

# Direct Variation

## Warm Up

**1.** Regina walked 9 miles in 3 hours. How many miles did she walk per hour?

**3 mi per hour**

**2.** To make 3 bowls of trail mix, Sandra needs 15 ounces of nuts. How many ounces of nuts does she need for 1 bowl of trail mix?

**5 oz**

## DAY 1

# IDENTIFYING

# Direct Variation

## What is it?

- Variation, in general, will concern two variables: say height and weight of a person
- When one of these changes, the other might be expected to change.

# Direct Variation

Let's focus on direct....

**MCC7.RP.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships

# Direct Variation

Fill in your Graphic Organizer with the following information.

## Direct variation

### What is it?

- When two variables change in the same sense; i.e. if one increases, so does the other.
  - Ex: As students increase, chaperones increase
  - Ex: As schools increase, buses increase

# Direct Variation

## Constant

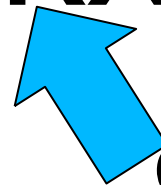
$K$  = it is constant for all ordered pairs

$$K = \frac{y}{x}$$

# Direct Variation

## Equation

$$y = kx$$



Constant

\*  $y$  and  $x$  are an ordered pair  $(x,y)$



# Direct Variation

## Table

$$y = \frac{1}{2}x$$

x	y
-4	-2
-2	-1
0	0
2	1
4	2

\* Plug each x-value in and solve for y.

# Direct Variation

Do not write, just watch

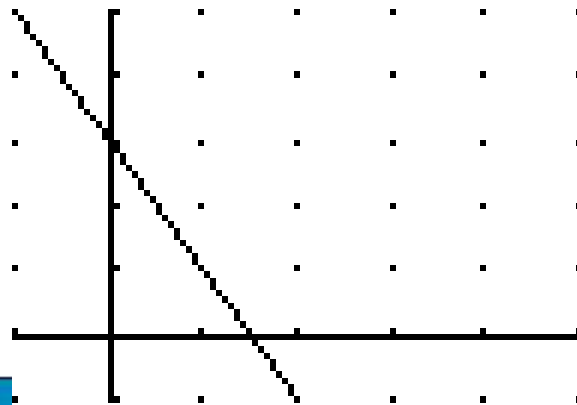
## Graph

- Always a line
- Must go through the origin

**Is**

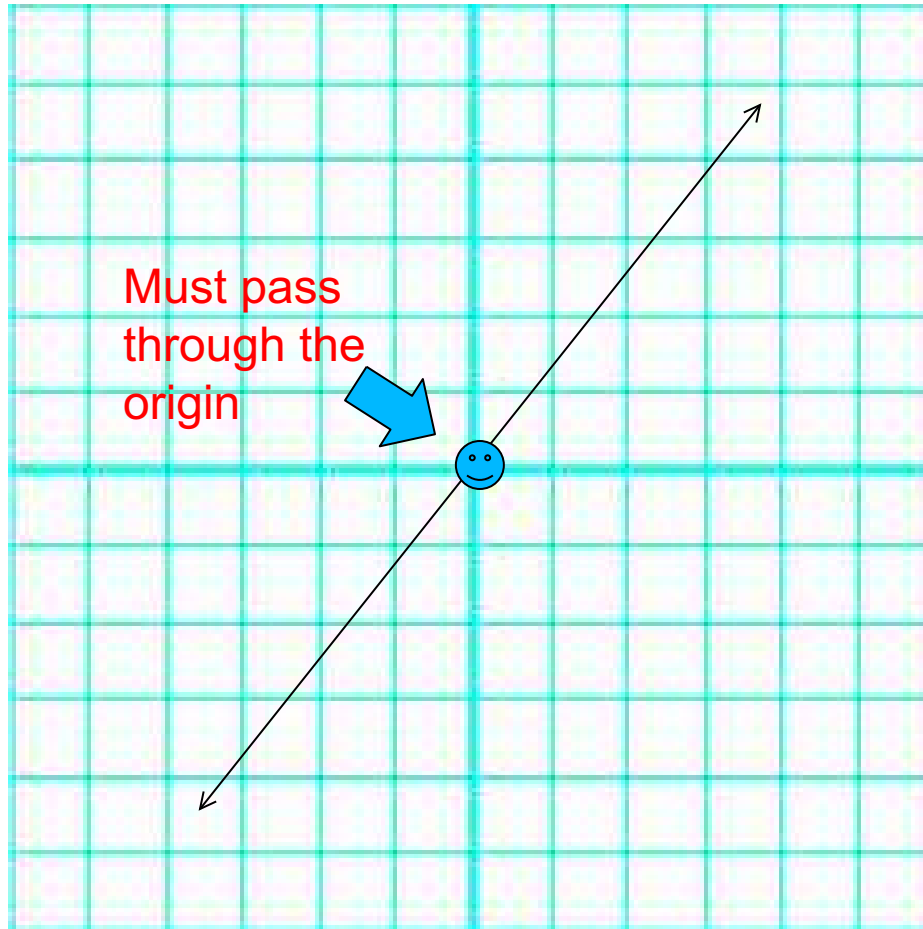


**Is Not**



# Direct Variation

## Graph



## 1

## EXPLORE

## Graphing Proportional Relationships

Most showerheads that were manufactured before 1994 use 5 gallons of water per minute. Is the relationship between the number of gallons of water and the number of minutes a proportional relationship?

Pg. 211

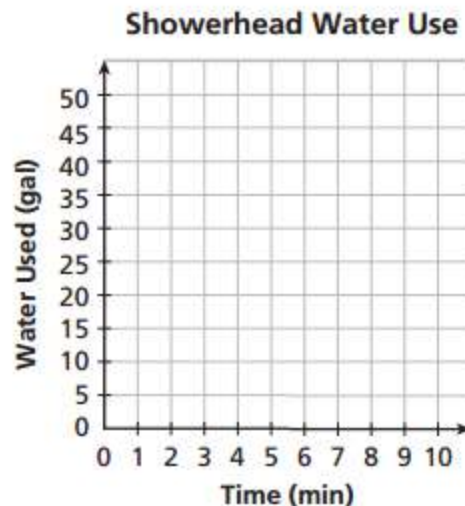
- A** Complete the table.

Time (min)	1	2	3		10
Water Used (gal)	5			35	

- B** Based on the table, is this a proportional relationship? Explain your answer.

- C** Plot the data from the table.

- D Draw Conclusions** If you continued the table to include 23 minutes, would the point (23, 125) be on this graph? Why or why not?



## REFLECT

Based on the table, is this a proportional relationship? Explain your answer.

**C** Plot the data from the table.

**D Draw Conclusions** If you continued the table to include 23 minutes, would the point (23, 125) be on this graph? Why or why not?

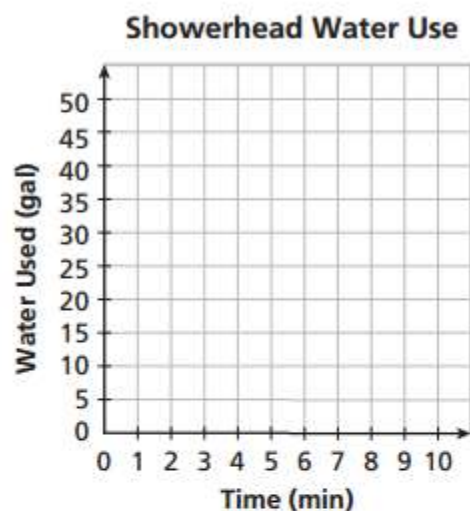
---



---



---



### REFLECT

**1a. What If...?** If a line was drawn through the plotted points, does it make sense that it would go through the origin? Explain.

---

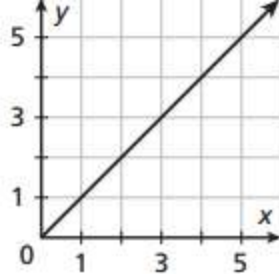


---

**1b.** Another showerhead uses less water per minute. How would its graph compare to the one you plotted?

---

In addition to using a table to determine if a relationship is proportional, you also can use a graph. A relationship is a proportional relationship if its graph is a straight line through the origin.



MCC7.RP.2a

2

**EXAMPLE**

**Identifying Proportional Relationships**

An Internet café charges a one-time \$5 service fee and then \$2 for every hour of use. Is this relationship a proportional relationship?

**A** Complete the table.

<b>Time (h)</b>	1	2	5		8
<b>Total Cost (\$)</b>	7			17	

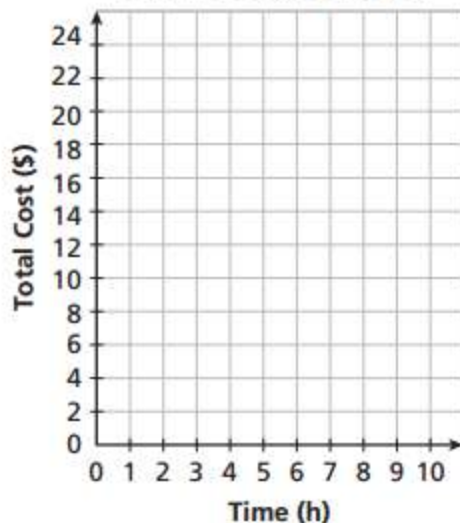
**B** Plot the data from the table and connect the points with a line.

**C** The graph of the data is a \_\_\_\_\_.

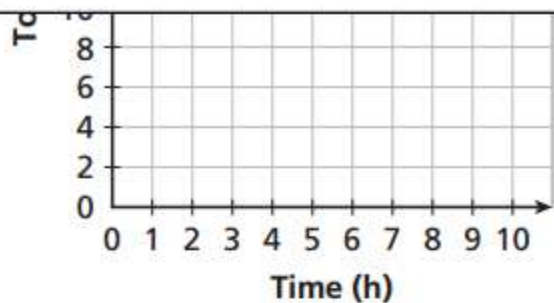
The line **does/does not** go through the origin.

So, the relationship is \_\_\_\_\_.

**Internet Café Charges**



So, the relationship is \_\_\_\_\_.



**TRY THIS!**

**2a.** Plot the data from the table and connect the points with a line.

<b>Canoe Rental (h)</b>	2	5	8	10
<b>Total Cost (\$)</b>	5	11	17	21

**2b.** Is this a proportional relationship? Explain.

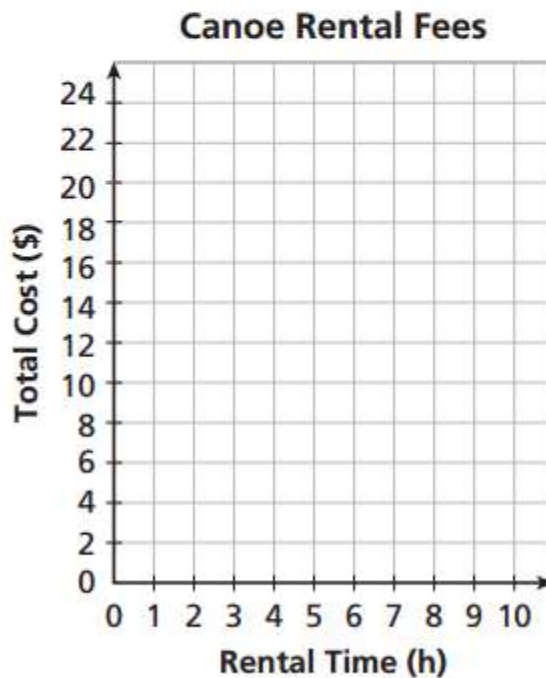
---



---



---



# Direct Variation

Where do graphs come  
from...

Equations!!

!



# Equation : $y = kx$

where  $k$  is called the **constant of proportionality** because the ratios of  $y$  and  $x$  are proportional.

- *Cannot not have anything else*
- *Must be positive*

- *Ex*  $y = 3x$

## Finding k

### Equation

$$y = kx$$

Divide both sides by 'x'

### Table

$$y \div x = k$$

**EXAMPLE** Writing an Equation for a Proportional Relationship

Two pounds of cashews cost \$5, 3 pounds of cashews cost \$7.50, and 8 pounds of cashews cost \$20. Show that the relationship between the number of pounds of cashews and the cost is a proportional relationship. Then write an equation for the relationship.

Make a table to find the common ratio. Then write an equation with the common ratio as the constant of proportionality.

**A** Complete the table.

<b>Number of Pounds</b>	2	3	8
<b>Cost (\$)</b>	5		

**B** Complete the ratios.  $\frac{\text{Cost}}{\text{Number of Pounds}} = \frac{5}{2} = \frac{\square}{\square} = \frac{\square}{\square} = \square$

The common ratio is \_\_\_\_\_.

**C** To write an equation, first tell what the variables represent.

Let  $x$  represent the number of pounds of cashews.

Let  $y$  represent the cost in dollars.

Use the common ratio as the constant of proportionality.

So, the equation for the relationship is \_\_\_\_\_.

**REFLECT**

**3a.** How can you use substitution to check your equation?

\_\_\_\_\_

**3b.** What is the unit cost (unit rate) for the cashews? How does the unit cost appear in your equation?

\_\_\_\_\_

**3c.** How can you use your equation to find the cost of 6 pounds of cashews?

# Direct Variation

## Additional Example 1A: Identifying a Direct Variation from an Equation

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

$$y + 8 = x$$

$$y + 8 = x$$

$$\underline{-8} = \underline{-8}$$

$$y = x - 8$$

*Solve the equation for  $y$ .  
Subtract 8 from both sides.*

The equation is not in the form  $y = kx$ , so  $y + 8 = x$  is not a direct variation.

# Direct Variation

## Additional Example 1B: Identifying a Direct Variation from an Equation

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

$$3y = 2x$$

$$\frac{3y}{3} = \frac{2x}{3}$$

$$y = \frac{2}{3}x$$

*Solve the equation for  $y$ .  
Divide both sides by 3.*

*Write  $\frac{2x}{3}$  as  $\frac{2}{3}x$ .*

The equation is in the form  $y = kx$ , so the original equation  $3y = 2x$  is a direct variation.

# Direct Variation

## Check It Out: Example 1A

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

$$y + 3 = 3x$$

$$\begin{array}{r} y + 3 = 3x \\ \underline{-3} \quad \underline{-3} \end{array}$$

*Solve the equation for  $y$ .  
Subtract 3 from both sides.*

$$y = 3x - 3$$

The equation is not in the form  $y = kx$ , so  $y + 3 = 3x$  is not a direct variation.

# Direct Variation

## Check It Out: Example 1B

**Tell whether each equation represents a direct variation. If so, identify the constant of variation.**

$$4y = 3x$$

$$\frac{4y}{4} = \frac{3x}{4}$$

$$y = \frac{3}{4}x$$

*Solve the equation for  $y$ .  
Divide both sides by 4.*

*Write  $\frac{3x}{4}$  as  $\frac{3}{4}x$ .*

The equation is in the form  $y = kx$ , so the original equation  $4y = 3x$  is a direct variation.

# Direct Variation

## Practice

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

1.  $Y = 7x$

3.  $y = 13x + 0$

2.  $3y = 2x + 5$

4.  $4y = 2x$

[Direct Variation Video](#)



## DAY 2

# FINDING K

## Warm Up

1. Describe the patterns you notice in the table.
2. Graph the table (Use graph paper if you have it)  
Graph on a coordinate plane.
3. What do you notice about the graph?

X value	Y value
0	0
1	2
3	4
5	6
7	8

# Direct Variation

## Finding $k$

Alisa and her parents are going on a vacation. The table shows the number of hours they drive and the miles they travel.

Hours ( $x$ )	2	2.5	3	4
Miles ( $y$ )	120	150	180	240

# Direct Variation

## Did you get it?

Can you find  $k$ ?  $Y = kx$  Solve for  $K$ . Divide both sides by  $x$ .

$$y/x = k$$

Hours ( $x$ )	2	2.5	3	4
Miles ( $y$ )	120	150	180	240

$$120 / 2 = 60$$

$$150 / 2.5 = 60$$

$$180 / 3 = 60$$

$$240 / 4 = 60$$

$$k = 60!!!$$

# Direct Variation

$$k = 60!!!$$

Can you rewrite an equation  $y = kx$

Hours (x)	2	2.5	3	4
Miles (y)	120	150	180	240

$$y = 60x$$

# Direct Variation

Try it!

Use this table

<b>Hours (x)</b>	<b>0</b>	<b>20</b>	<b>40</b>	<b>60</b>
<b>Wage (y)</b>	<b>0</b>	<b>40</b>	<b>80</b>	<b>120</b>

$$K = y/x$$

$$K = 40/20 = 2$$

$$K = 80/40 = 2$$

$$K = 120/60 = 2$$

Rewrite as an equation:  $y = kx$

$$Y = 2x$$

# Direct Variation

## Additional Example 2A: Identifying a Direct Variation from a Table

Tell whether each set of data represents a direct variation. If so, identify the constant of variation and then write the direct variation equation.

Price (c)	69	99	129
Weight (oz)	2	3	4

Find  $\frac{y}{x}$  for each ordered pair.

$$\frac{y}{x} = \frac{2}{69} \qquad \frac{y}{x} = \frac{3}{99} = \frac{1}{33} \qquad \frac{y}{x} = \frac{4}{129}$$

$k$  is not the same for each ordered pair.

The data does not represent a direct variation.

# Direct Variation

## Check It Out: Example 2A

Tell whether each set of data represents a direct variation. If so, identify the constant of variation and then write the direct variation equation.

Price (c)	5	10	15
Weight (lb)	2	3	4

Find  $\frac{y}{x}$  for each ordered pair.

$$\frac{y}{x} = \frac{2}{5}$$

$$\frac{y}{x} = \frac{3}{10}$$

$$\frac{y}{x} = \frac{4}{15}$$

$k$  is not the same for each ordered pair.

The data does not represent a direct variation.



# Direct Variation

## Check It Out: Example 2B

Tell whether each set of data represents a direct variation. If so, identify the constant of variation and then write the direct variation equation.

Meters	3	4	5
Miles	9	12	15

Find  $\frac{y}{x}$  for each ordered pair.

$$\frac{y}{x} = \frac{9}{3} = 3$$

$$\frac{y}{x} = \frac{12}{4} = 3$$

$$\frac{y}{x} = \frac{15}{5} = 3$$

$k = 3$  for each ordered pair.

The data represent a direct variation where  $k = 3$ .

The equation is  $y = 3x$

# Direct Variation

## Example

- Given that  $y$  and  $x$  are directly proportional, and  $y = 2$  when  $x = 5$ . Solve for  $k$ .

Formula:  $y = kx$

We first find value of  $k$ , using inverse operations.

$$y/x = k$$

- Substitute in the given values.  $y = 2$   $x = 5$

$$y/x = k \qquad 2/5 = k$$

# Direct Variation

Suppose  $y$  varies directly as  $x$ , and  $y = 16$  when  $x = 8$ . Find  $y$  when  $x = 16$ .

Step 1: Solve for  $k$ .

$y = kx$  Substitute the known values

$16 = k(8)$  Solve the one-step equation

$$2 = k$$

Step 2: Substitute the known values to solve for  $y$  when  $x = 16$

$$y = kx$$

$$y = 2(16)$$

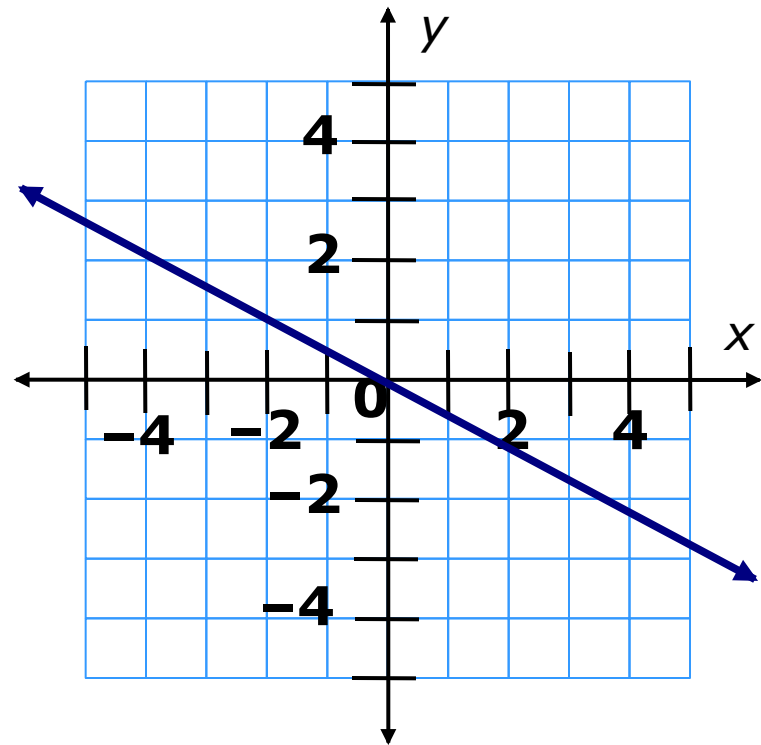
$$y = 32$$

# Direct Variation

## Additional Example 3: Identifying a Direct Variation from a Graph

**Tell whether each graph represents a direct variation. If so, identify the constant of variation and then write the direct variation equation.**

The graph is a line through  $(0, 0)$ . This is a direct variation. The Slope of the line is  $\frac{1}{2}$ , so  $k = \frac{1}{2}$ . The equation is  $y = \frac{1}{2}x$ .



# Direct Variation

## Helpful Hint

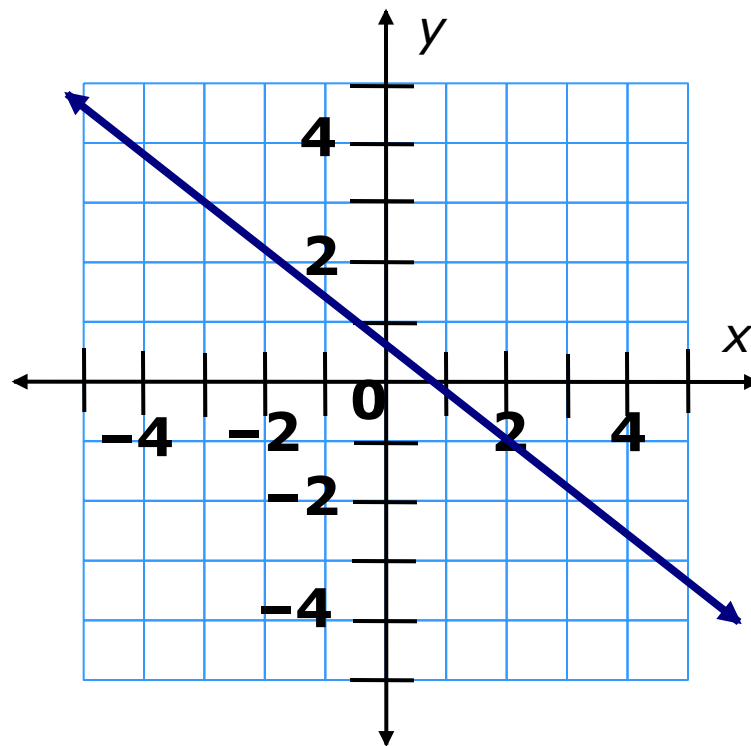
In a direct variation, the slope,  $k$ , represents a constant rate of change.

# Direct Variation

## Check It Out: Example 3

**Tell whether each graph represents a direct variation. If so, identify the constant of variation and then write the direct variation equation.**

The line does not pass through  $(0, 0)$ . This is not a direct variation.



$y = ax$ , where  $a$  is a positive number. The constant of proportionality,  $a$ , tells you how steep the graph of the relationship is. The greater the value of  $a$ , the steeper the line.

MCC7.RP.2d

### 4 EXAMPLE Analyzing Proportional Graphs

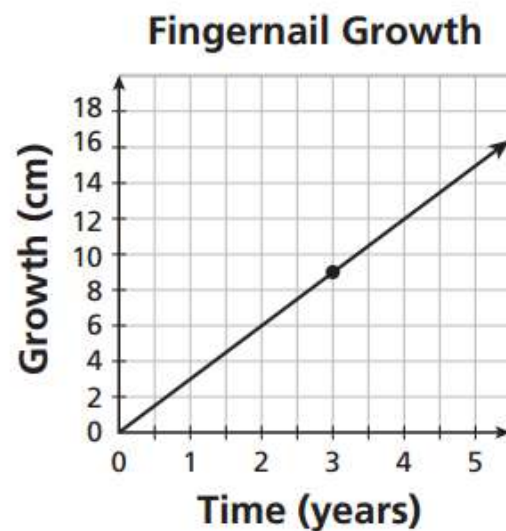
The graph shows the relationship between time in years and the number of centimeters a fingernail grows.

- A** What does the point (3, 9) represent?  
 \_\_\_\_\_  
 \_\_\_\_\_
- B** What is the constant of proportionality? \_\_\_\_\_
- C** Write an equation for the relationship. \_\_\_\_\_

#### REFLECT

- 4a.** What does the point (0, 0) on the graph represent?  
 \_\_\_\_\_
- 4b.** What is the rate at which a fingernail grows? How does this relate to the constant of proportionality?

Pg. 214



# Direct Variation

## Additional Example 4A: *Application*

A truck travels at a speed of 55 miles per hour.

Write a direct variation equation for the distance  $y$  the truck travels in  $x$  hours.

distance = 55 miles per hour times number of hours

*Use the formula  $y = kx$ .  $k = 55$*

$$y = 55 \cdot x$$

$$y = 55x$$



# Direct Variation

## Additional Example 4B: *Application*

**A truck travels at a speed of 55 miles per hour.**

**Graph the data.**

Make a table. Since time cannot be negative, use nonnegative number for  $t$ .



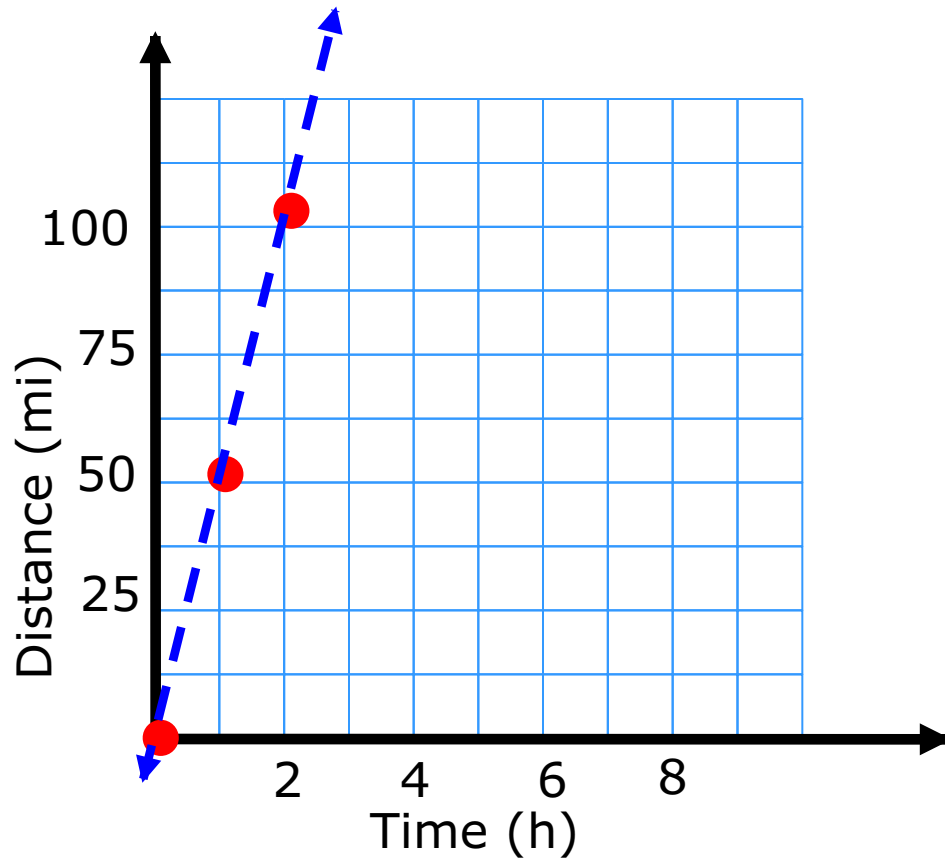
# Direct Variation

## Additional Example 4 Continued

Use the ordered pairs to plot the points on a coordinate plane. Connect the points in a straight line. Label the axes.

### Check

$y = 55x$  is in slope-intercept form with  $m = 55$  and  $b = 0$ . The graph shows a slope of 55 and a  $y$ -intercept of 0.



# Direct Variation

## Additional Example 4 Continued

**C.** How long does it take the truck to travel 660 miles?

Find the value of  $x$  when  $y = 660$

$$y = 55x$$

*Write the equation for the direct variation.*

$$\frac{660}{55} = \frac{55x}{55}$$

*Substitute 660 for  $y$ .*

*Divide both sides by 660.*

$$12 = x$$

It will take the truck 12 hours to travel 660 miles.

# Direct Variation

## Check It Out: Example 4A

A bicycle travels at a speed of 12 miles per hour.

Write a direct variation equation for the distance  $y$  the bike travels in  $x$  hours.

distance = 12 miles per hour times number of hours

*Use the formula  $y = kx$ .  $k = 12$*

$$y = 12 \cdot x$$

$$y = 12x$$

# Direct Variation

## Check It Out: Example 4B

**A bicycle travels at a speed of 12 miles per hour.**

**Graph the data.**

Make a table. Since time cannot be negative, use nonnegative number for  $x$ .



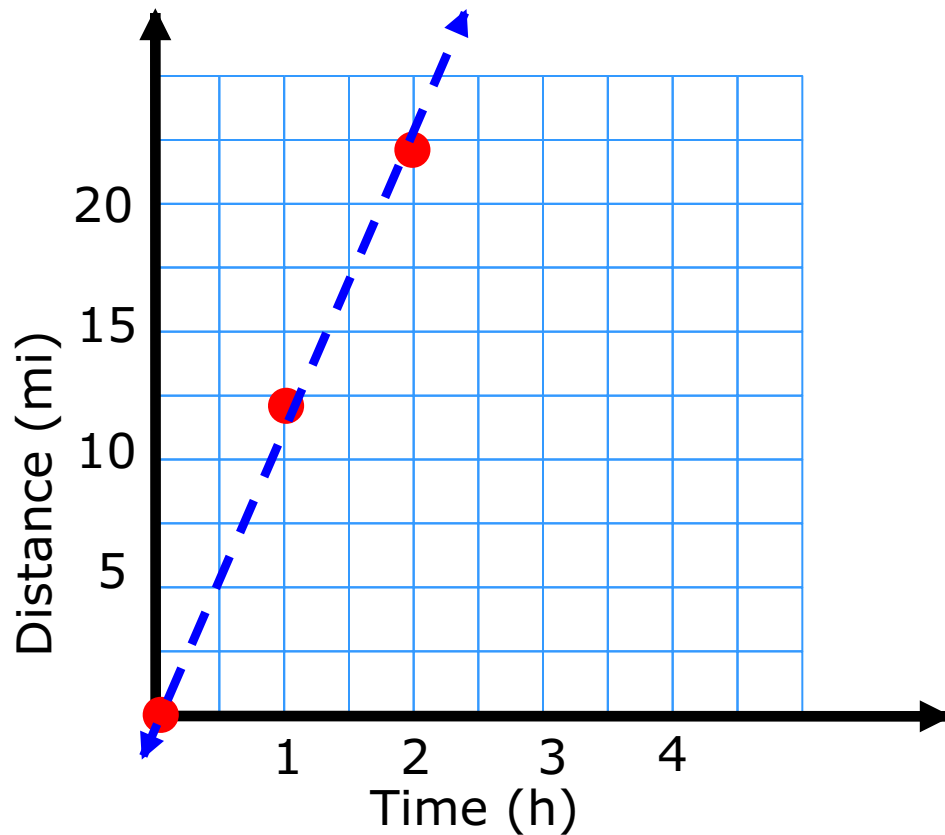
# Direct Variation

## Check It Out: Example 4 Continued

Use the ordered pairs to plot the points on a coordinate plane. Connect the points in a straight line. Label the axes.

### Check

$y = 12x$  is in slope-intercept form with  $m = 12$  and  $b = 0$ . The graph shows a slope of 12 and a  $y$ -intercept of 0.



# Direct Variation

## Check It Out: Example 4 Continued

**C.** How long does it take the bicycle to travel 96 miles?

Find the value of  $x$  when  $y = 96$

$$y = 12x$$

*Write the equation for the direct variation.*

$$\frac{96}{12} = \frac{12x}{12}$$

*Substitute 96 for  $x$ .*

*Divide both sides by 12.*

$$8 = x$$

It will take the bicycle 8 hours to travel 96 miles.

# Direct Variation

1. Fold paper in half, hotdog style. Label one side "Direct Variation" and the other "Not Direct Variation"
2. Separate by cutting the equations, tables, and word problems.
3. Place these in the appropriate column. Do NOT glue them yet!!!!!!

## Direct Variation

$$-3x + 2y = 0$$

$$y = 5x$$

$$3x - y = 0$$

You go to the movies and buy tickets for \$10.50 each.

x	y
4	8
7	14
10	20

x	y
-3	-2
3	2
9	6

x	y
-2	-2.8
3	4.2
8	-11.2

## Not Direct Variation

$$x - 3y = 6$$

$$8x + 4y = 12$$

$$y = \frac{3}{4}x - 7$$

$$y = 2x + 5$$

You go to the fair and pay \$5 admission and then \$0.50 per ride.



Homework:  
Workbook Pg. 217