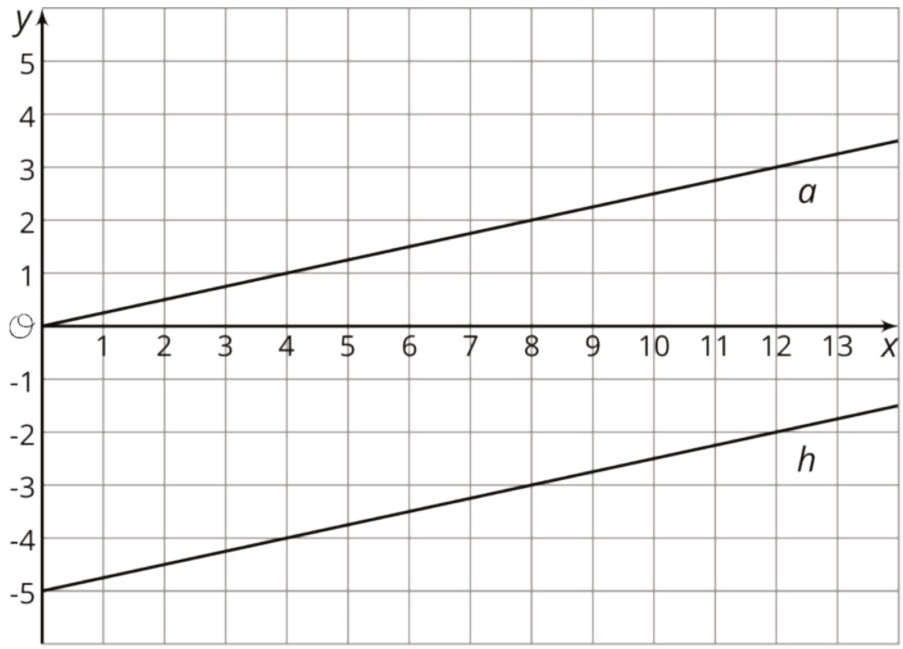
4. Write an equation for each line.

8.3: Translating a Line

This graph shows two lines.

Line a goes through the origin $(0, 0)$.

Line h is the image of line a under a translation.



1. Select all of the equations whose graph is line h.

   a. $y = \frac{1}{4}x - 5$

   b. $y = \frac{1}{4}x + 5$

   c. $\frac{1}{4}x - 5 = y$

   d. $y = -5 + \frac{1}{4}x$

   e. $-5 + \frac{1}{4}x = y$

   f. $y = 5 - \frac{1}{4}x$

2. Your teacher will give you 12 cards. There are 4 pairs of lines, A–D, showing the graph, a, of a proportional relationship and the image, h, of a under a translation. Match each line h with an equation and either a table or description. For the line with no matching equation, write one on the blank card.

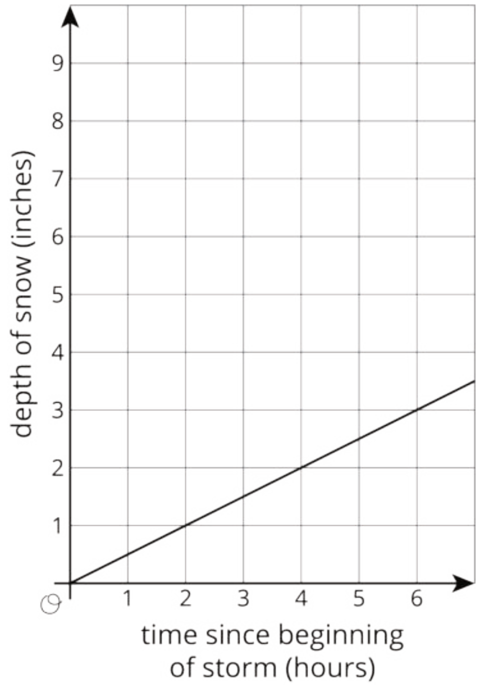
Are you ready for more?

A student says that the graph of the equation $y = 3(x + 8)$ is the same as the graph of $y = 3x$, only translated upwards by 8 units. Do you agree? Why or why not?

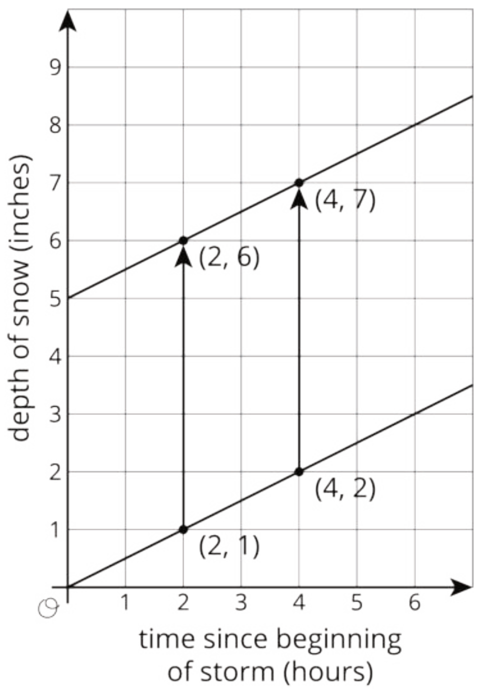
Lesson 8 Summary

During an early winter storm, the snow fell at a rate of $\frac{1}{2}$ inch per hour. We can see the rate of change, $\frac{1}{2}$, in both the equation that represents this storm, $y = \frac{1}{2}x$, and in the slope of the line representing this storm.

In addition to being a linear relationship between the time since the beginning of the storm and the depth of the snow, we can also call this as a proportional relationship since the depth of snow was 0 at the beginning of the storm.



During a mid-winter storm, the snow again fell at a rate of $\frac{1}{2}$ inch per hour, but this time there was already 5 inches of snow on the ground.



We can graph this storm on the same axes as the first storm by taking all the points on the graph of the first storm and translating them up 5 inches.

Two hours after each storm begins, 1 inch of new snow has fallen. For the first storm, this means there is now 1 inch of snow on the ground. For the second storm, this means there are now 6 inches of snow on the ground.

Unlike the first storm, the second is not a proportional relationship since the line representing the second storm has a vertical intercept of 5. The equation representing the storm, $y = \frac{1}{2}x + 5$, is of the form $y = mx + b$, where m is the rate of change, also the slope of the graph, and b is the initial amount, also the vertical intercept of the graph.

Lesson 8: Translating to $y = mx + b$

Cool Down: Similarities and Differences in Two Lines

Describe how the graph of $y = 2x$ is the same and different from the graph of $y = 2x - 7$. Explain or show your reasoning.

Unit 3 Lesson 8 Cumulative Practice Problems

1. Select **all** the equations that have graphs with the same y-intercept.

   A. $y = 3x - 8$

   B. $y = 3x - 9$

   C. $y = 3x + 8$

   D. $y = 5x - 8$

   E. $y = 2x - 8$

   F. $y = \frac{1}{3}x - 8$

2. Create a graph showing the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 5$. Explain how the graphs are the same and how they are different.

3. A cable company charges $70 per month for cable service to existing customers.

    a. Find a linear equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by an existing customer.

    b. For new customers, there is an additional one-time $100 service fee. Repeat the previous problem for new customers.

    c. When the two equations are graphed in the coordinate plane, how are they related to each other geometrically?

4. A mountain road is 5 miles long and gains elevation at a constant rate. After 2 miles, the elevation is 5500 feet above sea level. After 4 miles, the elevation is 6200 feet above sea level.

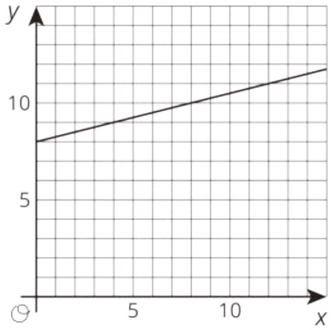
    a. Find the elevation of the road at the point where the road begins.

    b. Describe where you would see the point in part (a) on a graph where y represents the elevation in feet and x represents the distance along the road in miles.

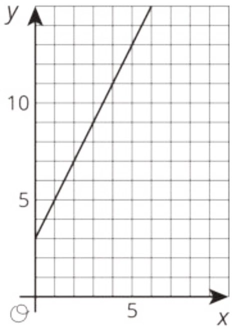
(From Unit 3, Lesson 6.)

5. Match each graph to a situation.

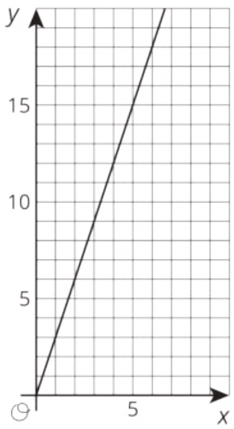
A



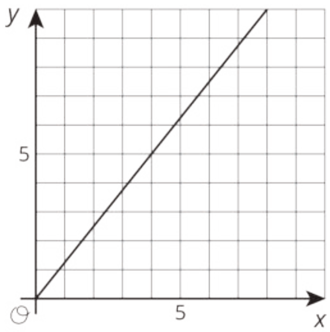
B



C



D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. The graph represents the perimeter, y, in units, for an equilateral triangle with side length of x units. The slope of the line is 3.

2. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$.

3. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$.

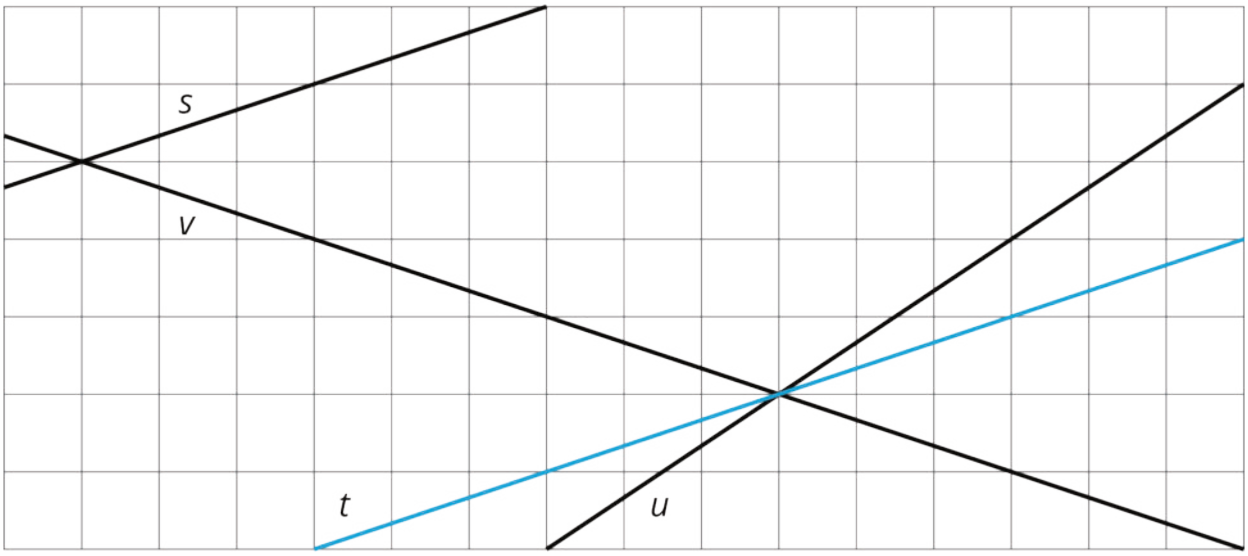
4. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2.

(From Unit 3, Lesson 6.)

Lesson 9: Slopes Don't Have to be Positive

9.1: Which One Doesn’t Belong: Odd Line Out

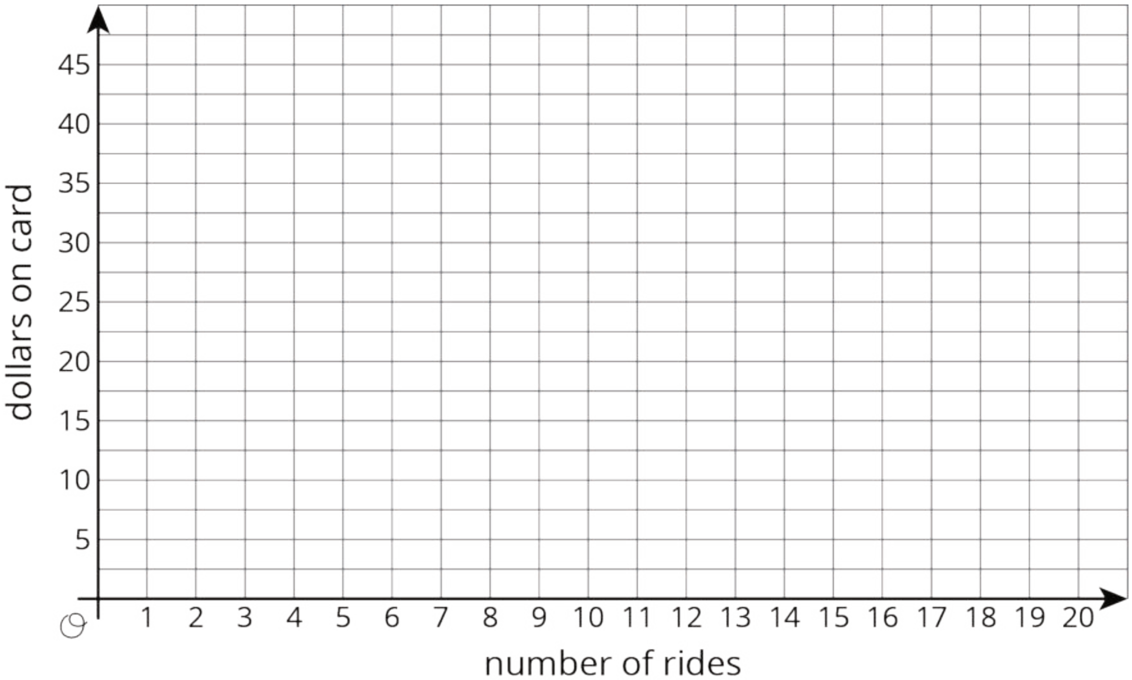
Which line doesn’t belong?



9.2: Stand Clear of the Closing Doors, Please

Noah put $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

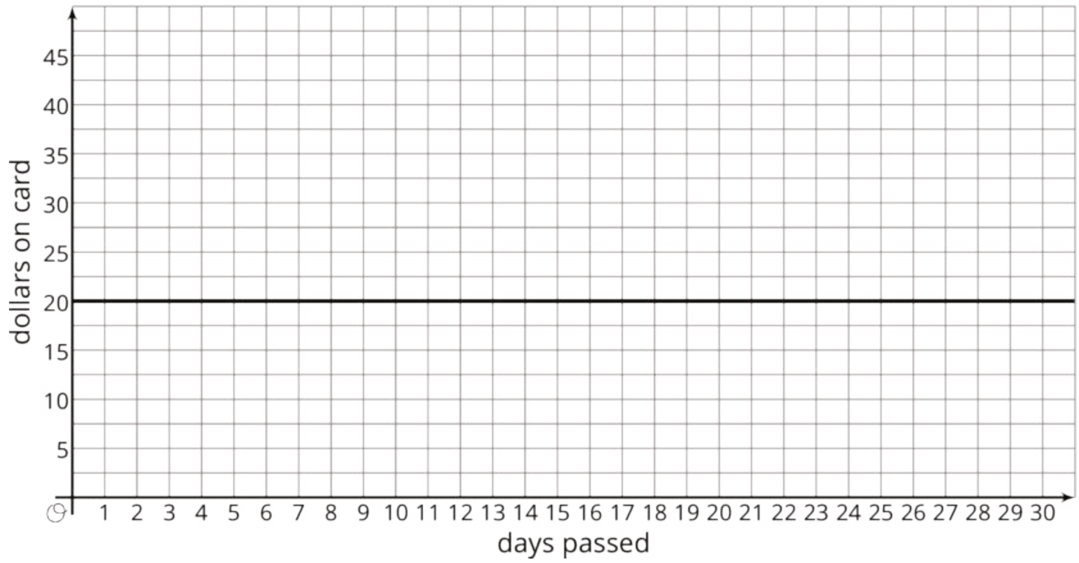
1. How much money, in dollars, is available on his card after he takes
    a. 0 rides?
    b. 1 ride?
    c. 2 rides?
    d. x rides?
2. Graph the relationship between amount of money on the card and number of rides.



3. How many rides can Noah take before the card runs out of money? Where do you see this number of rides on your graph?

9.3: Travel Habits in July

Here is a graph that shows the amount on Han's fare card for every day of last July.



1. Describe what happened with the amount on Han's fare card in July.

2. Plot and label 3 different points on the line.

3. Write an equation that represents the amount on the card in July, y, after x days.

4. What value makes sense for the slope of the line that represents the amounts on Han's fare card in July?

Are you ready for more?

Let's say you have taken out a loan and are paying it back. Which of the following graphs have positive slope and which have negative slope?

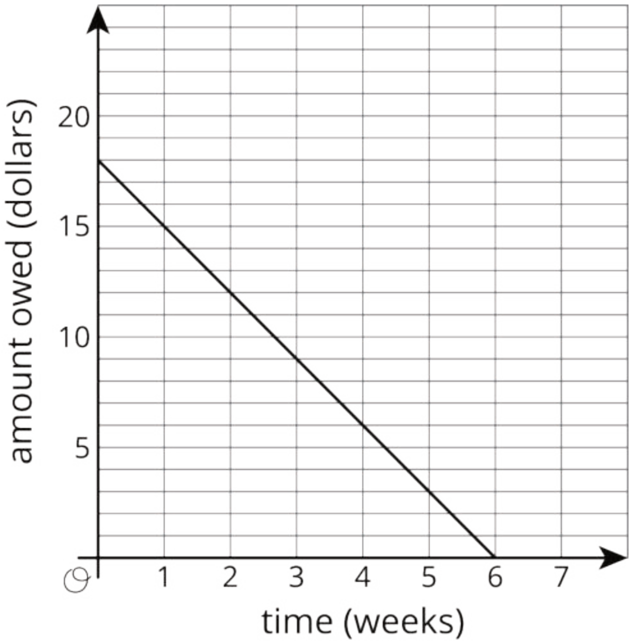
1. Amount paid on the vertical axis and time since payments started on the horizontal axis.

2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

9.4: Payback Plan

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.

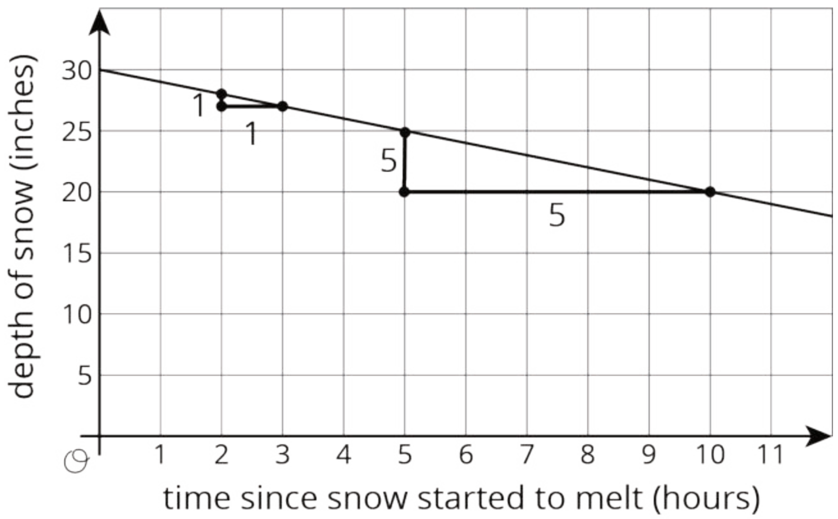


Answer and explain your reasoning for each question.

1. What is the slope of the line?
2. Explain how you know whether the slope is positive or negative.
3. What does the slope represent in this situation?
4. How much did Elena borrow?
5. How much time will it take for Elena to pay back all the money she borrowed?

Lesson 9 Summary

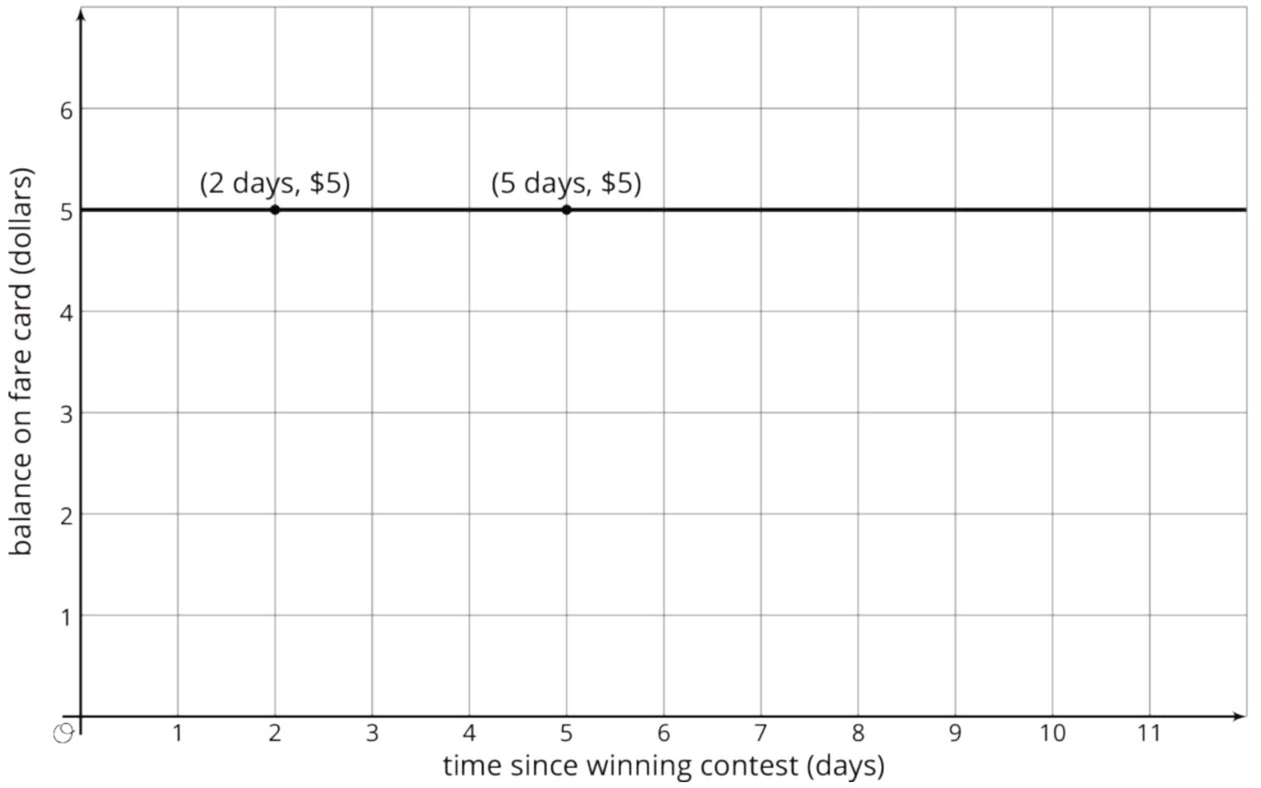
At the end of winter in Maine, the snow on the ground was 30 inches deep. Then there was a particularly warm day and the snow melted at the rate of 1 inch per hour. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of the graph is -1 since the rate of change is -1 inch per hour. That is, the depth goes *down* 1 inch per hour. The vertical intercept is 30 since the snow was 30 inches deep when the warmth started to melt the snow. The two slope triangles show how the rate of change is constant. It just also happens to be negative in this case since after each hour that passes, there is 1 inch *less* snow.

Graphs with negative slope often describe situations where some quantity is decreasing over time, like the depth of snow on warm days or the amount of money on a fare card being used to take rides on buses.

Slopes can be positive, negative, or even zero! A slope of 0 means there is no change in the y-value even though the x-value may be changing. For example, Elena won a contest where the prize was a special pass that gives her free bus rides for a year. Her fare card had $5 on it when she won the prize. Here is a graph of the amount of money on her fare card after winning the prize:



The vertical intercept is 5, since the graph starts when she has $5 on her fare card. The slope of the graph is 0 since she doesn't use her fare card for the next year, meaning the amount on her fare card doesn't change for a year. In fact, all graphs of linear relationships with slopes equal to 0 are horizontal—a rate of change of 0 means that, from one point to the next, the y-values remain the same.

Lesson 9: Slopes Don't Have to be Positive

Cool Down: The Slopes of Graphs

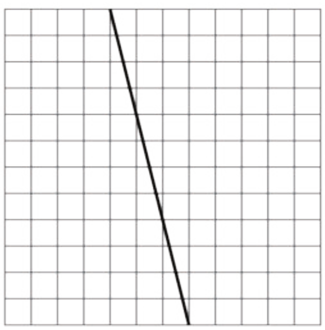
Each square on a grid represents 1 unit on each side.

1. Calculate the slope of graph D. Explain or show your reasoning.

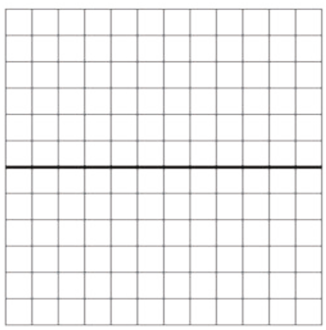
2. Calculate the slope of graph E. What situation could the graph represent?

3. On the blank grid F, draw a line that passes through the indicated point and has slope -2.

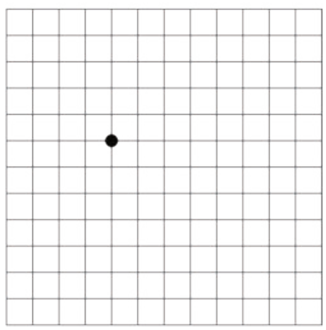
D



E



F



Unit 3 Lesson 9 Cumulative Practice Problems

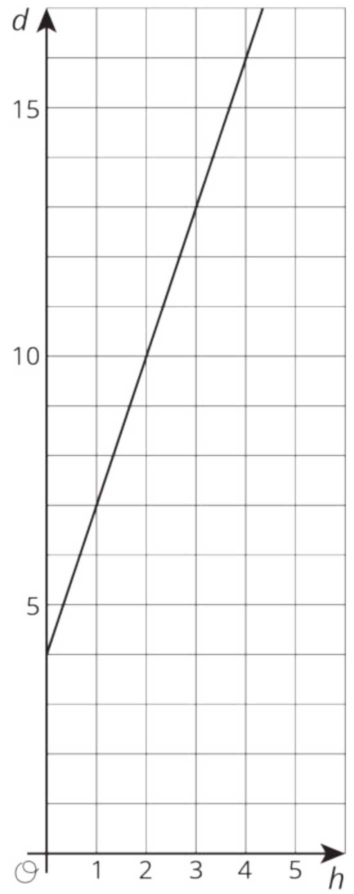
1. Suppose that during its flight, the elevation e (in feet) of a certain airplane and its time t, in minutes since takeoff, are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

    a. The plane is cruising at an altitude of 37,000 feet above sea level.

    b. The plane is descending at rate of 1000 feet per minute.

    c. The plane is ascending at a rate of 2000 feet per minute.

2. A group of hikers park their car at a trail head and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles, d, from the car after h hours of hiking.



   a. How far is the campsite from their car? Explain how you know.

   b. Write an equation that describes the relationship between d and h.

   c. After how many hours of hiking will they be 16 miles from their car? Explain or show your reasoning.

(From Unit 3, Lesson 7.)

3. Elena's aunt pays her \$1 for each call she makes to let people know about her aunt's new business.

The table shows how much money Diego receives for washing windows for his neighbors.

number of windows	number of dollars
27	30
45	50
81	90

Select **all** the statements about the situation that are true.

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.

B. Diego makes more money for washing each window than Elena makes for making each call.

C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.

D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.

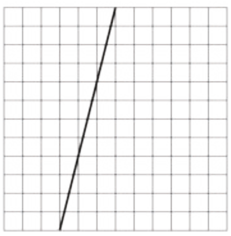
E. The equation $y = \frac{9}{10}x$, where y is number of dollars and x is number of windows, represents Diego's situation.

F. The equation $y = x$, where y is the number of dollars and x is the number of calls, represents Elena's situation.

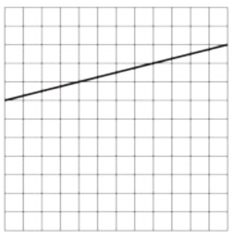
(From Unit 3, Lesson 4.)

4. Each square on a grid represents 1 unit on each side. Match the graphs with the slopes of the lines.

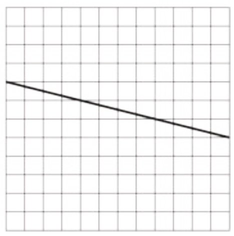
A



B



C



- $-\frac{1}{4}$
- $\frac{1}{4}$
- 4

Lesson 10: Calculating Slope

10.1: Number Talk: Integer Operations

Find values for a and b that make each side have the same value.

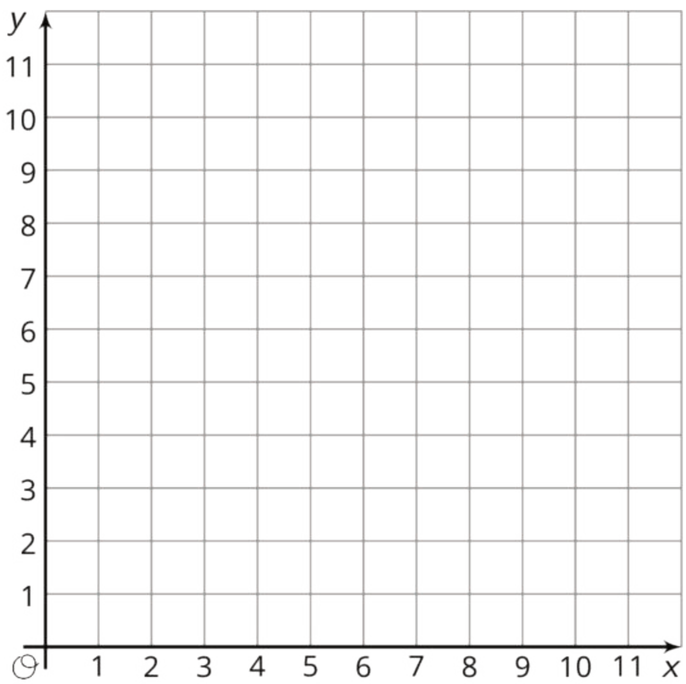
$\frac{a}{b} = -2$

$\frac{a}{b} = 2$

$a - b = -2$

10.2: Toward a More General Slope Formula

1. Plot the points $(1, 11)$ and $(8, 2)$, and use a ruler to draw the line that passes through them.
2. Without calculating, do you expect the slope of the line through $(1, 11)$ and $(8, 2)$ to be positive or negative? How can you tell?



3. Calculate the slope of this line.

Are you ready for more?

Find the value of k so that the line passing through each pair of points has the given slope.

1. $(k, 2)$ and $(11, 14)$, slope = 2

2. $(1, k)$ and $(4, 1)$, slope = -2

3. $(3, 5)$ and $(k, 9)$, slope = $\frac{1}{2}$

4. $(-1, 4)$ and $(-3, k)$, slope = $\frac{-1}{2}$

5. $(\frac{-15}{2}, \frac{3}{16})$ and $(\frac{-13}{22}, k)$, slope = 0

10.3: Making Designs

Your teacher will give you either a design or a blank graph. Do not show your card to your partner.

If your teacher gives you the design:

1. Look at the design silently and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or points that it goes through.
2. Describe each line, one at a time, and give your partner time to draw them.
3. Once your partner thinks they have drawn all the lines you described, only then should you show them the design.

If your teacher gives you the blank graph:

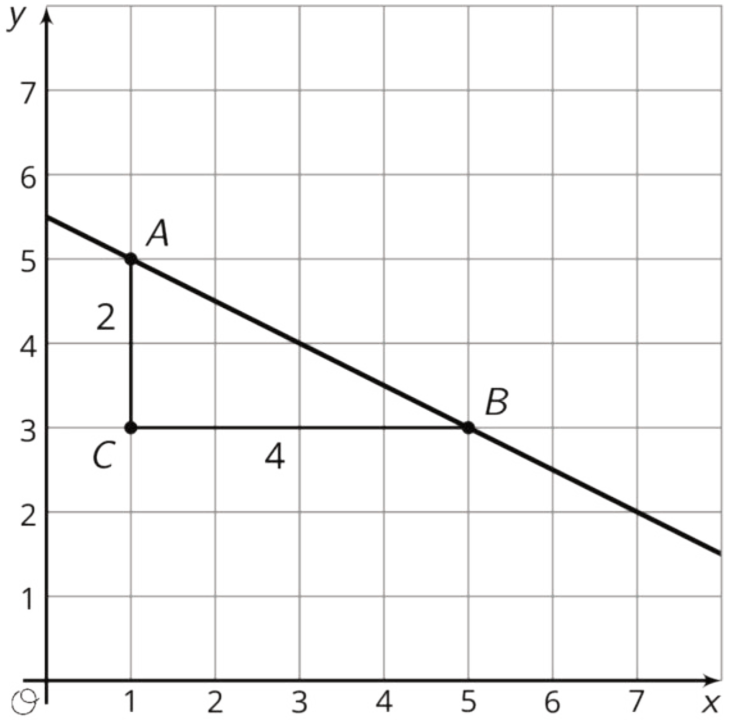
1. Listen carefully as your partner describes each line, and draw each line based on their description.
2. You are not allowed to ask for more information about a line than what your partner tells you.
3. Do not show your drawing to your partner until you have finished drawing all the lines they describe.

When finished, place the drawing next to the card with the design so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

Lesson 10 Summary

We learned earlier that one way to find the slope of a line is by drawing a slope triangle. For example, using the slope triangle shown here, the slope of the line is $-\frac{2}{4}$, or $-\frac{1}{2}$ (we know the slope is negative because the line is decreasing from left to right).

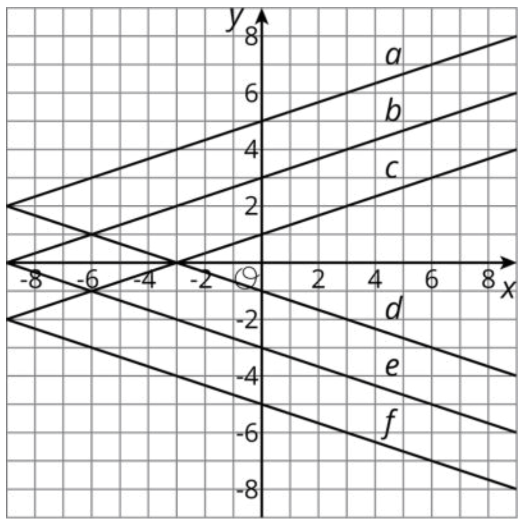


But slope triangles are only one way to calculate the slope of a line. Let's compute the slope of this line a different way using just the points $A = (1, 5)$ and $B = (5, 3)$. Since we know the slope is the vertical change divided by the horizontal change, we can calculate the change in the y-values and then the change in the x-values. Between points A and B, the y-value change is $3 - 5 = -2$ and the x-value change is $5 - 1 = 4$. This means the slope is $-\frac{2}{4}$, or $-\frac{1}{2}$, which is the same as what we found using the slope triangle.

Notice that in each of the calculations, We subtracted the value from point A from the value from point B. If we had done it the other way around, then the y-value change would have been $5 - 3 = 2$ and the x-value change would have been $1 - 5 = -4$, which still gives us a slope of $-\frac{1}{2}$. But what if we were to mix up the orders? If that had happened, we would think the slope of the line is *positive* $\frac{1}{2}$ since we would either have calculated $\frac{-2}{-4}$ or $\frac{2}{4}$. Since we already have a graph of the line and can see it has a negative slope, this is clearly incorrect. It we don't have a graph to check our calculation, we could think about how the point on the left, $(1, 5)$, is higher than the point on the right, $(5, 3)$, meaning the slope of the line must be negative.

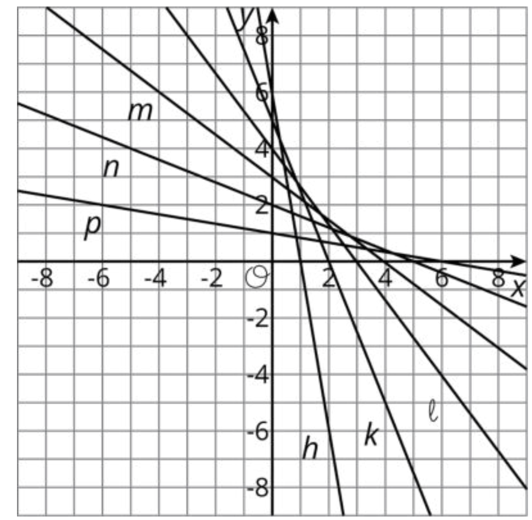
Making Designs

Design 1



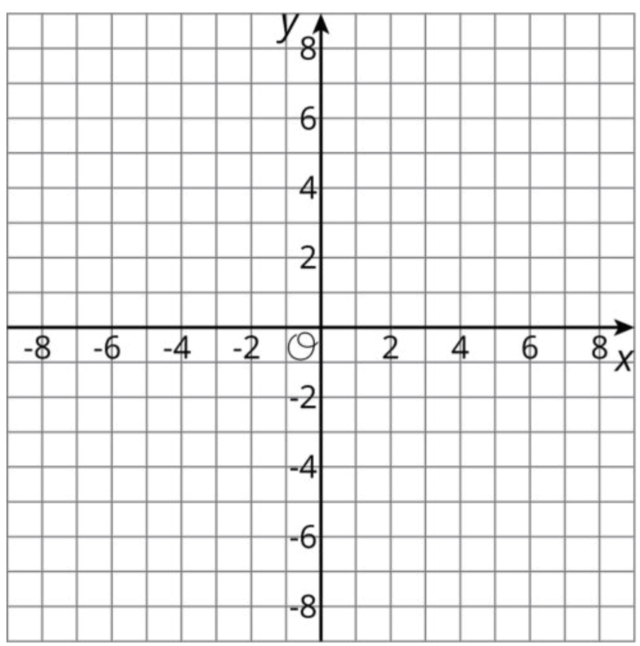
Making Designs

Design 2



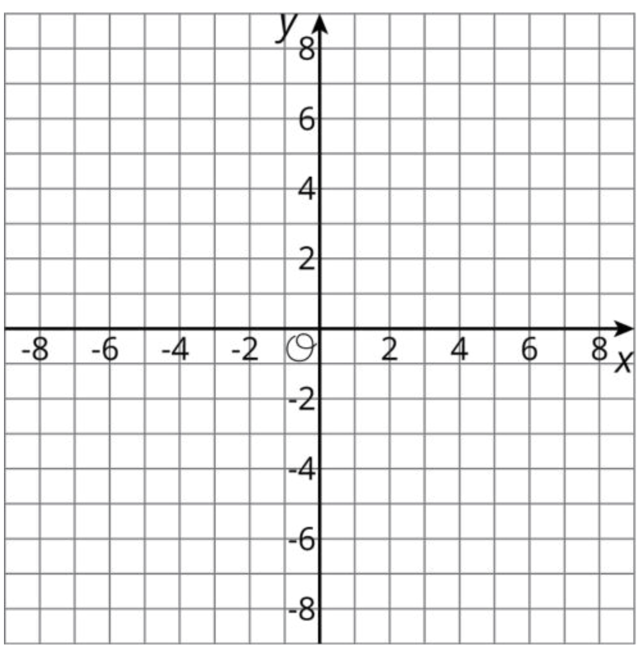
Making Designs

Design 1



Making Designs

Design 2



Lesson 10: Calculating Slope

Cool Down: Different Slopes

Without graphing, find the slope of the line that goes through

1. $(0, 5)$ and $(8, 2)$.

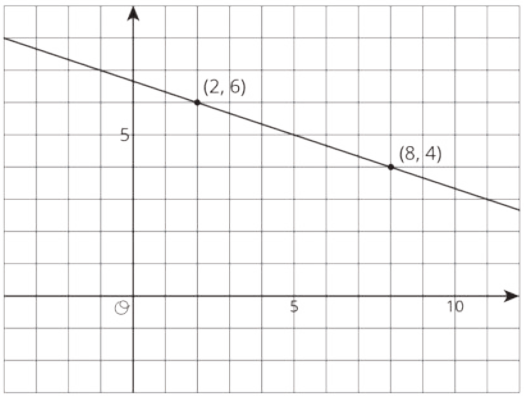
2. $(2, -1)$ and $(6, 1)$.

3. $(-3, -2)$ and $(-1, -5)$.

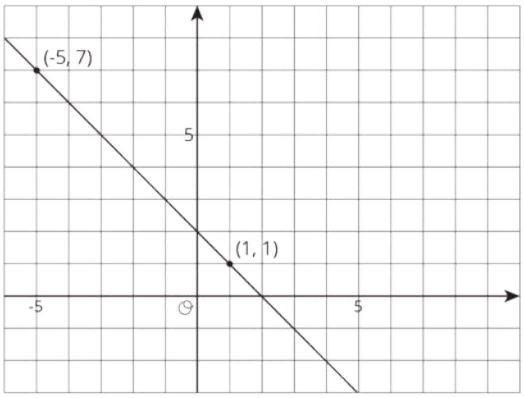
Unit 3 Lesson 10 Cumulative Practice Problems

1. For each graph, calculate the slope of the line.

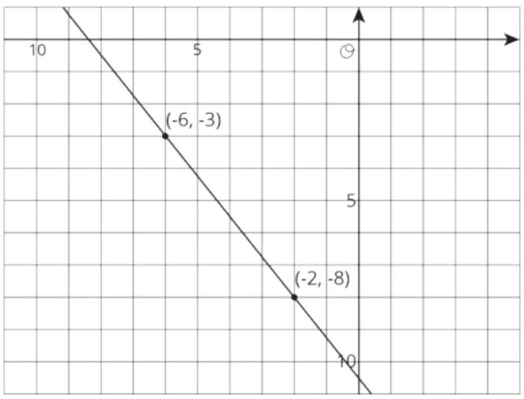
A



B



C

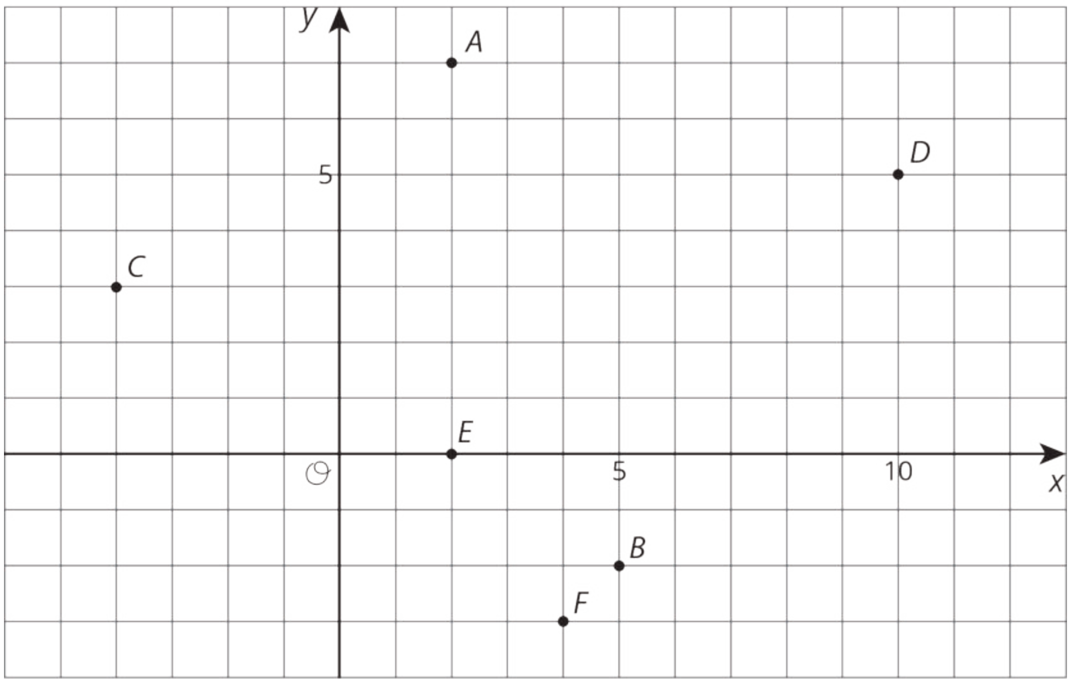


2. Match each pair of points to the slope of the line that joins them.

A. $(9, 10)$ and $(7, 2)$
B. $(-8, -11)$ and $(-1, -5)$
C. $(5, -6)$ and $(2, 3)$
D. $(6, 3)$ and $(5, -1)$
E. $(4, 7)$ and $(6, 2)$

1. 4
2. -3
3. $-\frac{5}{2}$
4. $\frac{6}{7}$

3. Draw a line with the given slope through the given point. What other point lies on that line?



a. Point A, slope = -3

b. Point A, slope = $\frac{-1}{4}$

c. Point C, slope = $\frac{-1}{2}$

d. Point E, slope = $\frac{-2}{3}$

4. Make a sketch of a linear relationship with a slope of 4 and a negative y-intercept. Show how you know the slope is 4 and write an equation for the line.

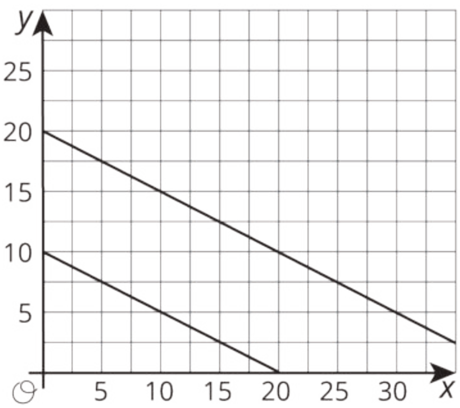
(From Unit 3, Lesson 8.)

Lesson 11: Equations of All Kinds of Lines

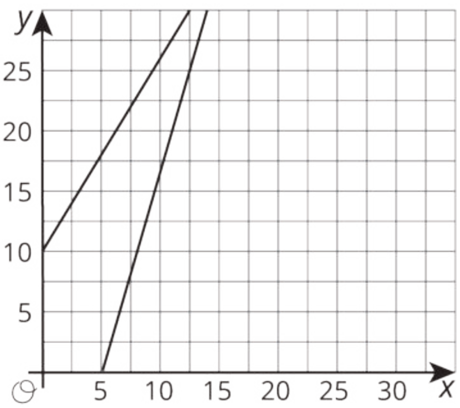
11.1: Which One Doesn’t Belong: Pairs of Lines

Which one doesn’t belong?

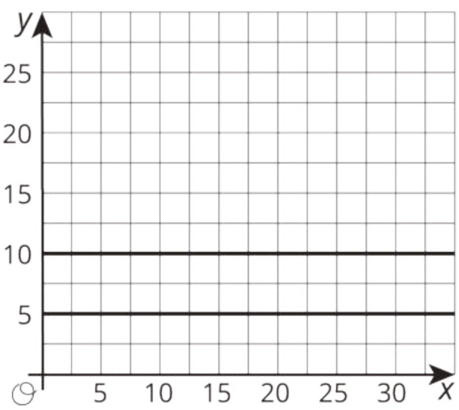
A



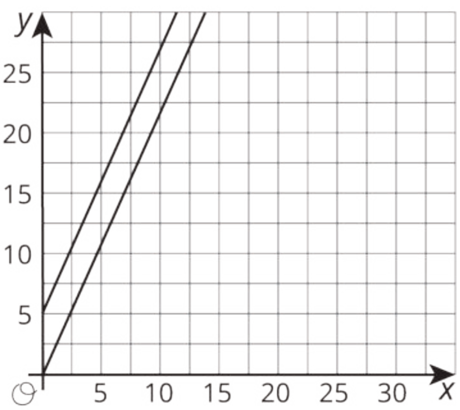
B



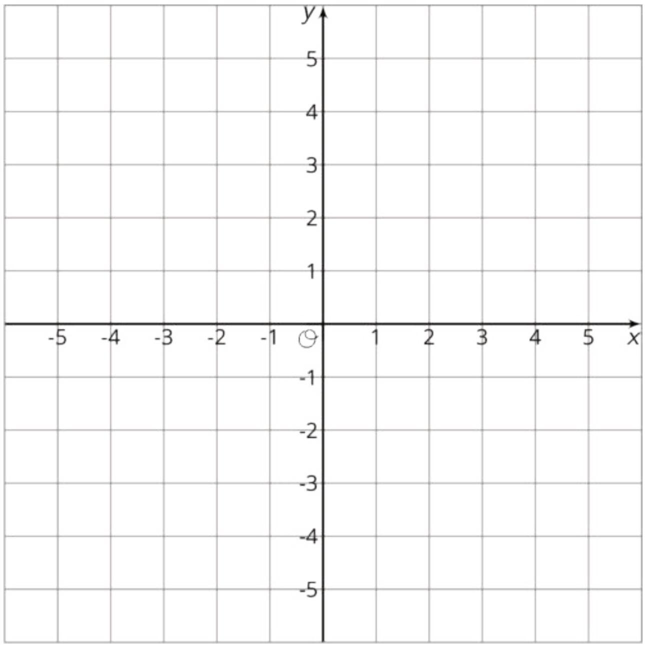
C



D



11.2: All the Same



1. Plot at least 10 points whose y-coordinate is -4. What do you notice about them?

2. Which equation makes the most sense to represent all of the points with y-coordinate -4? Explain how you know.

   $x = -4$ $\quad$ $y = -4x$ $\quad$ $y = -4$ $\quad$ $x + y = -4$

3. Plot at least 10 points whose x-coordinate is 3. What do you notice about them?

4. Which equation makes the most sense to represent all of the points with x-coordinate 3? Explain how you know.

   $x = 3$ $\quad$ $y = 3x$ $\quad$ $y = 3$ $\quad$ $x + y = 3$

5. Graph the equation $x = -2$.

6. Graph the equation $y = 5$.

Are you ready for more?

1. Draw the rectangle with vertices $(2, 1)$, $(5, 1)$, $(5, 3)$, $(2, 3)$.

2. For each of the four sides of the rectangle, write an equation for a line containing the side.

3. A rectangle has sides on the graphs of $x = \text{-}1$, $x = 3$, $y = \text{-}1$, $y = 1$. Find the coordinates of each vertex.

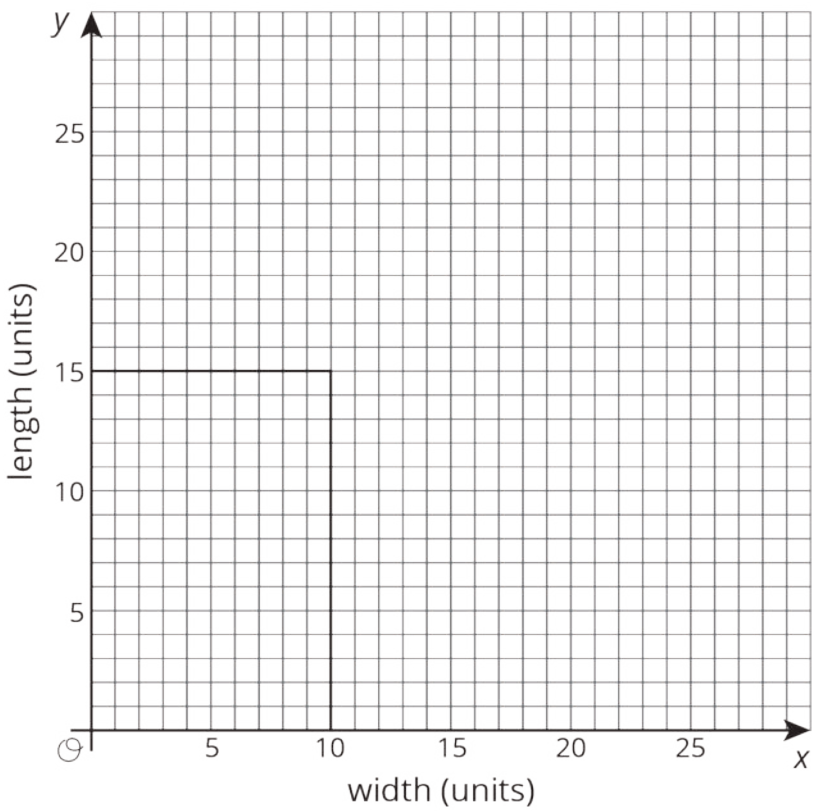
11.3: Same Perimeter

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, ℓ, and widths, w, of at least 10 such rectangles.

ℓ										
w										

2. The graph shows one rectangle whose perimeter is 50 units, and has its lower left vertex at the origin and two sides on the axes.

On the same graph, draw more rectangles with perimeter 50 units using the values from your table. Make sure that each rectangle has a lower left vertex at the origin and two sides on the axes.



3. Each rectangle has a vertex that lies in the first quadrant. These vertices lie on a line. Draw in this line and write an equation for it.

4. What is the the slope of this line? How does the slope describe how the width changes as the length changes (or vice versa)?

Lesson 11 Summary

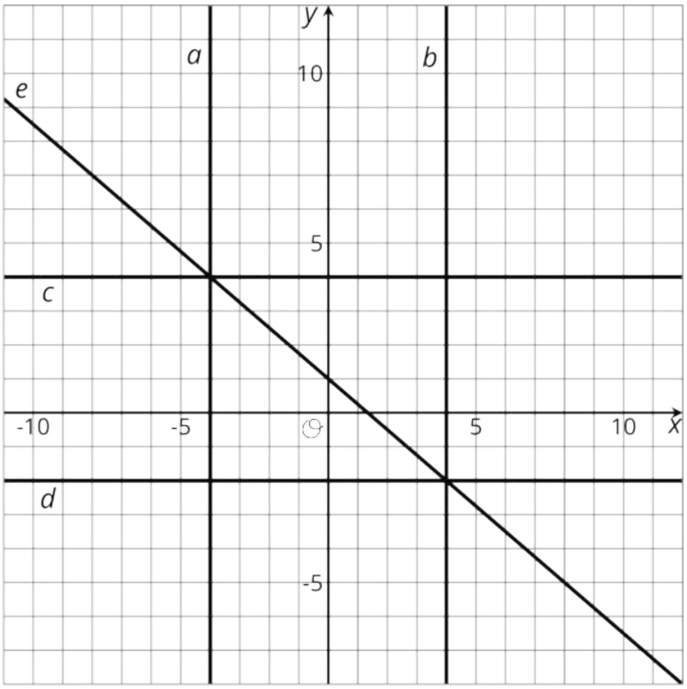
Horizontal lines in the coordinate plane represent situations where the y value doesn't change at all while the x value changes. For example, the horizontal line that goes through the point $(0, 13)$ can be described in words as "for all points on the line, the y value is always 13." An equation that says the same thing is $y = 13$.

Vertical lines represent situations where the x value doesn't change at all while the y value changes. The equation $x =$ -4 describes a vertical line through the point $(\text{-}4, 0)$.

Lesson 11: Equations of All Kinds of Lines

Cool Down: Line Design

Here are 5 lines on a coordinate grid:



Write equations for lines a, b, c, d, and e.

Unit 3 Lesson 11 Cumulative Practice Problems

1. Suppose you wanted to graph the equation $y = -4x - 1$.

    a. Describe the steps you would take to draw the graph.

    b. How would you check that the graph you drew is correct?

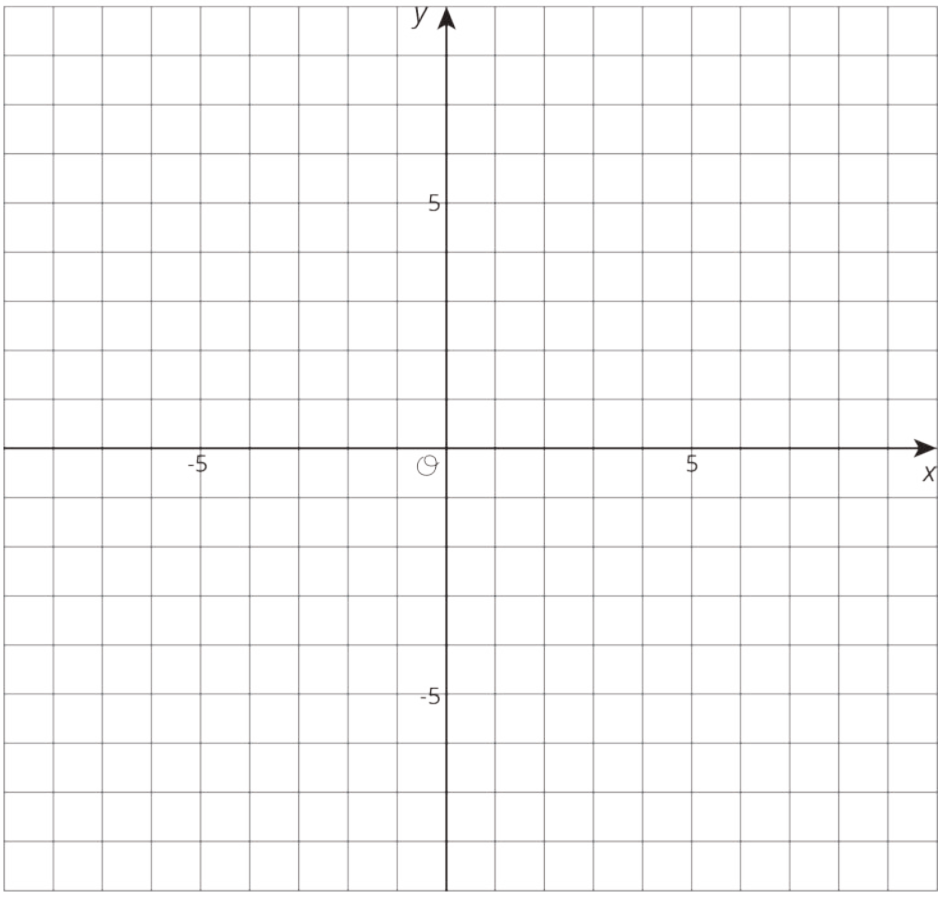
2. Draw the following lines and then write an equation for each.

a. Slope is 0, y-intercept is 5

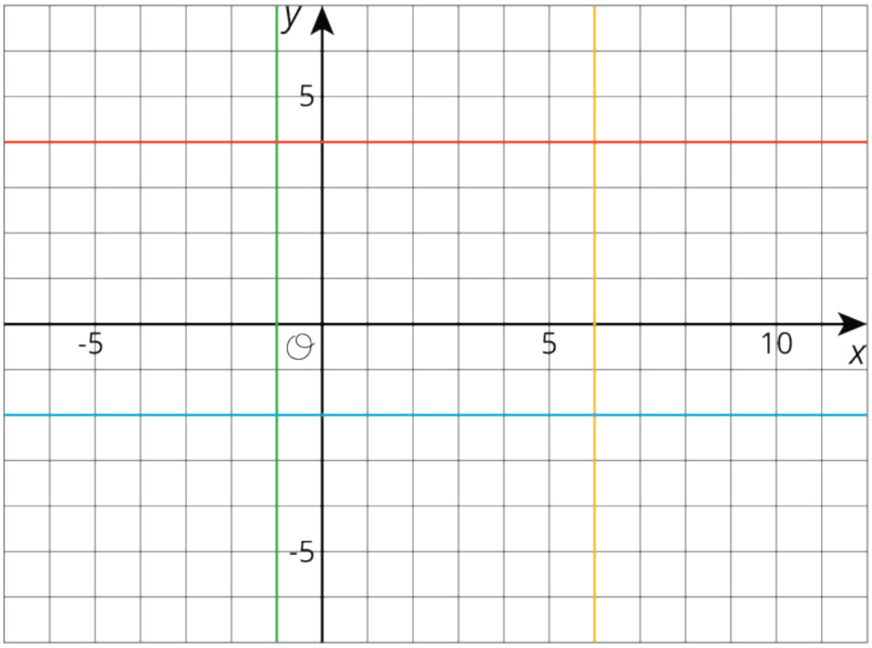
b. Slope is 2, y-intercept is -1

c. Slope is -2, y-intercept is 1

d. Slope is $-\frac{1}{2}$, y-intercept is -1



3. Write an equation for each line.



4. A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ of an inch. They have a choice of which type of paper to print the book on.

    a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the width of the book, y, if it has x hundred pages, printed on bond paper.

    b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the width of the book, y, if it has x hundred pages, printed on ledger paper.

    c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

(From Unit 3, Lesson 7.)

Lesson 12: Systems of Equations

12.1: Milkshakes

Diego and Lin are drinking milkshakes. Lin starts with 12 ounces and drinks $\frac{1}{4}$ an ounce per second. Diego starts with 20 ounces and drinks $\frac{2}{3}$ an ounce per second.

1. How long will it take Lin and Diego to finish their milkshakes?
2. Without graphing, explain what the graphs in this situation would look like. Think about slope, intercepts, axis labels, units, and intersection points to guide your thinking.
3. Discuss your description with your partner. If you disagree, work to reach an agreement.

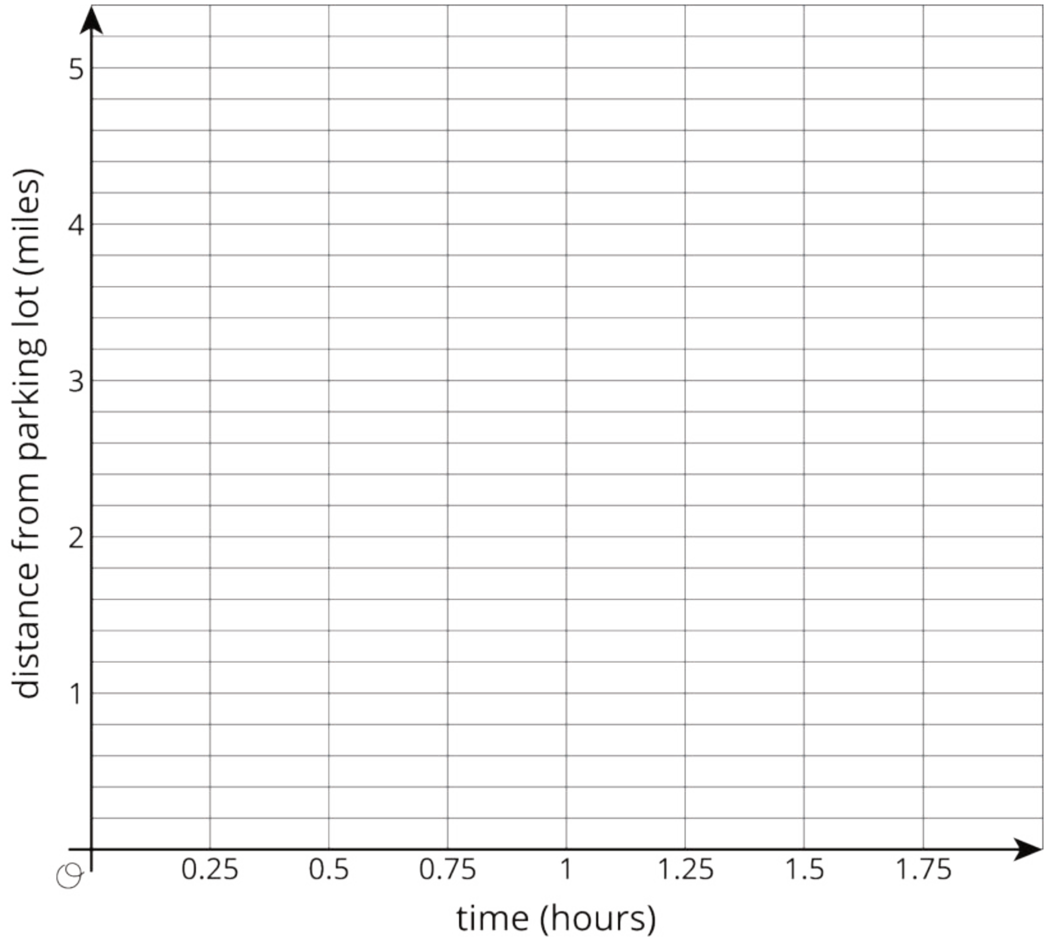
12.2: Passing on the Trail

There is a hiking trail near the town where Han and Jada live that starts at a parking lot and ends at a lake. Han and Jada both decide to hike from the parking lot to the lake and back, but they start their hikes at different times.

At the time that Han reaches the lake and starts to turn back, Jada is 0.6 miles away from the parking lot and hiking at a constant speed of 3.2 miles per hour towards the lake. Han's distance, d, from the parking lot can be expressed as $d = -2.4t + 4.8$, where t represents the time in hours since he left the lake.

1. What is an equation for Jada's distance from the parking lot as she heads toward the lake?

2. Draw both graphs: one representing Han's equation and one representing Jada’s equation. It is important to be very precise! Be careful, work in pencil, and use a ruler.



3. Find the point where the two graphs intersect each other. What are the coordinates of this point?

4. What do the coordinates mean in this situation?

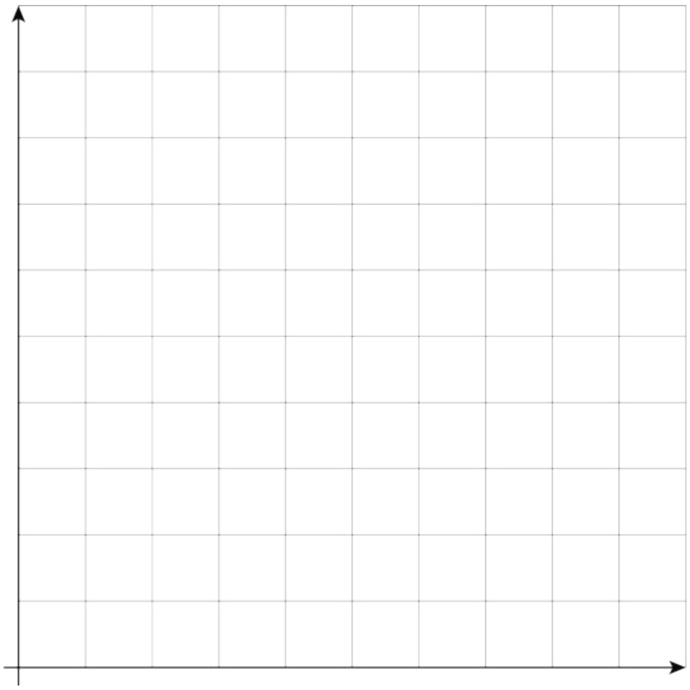
5. What has to be true about the relationship between these coordinates and Jada’s equation?

6. What has to be true about the relationship between these coordinates and Han’s equation?

12.3: Stacks of Cups

A stack of n small cups has a height, h, in centimeters of $h = 1.5n + 6$. A stack of n large cups has a height, h, in centimeters of $h = 1.5n + 9$.

1. Graph the equations for each cup on the same set of axes. Make sure to label the axes and decide on an appropriate scale.



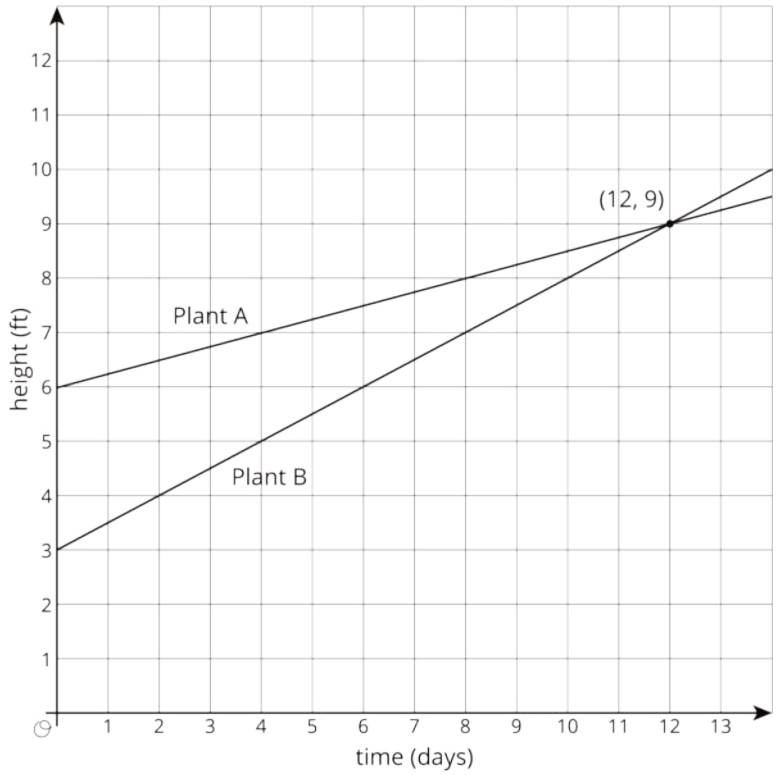
2. For what number of cups will the two stacks have the same height?

Lesson 12 Summary

A **system of equations** is a set of 2 (or more) equations where the variables represent the same unknown values. For example, suppose that two different kinds of bamboo are planted at the same time. Plant A starts at 6 ft tall and grows at a constant rate of $\frac{1}{4}$ foot each day. Plant B starts at 3 ft tall and grows at a constant rate of $\frac{1}{2}$ foot each day. We can write equations $y = \frac{1}{4}x + 6$ for Plant A and $y = \frac{1}{2}x + 3$ for Plant B, where x represents the number of days after being planted, and y represents height. We can write this system of equations.

$$\begin{cases} y = \frac{1}{4}x + 6 \\ y = \frac{1}{2}x + 3 \end{cases}$$

Solving a system of equations means to find the values of x and y that make both equations true at the same time. One way we have seen to find the solution to a system of equations is to graph both lines and find the intersection point. The intersection point represents the pair of x and y values that make both equations true. Here is a graph for the bamboo example:



The solution to this system of equations is $(12, 9)$, which means that both bamboo plants will be 9 feet tall after 12 days.

We have seen systems of equations that have no solutions, one solution, and infinitely many solutions.

- When the lines do not intersect, there is no solution. (Lines that do not intersect are *parallel*.)
- When the lines intersect once, there is one solution.
- When the lines are right on top of each other, there are infinitely many solutions.

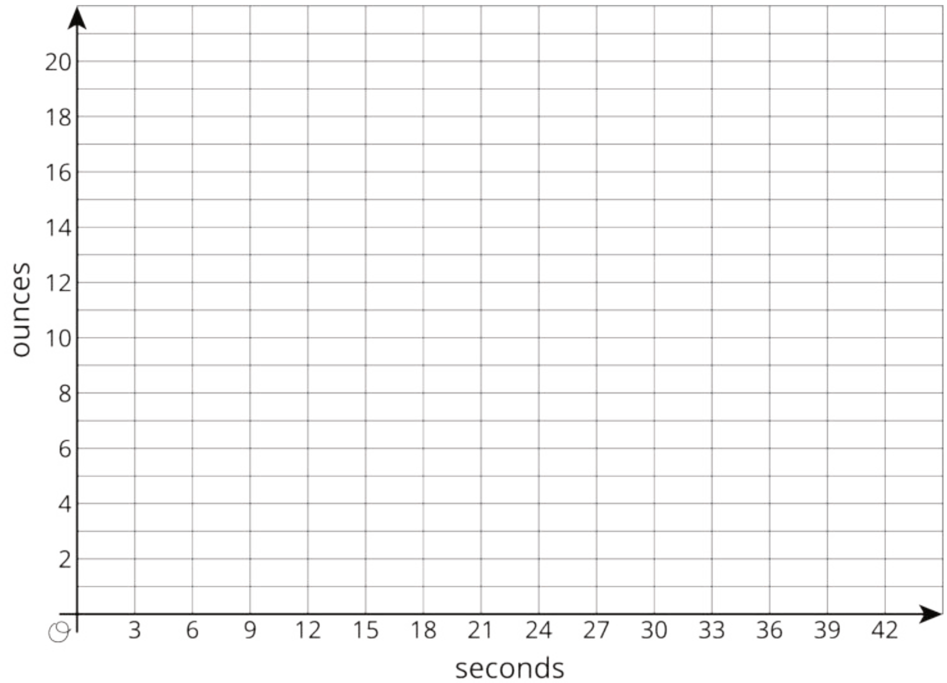
In future lessons, we will see that some systems cannot be easily solved by graphing, but can be easily solved using algebra.

Lesson 12: Systems of Equations

Cool Down: Milkshakes, Revisited

Determined to finish her milkshake before Diego, Lin now drinks her 12 ounce milkshake at a rate of $\frac{1}{3}$ an ounce per second. Diego starts with his usual 20 ounce milkshake and drinks at the same rate as before, $\frac{2}{3}$ an ounce per second.

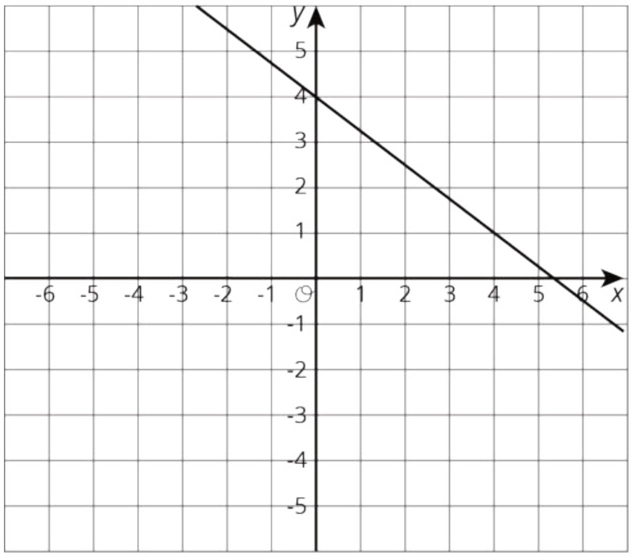
1. Graph this situation on the axes provided.



2. What does the graph tell you about the situation and how many solutions there are?

Unit 4 Lesson 12 Cumulative Practice Problems

1. Here is the graph for one equation in a system of equations:



   a. Write a second equation for the system so it has infinitely many solutions.

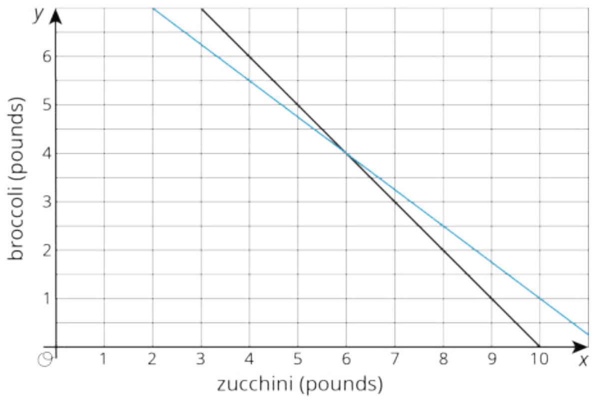
   b. Write a second equation whose graph goes through $(0, 1)$ so the system has no solutions.

   c. Write a second equation whose graph goes through $(0, 2)$ so the system has one solution at $(4, 1)$.

2. Create a second equation so the system has no solutions.

$$\begin{cases} y = \frac{3}{4}x - 4 \\ \end{cases}$$

3. Andre is in charge of cooking broccoli and zucchini for a large group. He has to spend all \$17 he has and can carry 10 pounds of veggies. Zucchini costs \$1.50 per pound and broccoli costs \$2 per pound. One graph shows combinations of zucchini and broccoli that weigh 10 pounds and the other shows combinations of zucchini and broccoli that cost \$17.



a. Name one combination of veggies that weighs 10 pounds but does not cost \$17.

b. Name one combination of veggies that costs \$17 but does not weigh 10 pounds.

c. How many pounds each of zucchini and broccoli can Andre get so that he spends all \$17 and gets 10 pounds of veggies?

(From Unit 4, Lesson 10.)

4. The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, by the equation

$$F = \frac{9}{5}C + 32$$

a. In the Sahara desert, temperatures often reach 50 degrees Celsius. How many degrees Fahrenheit is this?

b. In parts of Alaska, the temperatures can reach -60 degrees Fahrenheit. How many degrees Celsius is this?

c. There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. Use the expression from the equation, where F is expressed in terms of C, to solve for this temperature.

(From Unit 4, Lesson 9.)