8-3 Trigonometric Ratios

Warm Up

Lesson Presentation

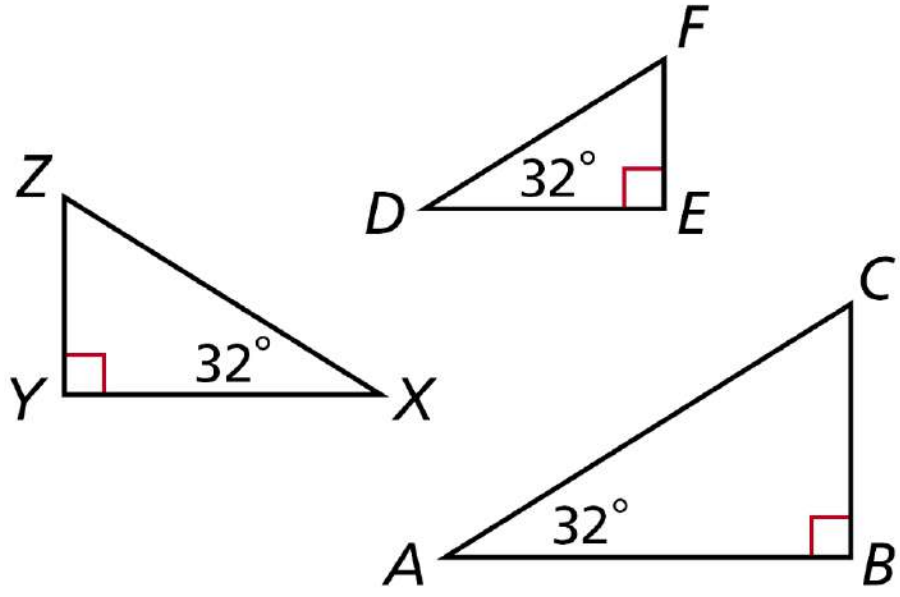
Lesson Quiz

Objectives

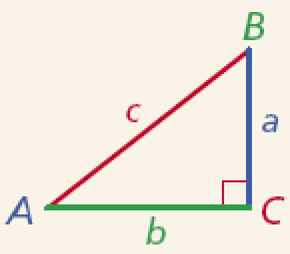
Find the sine, cosine, and tangent of an acute angle.

Use trigonometric ratios to find side lengths and angle measures in right triangles and to solve real-world problems.

By the AA Similarity Postulate, a right triangle with a given acute angle is similar to every other right triangle with that same acute angle measure. So $\Delta ABC \sim \Delta DEF \sim \Delta XYZ$, and $\frac{BC}{AC} = \frac{EF}{DF} = \frac{YZ}{XZ}$. These are *trigonometric ratios*. A **trigonometric ratio** is a ratio of two sides of a right triangle.



Trigonometric Ratios

DEFINITION	SYMBOLS	DIAGRAM
The **sine** of an angle is the ratio of the length of the leg opposite the angle to the length of the hypotenuse.	$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{a}{c}$ $\sin B = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{b}{c}$	
The **cosine** of an angle is the ratio of the length of the leg adjacent to the angle to the length of the hypotenuse.	$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{b}{c}$ $\cos B = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{a}{c}$	
The **tangent** of an angle is the ratio of the length of the leg opposite the angle to the length of the leg adjacent to the angle.	$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{a}{b}$ $\tan B = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{b}{a}$	

The trig functions can be summarized using the following mnemonic device:

SOHCAHTOA

$$\text{Sin} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tan} = \frac{\text{opposite}}{\text{adjacent}}$$

Calculator Tip

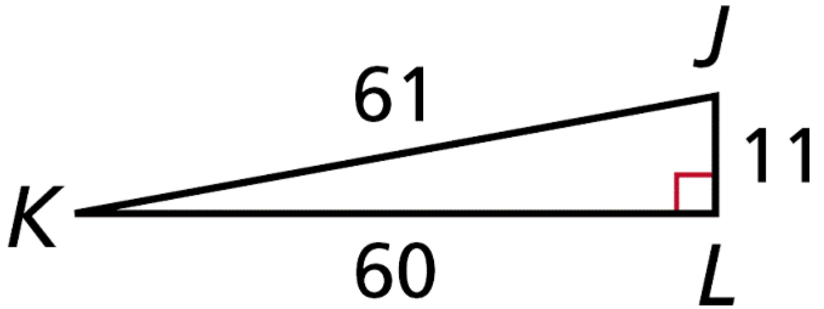
On a calculator, the trig functions are abbreviated as follows: sine → **sin**, cosine → **cos**, tangent → **tan**

Writing Math

In trigonometry, the letter of the vertex of the angle is often used to represent the measure of that angle. For example, the sine of $\angle A$ is written as sin A.

Example 1A: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.

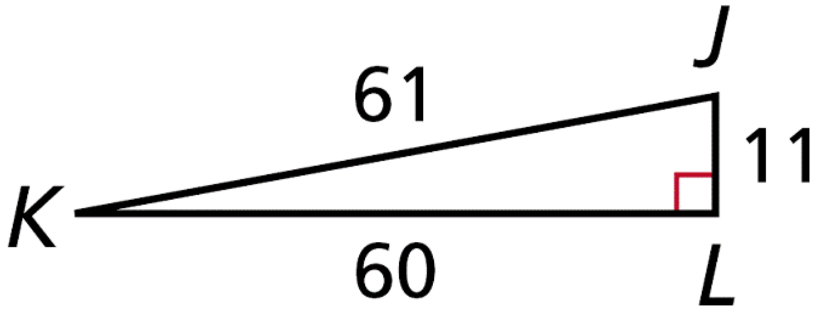


sin *J*

$\sin J = \frac{60}{61} \approx 0.98$ *The sine of an* $\angle$ *is* $\frac{\text{opp. leg}}{\text{hyp.}}$.

Example 1B: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



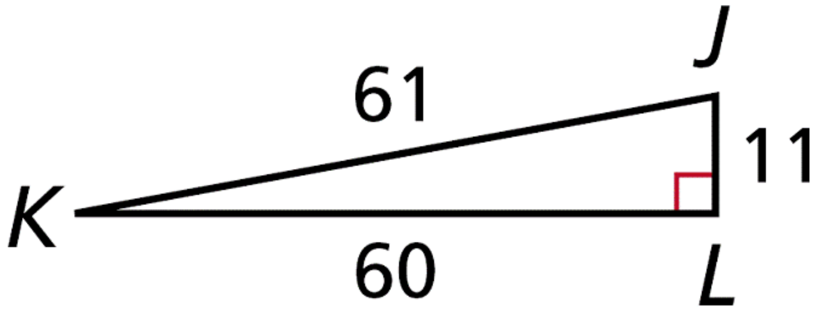
cos *J*

$\cos J = \frac{11}{61} \approx 0.18$

The cosine of an $\angle$ *is* $\frac{\text{adj. leg}}{\text{hyp.}}$.

Example 1C: Finding Trigonometric Ratios

Write the trigonometric ratio as a fraction and as a decimal rounded to the nearest hundredth.



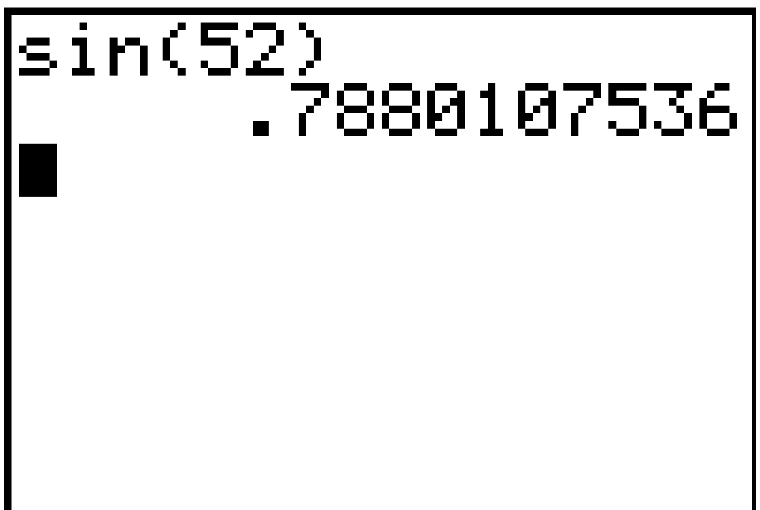
tan *K*

$\tan K = \frac{11}{60} \approx 0.18$ *The tangent of an $\angle$ is $\frac{\text{opp. leg}}{\text{adj. leg}}$.*

Example 3A: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

sin 52°



$\sin 52° \approx 0.79$

Caution!

Be sure your calculator is in degree mode, not radian mode.

Example 3B: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

cos 19°

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cos(19)
          .9455185756
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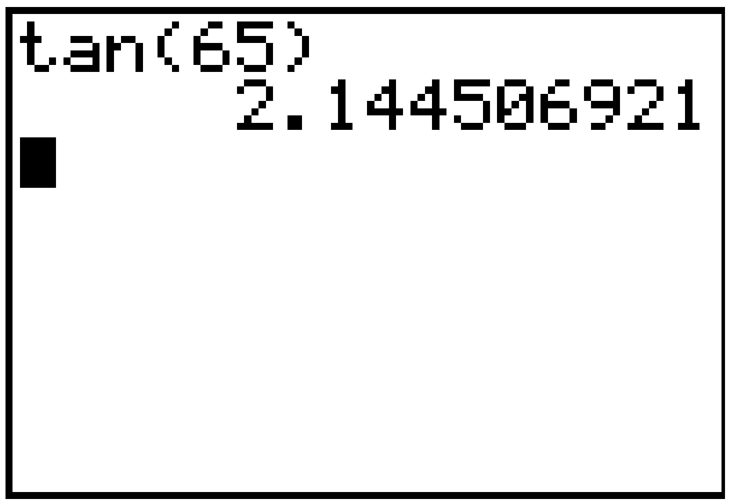
cos 19° ≈ 0.95

Example 3C: Calculating Trigonometric Ratios

Use your calculator to find the trigonometric ratio. Round to the nearest hundredth.

tan 65°

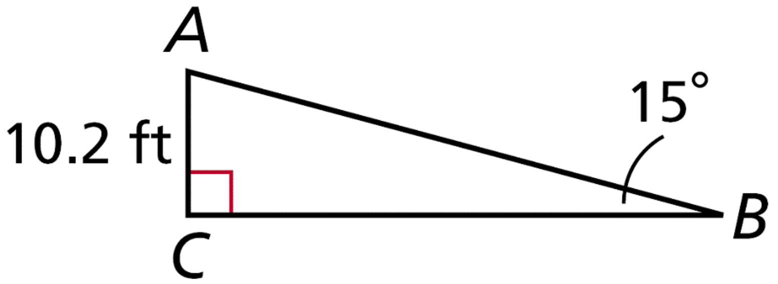


tan 65° ≈ 2.14

The hypotenuse is always the longest side of a right triangle. So the denominator of a sine or cosine ratio is always greater than the numerator. Therefore the sine and cosine of an acute angle are always positive numbers less than 1. Since the tangent of an acute angle is the ratio of the lengths of the legs, it can have any value greater than 0.

Example 4A: Using Trigonometric Ratios to Find Lengths

Find the length. Round to the nearest hundredth.

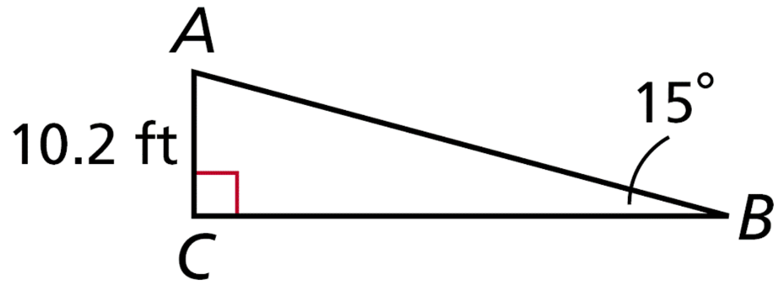


BC

$\overline{BC}$ is adjacent to the given angle, ∠*B*. You are given AC, which is opposite ∠*B*. Since the opposite and adjacent legs are involved, use a tangent ratio.

O and A → tangent

Example 4A Continued



$$\tan B = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{AC}{BC}$$ *Write a trigonometric ratio.*

$$\tan 15° = \frac{10.2}{BC}$$ *Substitute the given values.*

$$BC = \frac{10.2}{\tan 15°}$$ *Multiply both sides by BC and divide by tan 15°.*

$$BC \approx 38.07 \text{ ft}$$ *Simplify the expression.*

When problem solving, you may be asked to find a missing side of a right triangle. You also may be asked to find a missing angle.

If you look at your calculator, you should be able to find the inverse trig functions. These can be used to find the measure of an angle that has a specific sine, cosine, or tangent.

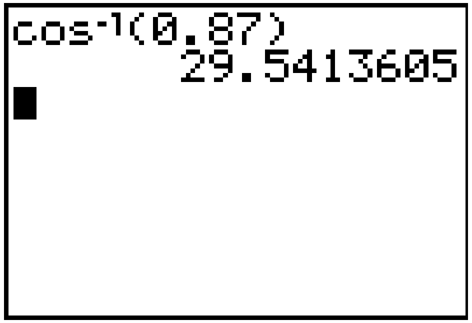
If you know the sine, cosine, or tangent of an acute angle measure, you can use the inverse trigonometric functions to find the measure of the angle.

Inverse Trigonometric Functions
If $\sin A = x$, then $\sin^{-1} x = m\angle A$.
If $\cos A = x$, then $\cos^{-1} x = m\angle A$.
If $\tan A = x$, then $\tan^{-1} x = m\angle A$.

Example 2: Calculating Angle Measures from Trigonometric Ratios

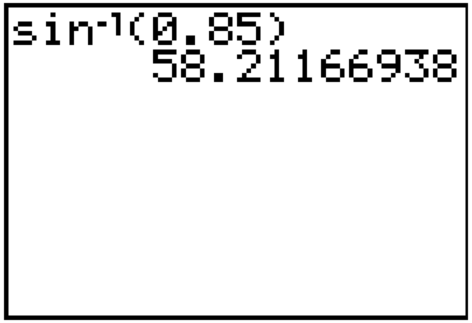
Use your calculator to find each angle measure to the nearest degree.

A. $\cos^{-1}(0.87)$



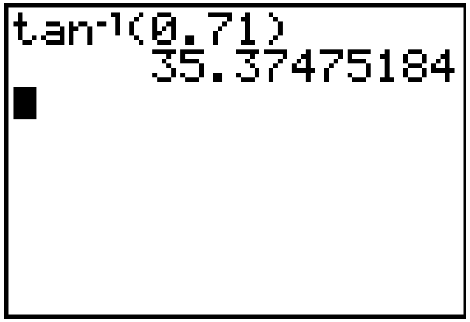
$\cos^{-1}(0.87) \approx 30°$

B. $\sin^{-1}(0.85)$



$\sin^{-1}(0.85) \approx 58°$

C. $\tan^{-1}(0.71)$



$\tan^{-1}(0.71) \approx 35°$

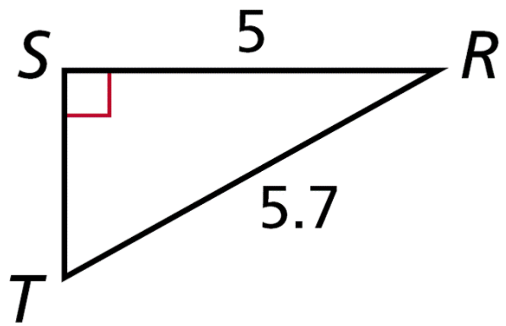
Using given measures to find the unknown angle measures or side lengths of a triangle is known as *solving a triangle*. To solve a right triangle, you need to know two side lengths or one side length and an acute angle measure.

Caution!

Do not round until the final step of your answer. Use the values of the trigonometric ratios provided by your calculator.

Example 3: Solving Right Triangles

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Method: By the Pythagorean Theorem,

$$RT^2 = RS^2 + ST^2$$

$$(5.7)^2 = 5^2 + ST^2$$

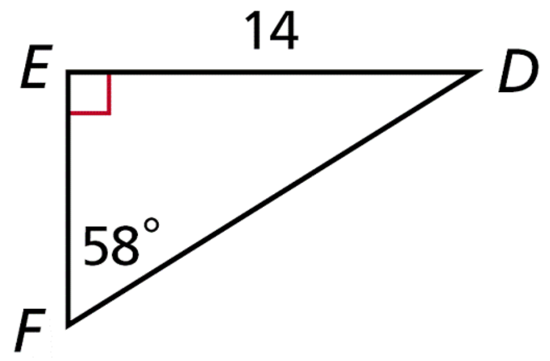
So $ST = \sqrt{7.49} \approx 2.74$.

$$m\angle R = \cos^{-1}\left(\frac{5}{5.7}\right) \approx 29°$$

Since the acute angles of a right triangle are complementary, $m\angle T \approx 90° - 29° \approx 61°$.

Check It Out! Example 3

Find the unknown measures. Round lengths to the nearest hundredth and angle measures to the nearest degree.



Since the acute angles of a right triangle are complementary, $m\angle D = 90° - 58° = 32°$.

$\tan 32° = \dfrac{EF}{14}$, so $EF = 14 \tan 32°$. $EF \approx 8.75$

$DF^2 = ED^2 + EF^2$

$DF^2 = 14^2 + 8.75^2$

$DF \approx 16.51$