#1 (2004 ABB4c)

AP Scoring Rubric

(c) $g'(x) = f(x) + xf'(x)$

$g'(2) = f(2) + 2f'(2) = 6 + 2(-1) = 4$

$g(2) = 2f(2) = 12$

Tangent line is $y = 4(x-2) + 12$

$3 : \begin{cases} 2 : g'(x) \\ 1 : \text{tangent line} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 4(c)

$g(x) = x\,f(x)$

$g'(x) = (x)'\,f(x) + x\,f'(x)$

$g'(2) = f(2) + 2f'(2)$

$= 6 + 2\cdot(-1)$

$= 4$

$g(2) = 2\cdot f(2)$

$= 12$

$(2, 12)$

$4 = \frac{y-12}{x-2}$

$4x - 8 + 12 = y$

$y = 4x + 4$

#2 (2007B- AB3 a&b)

AP Scoring Rubric

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286 When $v = 20$ mph, the wind chill is decreasing at $0.286\ ^\circ\text{F}/\text{mph}$.	$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$
(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254. $W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.	$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 3(a)

$$W(v) = 55.6 - 22.1\, V^{0.16}$$

$$W'(v) = -22.1\,(0.16)\, V^{0.16-1}$$

$$= -3.536\, V^{-0.84}$$

$$W'(20) \approx -3.536\,(20)^{-0.84}$$

$$\approx -0.286\ ^\circ F/mph$$

It means that the wind chill is decreasing at a rate of 0.286 °F/mph when V=20 mph.

Work for problem 3(b)

$$\frac{F(60)-F(5)}{60-5} = \frac{13.0503-27.0091}{60-5} = \boxed{-.254}$$

$$W'(v) = -22.1\,(.16)v^{-.84} = -.254$$

$$\frac{1}{-.84}\left(v^{-.84}\right) = (.7177)^{1/-.84}$$

$$\boxed{v = 23.011}$$

#3 (2008 AB6a).

AP Scoring Rubric

(a) $f(e^2) = \frac{\ln e^2}{e^2} = \frac{2}{e^2},\ f'(e^2) = \frac{1-\ln e^2}{(e^2)^2} = -\frac{1}{e^4}$ An equation for the tangent line is $y = \frac{2}{e^2} - \frac{1}{e^4}(x - e^2)$.	$2: \begin{cases} 1: f(e^2) \text{ and } f'(e^2) \\ 1: \text{answer} \end{cases}$

Actual Solution Receiving Full Credit

6A1

Work for problem 6(a)

$$f(e^2) = \frac{\ln(e^2)}{e^2} = \frac{2}{e^2}$$

$$y - \frac{2}{e^2} = \left(\frac{1-\ln(e^2)}{e^4}\right)(x-e^2)$$

$$y - \frac{2}{e^2} = -\frac{1}{e^4}(x-e^2)$$

#4 20011B- AB2c

AP Scoring Rubric

(c) $r'(3) = 50$ The rate at which water is draining out of the tank at time $t = 3$ hours is increasing at 50 liters/hour^2.	$2: \begin{cases} 1: r'(3) \\ 1: \text{meaning of } r'(3) \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 2(c)

$$r'(3) = \frac{d}{dt}\left(\frac{600t}{t+3}\right)$$

$$= \frac{(t+3)(600) - (600t)(1)}{(t+3)^2}$$

$$= \frac{3600 - 1800}{36}$$

$$= 50$$

The rate of at which water is draining is increasing at 50 liters/h^2 at $t=3$.

#5 . (2009 AB3a)

AP Scoring Rubric

(a) $\lim\limits_{h\to 0^-} \frac{f(h)-f(0)}{h} = \frac{2}{3}$

$\lim\limits_{h\to 0^+} \frac{f(h)-f(0)}{h} < 0$

Since the one-sided limits do not agree, f is not differentiable at $x = 0$.

$2 : \begin{cases} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

Actual Solution Receiving Full Credit

Work for problem 3(a)

f is differentiable at $x=0 \iff \lim\limits_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists

Now $\lim\limits_{x\to 0^+} \frac{f(x)-f(0)}{x} < 0$ because $f(x) < f(0)$ at $x > 0$

$\lim\limits_{x\to 0^-} \frac{f(x)-f(0)}{x} > 0$ because $f(x) < f(0)$ at $x < 0$

so $\lim\limits_{x\to 0^+} \frac{f(x)-f(0)}{x} \neq \lim\limits_{x\to 0^-} \frac{f(x)-f(0)}{x}$

$\Rightarrow \lim\limits_{x\to 0} \frac{f(x)-f(0)}{x}$ does not exist $\Rightarrow$ f is not differentiable at $x=0$

Do not write bey

#6 (2011 ABB3c)

(c) $g'(x) = -1$

Thus a line perpendicular to the graph of g has slope 1.

$f'(x) = \frac{1}{2\sqrt{x}}$

$\frac{1}{2\sqrt{x}} = 1 \Rightarrow x = \frac{1}{4}$

The point P has coordinates $\left(\frac{1}{4}, \frac{1}{2}\right)$.

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}}$ $g'(x) = -1$

Since the line is perpendicular to $g(x)$, when $g'(x) = -1$.

the slope of the line is 1

Therefore $\frac{1}{2\sqrt{x}} = 1$ $\therefore x = \frac{1}{4}$

So $f(\frac{1}{4}) = \frac{1}{2}$ the coordinates of point P is $(\frac{1}{4}, \frac{1}{2})$

Work for problem 3(c)

$f'(x) = \frac{1}{2\sqrt{x}}$ $g'(x) = -1$

$\frac{1}{2\sqrt{x}} = 1$ $1 = 2\sqrt{x}$

$\sqrt{x} = 1/2 \Rightarrow x = \frac{1}{4}$

$f(1/4) = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow P(\frac{1}{4}, \frac{1}{2})$