AP Calculus – Unit 2 – Intro to Derivatives Study Guide

Definition

If $y = f(x)$ then the derivative is defined to be

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or

$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

The derivative at a point x=a where is defined as $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Notation

If $y = f(x)$ then all of the following are equivalent notations for the derivative.

$$f'(x) = y' = \frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx}\left(f(x)\right) = Df(x)$$

If $y = f(x)$ all of the following are equivalent notations for derivative evaluated at $x=a$.

$$f'(a) = y'\big|_{x=a} = \frac{df}{dx}\bigg|_{x=a} = \frac{dy}{dx}\bigg|_{x=a} = Df(a)$$

Interpretation of the Derivative

If $y = f(x)$ then,

1.) $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x=a$ and the equation of the tangent line at $x=a$ is given by $y = f(a) + m(x-a)$. This formula can also be written as $y = y_1 + m(x - x_1)$ where $m = f'(x_1)$
.

2.) $f'(a)$ is the instantaneous rate of change of $f(x)$ at $x = a$. This is sometimes referred to as the slope of the curve f at $x = a$. Average rate of change can be found using the formula for the slope of a line $\left(\frac{\Delta x}{\Delta y}\right)$.

3.) If $f(x)$ is the position of an object at time x then $f(a)$ is the velocity of the object at $x=a$.

Differentiability

A function is not differentiable at a point x=a (meaning the derivative does not exist at x=a) if the function 1.) is not continuous at x=a (vertical asymptotes, POD's, etc.) 2.) has a sharp point at x=a (change in slope (+/-) with no horizontal tangent) 3.) has a vertical tangent has a vertical tangent at x=a (slope of the tangent is undefined). If a function is differentiable then the function is also continuous at x=a. If the function is continuous, it may be differentiable but that is not certain.

Facts about lines…

1.) Slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

2.) Horizontal Line $m = 0$ with equation $y = a$. For vertical lines, slope is undefined and the equation is $x = a$.

3.) Parallel lines have the same slope.
Normal/Perpendicular lines have slopes that are negative reciprocals. $m_1 = -\frac{1}{m_2}$.

Facts about motion…

1.) "at rest" $v(t) = 0$

2.) direction change $v(t) = 0$ & v(t) changes sign

3.) speed $|v(t)|$

4.)" moving right" – v(t) >0

5.) "moving left" – v(t)<0

4.) Units – distance / time for velocity.

5.) Acceleration is the derivative of velocity.
Units for acceleration - distance/ time^2

Constant Rule $\frac{d}{dx}[c]=0$	Constant Multiple Rule $\frac{d}{dx}[cu]=cu'$
Power Rule	$\frac{d}{dx}[x^n]=nx^{n-1}$
Sum and Difference Rules	$\frac{d}{dx}[f\pm g]=f'\pm g'$
Product Rule	$\frac{d}{dx}[fg]=gf'+fg'$
Quotient Rule	$\frac{d}{dx}\left[\frac{f}{g}\right]=\frac{gf'-fg'}{g^2}$
Trig Functions	
$\frac{d}{dx}[\sin x]=\cos x$	$\frac{d}{dx}[\cos x]=-\sin x$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\frac{d}{dx}[\cot x]=-\csc^2 x$