

Direct Variation

Warm Up

Use the point-slope form of each equation to identify a point the line passes through and the slope of the line.

1. $y - 3 = -\frac{1}{7}(x - 9)$ $(9, 3), -\frac{1}{7}$

2. $y + 2 = \frac{2}{3}(x - 5)$ $(5, -2), \frac{2}{3}$

3. $y - 9 = -2(x + 4)$ $(-4, 9), -2$

4. $y - 5 = -\frac{1}{4}(x + 7)$ $(-7, 5), -\frac{1}{4}$

Direct Variation

Essential Question

How can you identify a direct variation?

Standard

MCC8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Direct Variation

A **direct variation** is a linear function that can be written as $y = kx$, where k is a nonzero constant called the **constant of variation**.

Direct Variation

Reading Math


The constant of variation is also called the constant of proportionality.


Name: _____

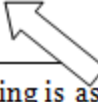
What is a Direct Variation?

- The equation that represents a direct variation is _____, where k is the _____ (this is the factor that changes the quantities.)
- The equation can be changed to $\frac{Y}{x} = k$ to find the constant of proportionality.
- Let's say you know the coordinate (2,4). This is the same as saying that x is _____ and y is _____. Substitute your coordinate into the direct variation equation to solve for the constant of proportionality.

Relationship: $\frac{Y}{x} = k$ $\frac{4}{2} = k$ $k = 2$ $y = 2x$

Solving for k
(constant of
proportionality)
 

This is the
constant of
proportionality
 

Writing is as
an equation
 

- A direct variation equation can be represented by a proportion: A direct proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ can represent 2 different coordinates (x_1, y_1) and (x_2, y_2)
- A direct variation graph _____ passes through the _____ (0,0).

- The equation can be changed to $\frac{y}{x} = k$ to find the constant of proportionality.
- Let's say you know the coordinate (2,4). This is the same as saying that x is ____ and y is _____. Substitute your coordinate into the direct variation equation to solve for the constant of proportionality.

Relationship: $\frac{y}{x} = k$ $\frac{4}{2} = k$ $k = 2$ $y = 2x$

Solving for k
(constant of proportionality)

This is the
constant of
proportionality

Writing is as
an equation

- A direct variation equation can be represented by a proportion: A direct proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ can represent 2 different coordinates (x_1, y_1) and (x_2, y_2)
- A direct variation graph _____ passes through the _____ (0,0).

Direct Variation In Context

A direct variation in context reveals how an increase in 1 variable will result in an _____ in another variable. It can also reveal how a decrease in 1 variable will result in a _____ in another variable. *A direct variation can represent an increase in 1 variable and a decrease in another variable.*



- The more time I drive at a constant rate, the more miles I go.
- If I increase a recipe for more people, the more of an ingredient I need.
- The more hours I work, the more money I make.
- The more CD's I purchase, the more money it costs.
- If you buy half as much cheese at the deli, you pay half as much.

Increase in one variable results in decrease in another.

Ex.

- The more time you spend working out, the less you weigh.
- The less time you work out, the more you weigh.
-

Direct Variation Proportions

A direct variation proportion has ratios that are equivalent. A coordinate (x,y) is actually the ratio of $\frac{y}{x}$. A direct proportion $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ can represent 2 different coordinates (x_1, y_1) and (x_2, y_2) or 2 different coordinates can represent a direct proportion. Write 1-2 examples in your foldable.

For example:

$$\frac{3}{12} = \frac{2}{8} \text{ is the same as the coordinates } (12, 3), (8, 2)$$

$$\frac{9}{3} = \frac{15}{5} \text{ is the same as the coordinates } (3, 9), (5, 15)$$

Direct Variation

Direct Variation Tables

Look at the following tables that represent direct variation.

X	Y
1	2
2	4
3	6

X	Y
10	5
6	3
4	2

Y divided by X = k (constant)

What is the constant for each of the tables?

Table 1:

Table 2:

NOT Direct Variation Tables

Look at the following tables that do not represent a direct variation. Find what all of the tables have in common.

X	Y
1	3

Explain why each of the tables are not direct variation.

X	Y
10	5
6	3
4	2

Table 1:

Table 2:

NOT Direct Variation Tables

Look at the following tables that do not represent a direct variation. Find what all of the tables have in common.

X	Y
1	3
2	4
3	5

X	Y
10	6
20	4
30	2

Explain why each of the tables are not direct variation.

Table 1:

Table 2:

Direct Variation Equations



The following are examples of direct variation equations. The number in front of the x variable is called the constant of proportionality or variation. $Y=kx$

Why are the following equations direct variation?

$$y = \frac{1}{2}x$$

$$y = x$$

$$y = 2x$$

$$y = -2.5x$$

NON-Direct Variation Equations

The following are non-examples of direct variation equations.

Why are the following NOT equations direct variation?

$$y = \frac{1}{2}x - 2$$

$$y = x + 3$$

$$y = 2$$

$$y = 2.5x - 4$$

[Video](#)

Direct Variation

Additional Example 1A: Determining Whether a Data Set Varies Directly

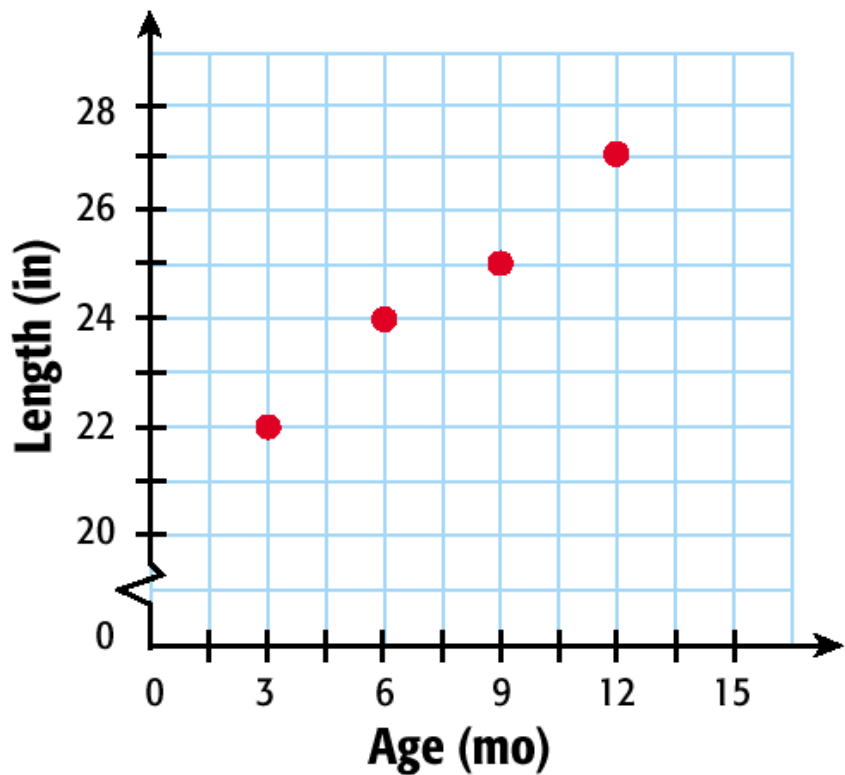
Determine whether the data set shows direct variation.

Adam's Growth Chart				
Age (mo)	3	6	9	12
Length (in.)	22	24	25	27

Direct Variation

Additional Example 1A Continued

Make a graph that shows the relationship between Adam's age and his length. The graph is not linear.



Direct Variation

Additional Example 1A Continued

You can also compare ratios to see if a direct variation occurs.

$$\frac{22}{3} \stackrel{?}{=} \frac{27}{12}$$

81 $81 \neq 264$
The ratios are not proportional.
 264

The relationship of the data is not a direct variation.

Direct Variation

Additional Example 1B: Determining Whether a Data Set Varies Directly

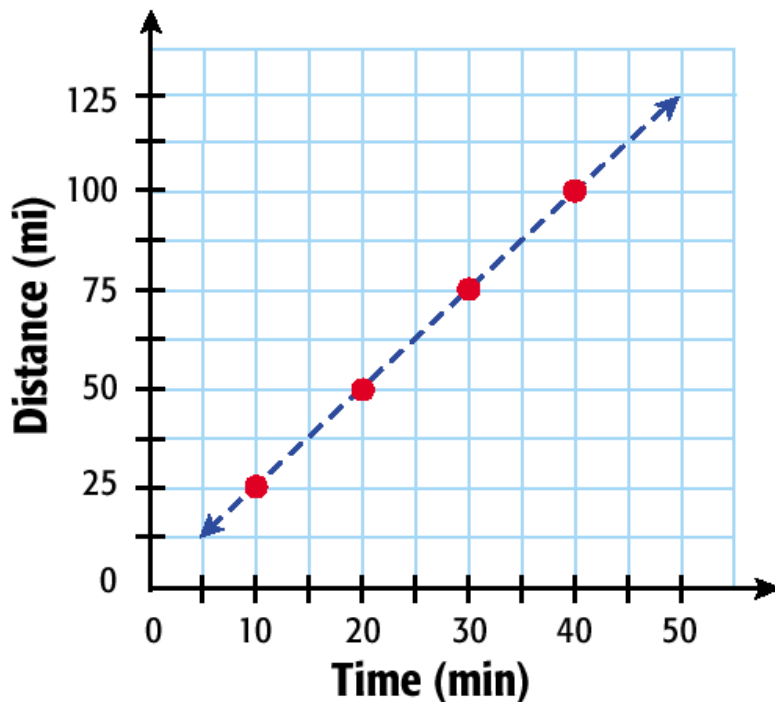
Determine whether the data set shows direct variation.

Distance Traveled by Train				
Time (min)	10	20	30	40
Distance (mi.)	25	50	75	100

Direct Variation

Additional Example 1B Continued

Make a graph that shows the relationship between the number of minutes and the distance the train travels.



Plot the points.

The points lie in a straight line.

$(0, 0)$ is included.

Direct Variation

Additional Example 1B Continued

You can also compare ratios to see if a direct variation occurs.

$$\frac{25}{10} = \frac{50}{20} = \frac{75}{30} = \frac{100}{40} \quad \textit{Compare ratios.}$$

The ratios are proportional. The relationship is a direct variation.

Direct Variation

Additional Example 2A: Finding Equations of Direct Variation

Find each equation of direct variation, given that y varies directly with x .

y is 54 when x is 6

$$y = kx$$

y varies directly with x .

$$54 = k \cdot 6$$

Substitute for x and y .

$$9 = k$$

Solve for k .

$$y = 9x$$

Substitute 9 for k in the original equation.

Direct Variation

Additional Example 2B: Finding Equations of Direct Variation

x is 12 when y is 15

$$y = kx$$

y varies directly with x .

$$15 = k \cdot 12$$

Substitute for x and y .

$$\frac{5}{4} = k$$

Solve for k .

$$y = \frac{5}{4}x$$

Substitute $\frac{5}{4}$ for k in the original equation.

Direct Variation

Check It Out: Example 2A

Find each equation of direct variation, given that y varies directly with x .

y is 24 when x is 4

$$y = kx \quad y \text{ varies directly with } x.$$

$$24 = k \cdot 4 \quad \text{Substitute for } x \text{ and } y.$$

$$6 = k \quad \text{Solve for } k.$$

$$y = 6x \quad \text{Substitute 6 for } k \text{ in the original equation.}$$

Direct Variation

Check It Out: Example 2B

x is 28 when y is 14

$$y = kx$$

y varies directly with x .

$$14 = k \cdot 28$$

Substitute for x and y .

$$\frac{1}{2} = k$$

Solve for k .

$$y = \frac{1}{2}x$$

Substitute $\frac{1}{2}$ for k in the original equation.



Essential question: How can you identify a direct variation?

A **direct variation** is a linear function that has an equation of the form $y = kx$, in which k is a fixed nonzero constant called the *constant of variation*.

1 EXAMPLE Identifying Direct Variations

Does each equation represent a direct variation between x and y ?

A $-3x + y = 0$

Write the equation in $y = kx$ form, if possible.

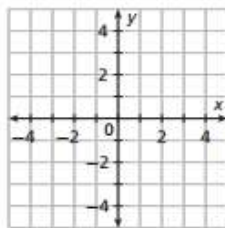
$$\begin{array}{r} -3x + y = 0 \\ + \quad + \\ \hline y = \end{array}$$

Make a table of values. Then find the ratios of y to x .

x	-1	1	2	3
y				

$$\frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \square$$

Graph the equation.



Can $-3x + y = 0$ be written as $y = kx$? _____

Do the ratios of y to x form a constant? _____

Is the graph a straight line passing through the origin? _____

Does $-3x + y = 0$ represent a direct variation between x and y ? _____

The constant of variation is _____.

B $-3x + y = 2$

Write the equation in $y = kx$ form, if possible.

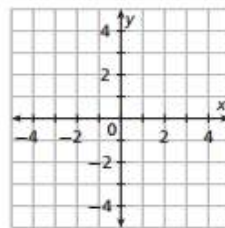
$$\begin{array}{r} -3x + y = 2 \\ + \quad + \\ \hline y = \end{array}$$

Make a table of values. Then find the ratios of y to x .

x	-1	1	2	3
y				

$$\frac{\square}{\square} \neq \frac{\square}{\square} \neq \frac{\square}{\square} \neq \frac{\square}{\square}$$

Graph the equation.



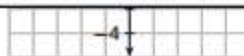
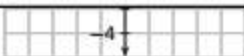
Can $-3x + y = 2$ be written as $y = kx$? _____

Do the ratios of y to x form a constant? _____

Is the graph a straight line passing through the origin? _____

Does $-3x + y = 2$ represent a direct variation between x and y ? _____

Pg. 225



- Can $-3x + y = 0$ be written as $y = kx$? _____ Can $-3x + y = 2$ be written as $y = kx$? _____
- Do the ratios of y to x form a constant? _____ Do the ratios of y to x form a constant? _____
- Is the graph a straight line passing through the origin? _____ Is the graph a straight line passing through the origin? _____
- Does $-3x + y = 0$ represent a direct variation between x and y ? _____ Does $-3x + y = 2$ represent a direct variation between x and y ? _____
- The constant of variation is _____.

REFLECT

- 1a.** Does the equation $2.5x - y = 0$ represent a direct variation between x and y ? If so, what is the constant of variation? What is the slope of the line?
- _____
- 1b.** Is the equation for finding the perimeter of a square P given the length of a side s an example of direct variation? Explain.
- _____
- _____
- _____

PRACTICE

Tell whether the function represented by each table is or is not a direct variation. Explain your reasoning.

1.

x	1	2	3	4
y	-3	-6	-9	-12

2.

x	1	2	3	4
y	4	5	6	7

12. Is the equation $s = 2t$ an example of direct variation? Explain.

PRACTICE

Tell whether the function represented by each table is or is not a direct variation. Explain your reasoning.

1.

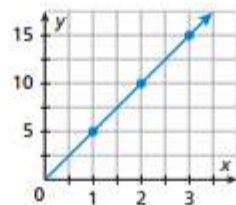
x	1	2	3	4
y	-3	-6	-9	-12

2.

x	1	2	3	4
y	4	5	6	7

Use the graph for 3–5.

3. Does the graph represent a direct variation between x and y ? _____
4. Write an equation for the graph. _____
5. What is the value of y when $x = 6$? _____



6. The circumference of a circle varies directly with the length of its diameter. How would you describe the relationship between circumference C and diameter d ? What equation can you write to show this relationship?

7. Explain how the constant of variation k affects the appearance of the graph of an equation that is a direct variation.

8. **Error Analysis** A classmate claims that the score you'll get on a test varies directly with the amount of time you spend studying. What do you think?
