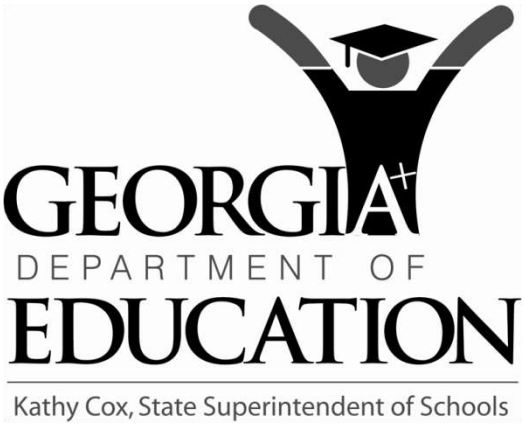
MATHEMATICS

Grade 7 Mathematics Frameworks

Unit 3
Rational Reasoning

Student Edition



Unit 3
RATIONAL REASONING

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Grade 7 Mathematics
Rational Reasoning

OVERVIEW:

In this unit, students will explore positive and negative numbers and extend many previously learned number and operations concepts to include the use of all rational numbers (natural numbers, whole numbers, integers and other rational numbers.) Students will also solve problems by defining variables, writing and solving one-step and two-step equations, and interpreting the solution of the equations in the context of the original problem. In Grade 6 Mathematics, students studied one-step equations with positive rational numbers.

Students will begin the unit with the task *What Is Your Sign.* This task allows students to explore and develop a deeper understanding of positive and negative numbers. The discussions should focus on comparing and ordering rational numbers, plotting rational numbers on a number line and on a coordinate plane, exploring the relationship between a number and its additive inverse through the concept of absolute value, and developing algorithms for computing with positive and negative rational numbers. Writing and solving one-step and two-step equations and simplifying and evaluating algebraic expressions involving positive and negative rational numbers is an additional focus of this unit. *Working with Integers* will help students strengthen their understanding of algebraic expressions which is used extensively in Grade 8 Mathematics and throughout high school mathematics.

It is important for students to investigate, discover and formalize the aforementioned concepts by using models, diagrams, manipulatives, and patterns. The properties of real numbers (commutative, associative, distributive, inverse and identity) and the order of operations are vital to student's understanding of simplifying and evaluating simple numeric and algebraic expressions. Students will continually study more complex equations and algebraic expressions throughout middle school and high school mathematics.

ENDURING UNDERSTANDINGS:

- Negative numbers are used to represent quantities that are less than zero such as temperatures, scores in games or sports, and loss of income in business.
- Absolute value is useful in ordering and graphing positive and negative numbers.
- Computation with positive and negative numbers is often necessary to determine relationships between quantities.
- Models, diagrams, manipulatives and patterns are useful in developing and remembering algorithms for computing with positive and negative numbers.
- Properties of real numbers hold for all rational numbers.
- Positive and negative numbers are often used to solve problems in everyday life.

ESSENTIAL QUESTIONS:

- When are negative numbers used and why are they important?
- Why is it useful for me to know the absolute value of a number?
- What strategies are most useful in helping me develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers?
- What properties and conventions do I need to understand in order to simplify and evaluate algebraic expressions?
- Why is graphing on a coordinate plane helpful?

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS:

M7N1. Students will understand the meaning of positive and negative rational numbers and use them in computation.

a. Find the absolute value of a number and understand it as a distance from zero on a number line.
b. Compare and order rational numbers, including repeating decimals.
c. Add, subtract, multiply, and divide positive and negative rational numbers.
d. Solve problems using rational numbers.

M7A1. Students will represent and evaluate quantities using algebraic expressions.

a. Translate verbal phrases to algebraic expressions.
b. Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate.
c. Add and subtract linear expressions.

M7A2. Students will understand and apply linear equations in one variable.

a. Given a problem, define a variable, write an equation, solve the equation, and interpret the solution.
b. Use the addition and multiplication properties of equality to solve one- and two-step linear equations.

M7A3. Students will understand relationships between two variables.

a. Plot points on a coordinate plane.
b. Represent, describe, and analyze relations from tables, graphs, and formulas.
c. Describe how change in one variable affects the other variable.

RELATED STANDARDS:

M7P1. Students will solve problems (using appropriate technology)
a. Build new mathematical knowledge through problem solving,
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

M7P2. Students will reason and evaluate mathematical arguments.
a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

M7P3. Students will communicate mathematically
a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

M7P4. Students will make connections among mathematical ideas and to other disciplines
a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

M7P5. Students will represent mathematics in multiple ways
a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

SELECTED TERMS AND SYMBOLS:

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

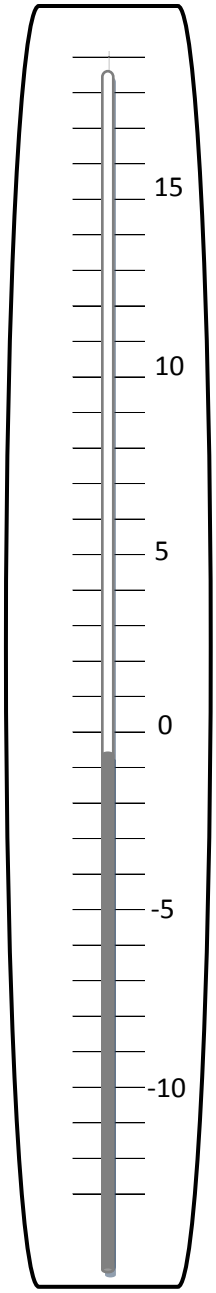
- **Absolute value:** The distance between a number and zero on the number line. The symbol for absolute value is shown in this equation $|-8| = 8$.
- **Associative property:** In addition or multiplication, the result of the expression will remain the same regardless of grouping. Examples: $a + (b + c) = (a + b) + c$; $a(bc) = (ab)c$
- **Commutative property:** The sum or product of numbers is the same no matter how the numbers are arranged.
  Examples: $a + b = b + a$; $ab = ba$
- **Distributive property:** The sum of two addends multiplied by a number will be the sum of the product of each addend and the number. Example: $a(b + c) = ab + ac$
- **Integers:** The set of whole numbers and their opposites {… -3, -2, -1, 0, 1, 2, 3, …}
- **Natural numbers:** The set of numbers {1, 2, 3, 4, …}. Natural numbers can also be called counting numbers.
- **Negative Numbers:** The set of numbers less than zero.
- **Opposite Numbers:** Two different numbers that have the same absolute value. Example: 4 and –4 are opposite numbers because both have an absolute value of 4.
- **Positive Numbers:** The set of numbers greater than zero.
- **Rational Numbers:** The set of numbers that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
- **Sign:** a symbol that indicates whether a number is positive or negative. Example: in –4, the (–) sign shows this number is read "negative four".
- **Whole numbers:** The set of all natural numbers and the number zero.

You may visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see definitions and specific examples of many terms and symbols used in the seventh-grade GPS.

TASKS:

The collection of the following tasks represents the level of depth, rigor, and complexity expected of all eighth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning.

What Is Your Sign?



1. Which temperature is colder, -10 (ten below zero) or 0 (zero)?

   Plot both numbers on the number line.

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2. Which temperature is colder, -5 (five below zero) or 0 (zero)?

   Plot both numbers on the number line.

---

3. Which temperature is warmer, -10 (ten below zero) or -5 (five below zero)?

   Plot both numbers on the number line.

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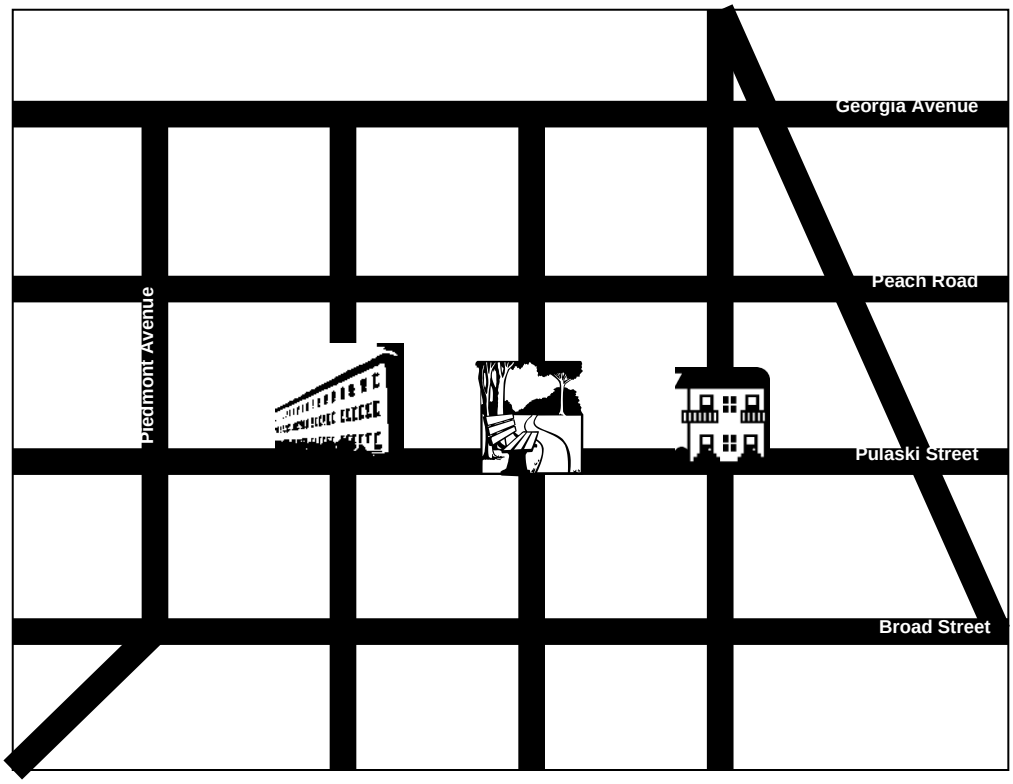
4. Which temperature is warmer, -10 (ten below zero) or 15 (fifteen degrees)?

   Plot both numbers on the number line.

5. What do you notice about negative numbers?

Part II.



1. If the park is located at zero on the number line, plot the location of the house and school if they are located one unit from the park. What do you notice about the placement of your plots on the number line?

2. Plot the location of the house and school if they are two units from the park. What do you notice about the placement of your plots on the number line?

3. Plot the location of the house and school if they are five units from the park. What do you notice about the placement of your plots on the number line?

4. Plot the location of the house and school if they are nine units from the park. What do you notice about the placement of your plots on the number line?

The distance between a number and zero on the number line is called **absolute value**. The symbol for absolute value is shown in this equation |8| = 8 and |-8| = 8.

5. Explain |4|.

6. Explain |-7|.

7. Explain |8|.

8. Explain |-21|.

9. Explain |*d*|.

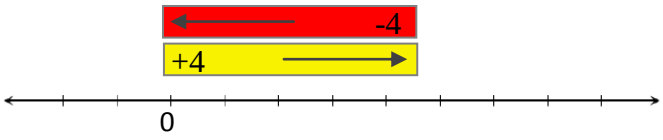
Opposite Numbers: Two different numbers that have the same absolute value. Example: 4 and –4 are opposite numbers because both have an absolute value of 4.

Part III.

Number lines and counters are useful in demonstrating understanding of operations with integers.

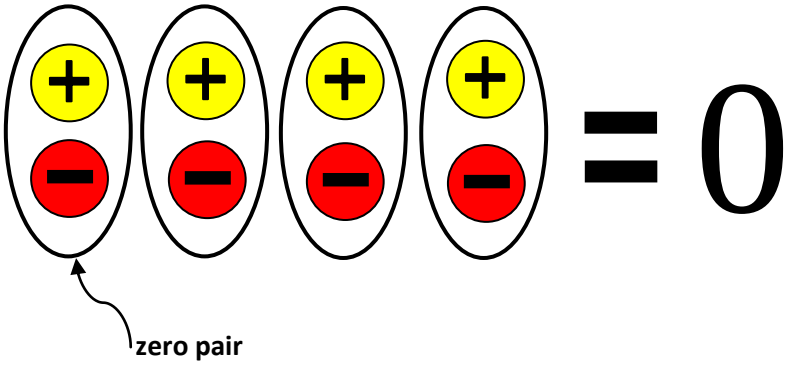
Example: 4 + (-4)

Number Line



Number Line

Counters



1. Find four pairs of integers with a sum of 5. Explain your process.
(Use positive integers only)

Pair		Sum	Equation

2. Find four pairs of integers with a sum of 5. Explain your process.
(Use one positive integer and one negative integer for each pair of integers)

Pair		Sum	Equation

3. Find a pair of negative integers with a sum of 5. What do you notice about the result? Explain your findings.

4. What do you notice when adding integers?

5. Explain (-2) + 5.

6. Explain 7 + 8.

7. Explain (-3) + (-14).

8. Explain 32 + -42.

9. Explain $a + b$ if both a and b are positive numbers.

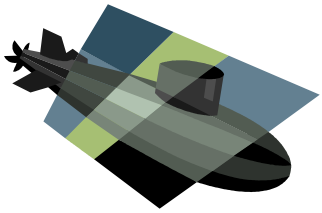
10. Explain (-*a*) + (-*b*) if (-*a*) and (-*b*) both represent negative numbers.

11. Explain *a* + (-*b*) if *a* represents any positive number and (-*b*) represents any negative number.

12. Explain *b* + (-*a*) if *b* represents any positive number and (-*a*) represents any negative number.

13. Explain 3 + 62 + -7.

14. Explain *a* + *b* + -*c* if –*c* represents a negative number.



Helicopters and Submarines

You are an engineer in charge of testing new equipment that can detect underwater submarines from the air.

<u>Part 1:</u> The first three hours

During this part of the test, you are in a helicopter 250 feet above the surface of the ocean. The helicopter moves horizontally to remain directly above a submarine. The submarine begins the test positioned at 275 feet below sea level.

- After one hour, the submarine is 325.8 feet below sea level.
- After two hours, the submarine dives another 23 feet.
- After three hours, the submarine dives again, descending by an amount equal to the average of the first two dives.

Make a table/chart with five columns (Time, Position of Submarine, Position of Helicopter, Distance between Helicopter and Submarine, and a Mathematical Sentence showing how to determine this distance) and four rows (start, one hour, two hours, three hours).

Make a graphical display which shows the positions of the submarine and helicopter using the information in your table/graph.

Part 2: The next three hours

The equipment in the helicopter is able to detect the submarine within a total distance of 750 feet.

For each scenario, determine the maximum or minimum location for the other vehicle in order for the helicopter to detect the submarine; and write a mathematical sentence to show your thinking.
Determine the ordered pairs for these additional hours and include them on your graph.

- At the end of the fourth hour, the helicopter remains at 250 feet.

- At the end of the fifth hour, the submarine returns to the same depth that it was at the end of the third hour.

- At the end of the sixth hour, the submarine descends to three times its second hour position.

Using the Number Line

Graph these numbers on the number line and then answer questions 1 – 4.

$$-\frac{1}{7},\ 1\frac{2}{3},\ -1\frac{6}{8},\ 0,\ 1.8,\ 1\frac{2}{5},\ \text{-0.2},\ -1.2$$

1. How did you scale your number line? Explain why you chose this increment.

2. Which number has the larger absolute value 1.8 or $-1\frac{6}{8}$? How do you know?

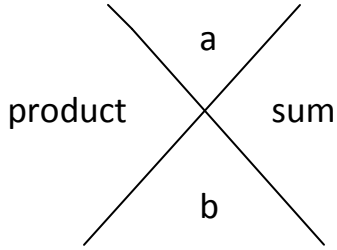
3. Look at the fractions and mixed numbers in this list. Which of these numbers, when written as decimals, are repeating decimals? Which form terminating decimals? Can you tell, without dividing, which fractions will repeat and which will terminate? How do you know?

4. Compare your number line with a partner.
    a. Did you both use the same increment? Is one choice better than the other? Why or why not?
    b. Explain how you placed your numbers. Are your numbers in the same order? If not, decide who is correct and why.

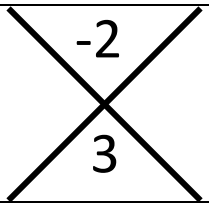
Sums and Products

Each of the puzzles in this task has a space for two different rational numbers, their sum, and their product. You are given the position of each value and a sample puzzle. Your job is to find the missing numbers. For each space you fill, write a number sentence that results in the missing value. Show how you found your sentence using any of the methods you have learned-facts families, number lines, positive/negative counters, area models, equations, etc.

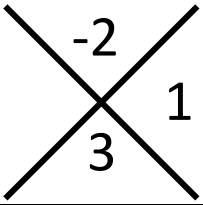
For each puzzle, values should be placed in the following positions.



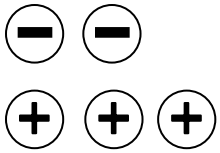
Suppose you are given that *a* is -2 and *b* is 3.



The problem you use to find the sum might read: -2 + 3 = 1. Here is a set of positive and negative counters and a number line. Use it to illustrate the number sentence.

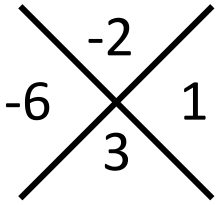


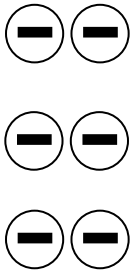
Counters

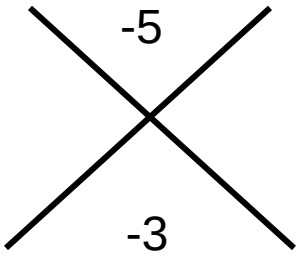


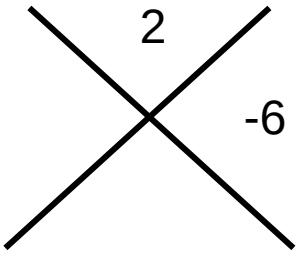
Number line

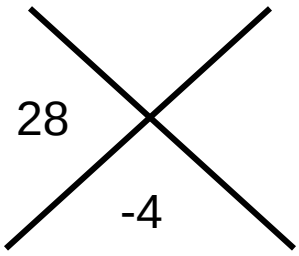
The problem you use to find the product might read: 3 • -2 = -6. Here is a set of positive and negative counters and a number line. Use it to illustrate the number sentence.

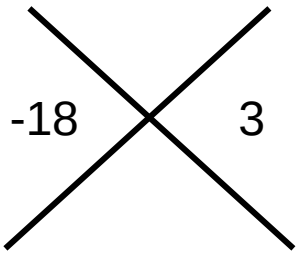


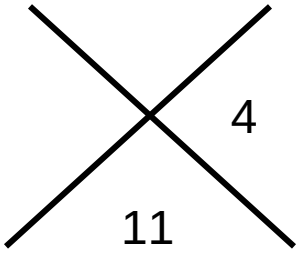
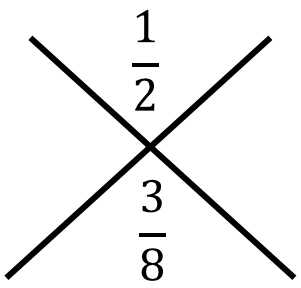
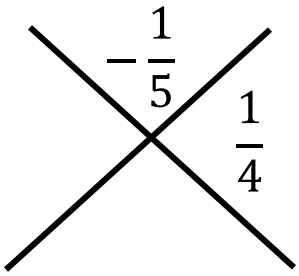
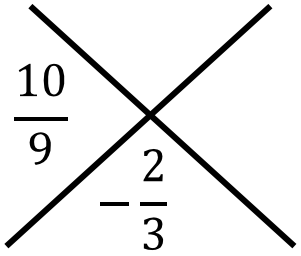
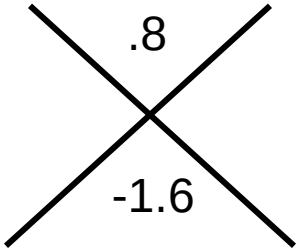
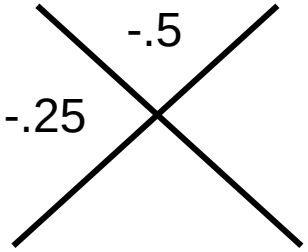
Counters	Number line
	

1. 

2. 

3. 

4. 

5. 	6. 
7. 	8. 
9. 	10. 

Connect the Dots

Do you remember the activity books from elementary school? There were dots on a page numbered one to a larger number and you connected the dots to form a picture. Your task is to create a series of points on the coordinate plane that, when connected, will form a picture. You must then create a script that tells someone how to connect the dots. Your picture must have a minimum of eight points and there must be at least one point in each of the four quadrants. The example below is a simple version of what you could create.

Script:

- Start at (-1,1) and draw a line segment to (-4, 1).
- Connect (-4, 1) to (-3,-1).
- Connect this dot to (3, -1) and continue with a line segment to (4,1).
- Connect this dot to (-1, 1).
- Continuing from (-1, 1), connect to (-1, 6).
- This will then be connected to (-4, 2).
- Connect this to (0,2).
- Finish the drawing by drawing a line segment from here to (-1,6).

Always, Sometimes, Never

In the table below, you are given ten statements. You are to choose whether each statement is always true (A), sometimes true (S), or never true (N).
If the statement is always true, you should be able to give a rule or a property that justifies your claim.
If the statement is sometimes true, you should be able to give an example showing that it can be true and a counterexample showing that it can be false.
If the statement is never true, you should again be able to give a rule or a property that is contradicted by the statement.

1. $3a > 3$

2. $a + 2 = 2 + a$

3. $-2b < 0$

4. $a + -a = 0$

5. $a - 6 > a$

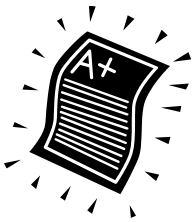
6. $5a + 5b = 5(a + b)$

7. $a - 2 = 2 - a$

8. $a + 10 > 10$

9. If two different numbers have the same absolute value, their sum is zero.

10. If the sum of two numbers is negative, their product is negative.



Making a Test

In this task, your job is to create a multiple choice test. You are given ten problems. For each problem, you are to write four possible "answer" choices. Your choices should include exactly one correct answer and at least one choice that contains common errors or misconceptions.

On a separate sheet of paper, for each problem, show how you got the correct answer; and explain the choices you made based on common errors or misconceptions.

So, here is the test!

Evaluating and Simplifying Algebraic Expressions

1. Which of the following is the value of $3a - 2b$ when $a = 2$ and $b = -6$?

2. Which of the following is the value of $3a^2 - 4$ when $a = -5$?

3. Which of the following is the value of $2(4 - b)^2 - b$ when $b = 5$?

4. Which of the following is the value of $\frac{(10a-6)}{4}$ when $a = \frac{2}{5}$?

5. Which of the following is the value of $(a + 7) - 3b$ when $a = -3$ and $b = 2$?

6. Which of the following shows the expression -3m – 2 + 5m – 8 in simplest possible form with no parentheses and no like terms?

7. Which of the following shows the expression 4(3a – 5) + 2(-3a + 7) in simplest possible form with no parentheses and no like terms?

8. Which of the following shows the expression $\frac{2}{3}\left(\frac{1}{2}x + 9\right) + 3$ in simplest possible form with no parentheses and no like terms?

9. Which of the following shows the expression $\frac{x}{4} - \frac{1}{8} - \frac{x}{6} + \frac{1}{6}$ in simplest possible form with no parentheses and no like terms?

10. Which of the following shows the expression $2ab + a - ab - 7$ in simplest possible form with no parentheses and no like terms?

Working with Integers

Adapted from Achieve

Even and Odd Numbers

1. The algebraic expression 2*n* is often used to represent an even number. Why do you think this is true? Illustrate your explanation with pictures and/or models.

   If 2*n* represents an even number, could *n* represent any number? Why or why not.

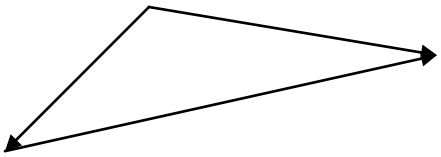
2. Write an algebraic expression that could represent an odd number. Explain your thinking and illustrate with a picture or a model.

3. What kind of number do you get when you add two even numbers? Justify your answer two different ways, by using models and by using algebraic expressions.

4. What kind of number do you get when you add two odd numbers? Justify your answer two different ways, by using models and by using algebraic expressions.

Consecutive Integers I

The lengths of the sides of the triangle below are consecutive integers.



1. What are consecutive integers? Give examples.

2. How can you represent the lengths of the sides of the triangle in terms of one variable? Explain your thinking.

Consecutive Integers II

Suppose three consecutive integers have a sum of -195.

1. If x denotes the middle integer, how can you represent the other two in terms of x?

2. Write an equation in x that can be used to find the integers.

3. Show that the sum of any three consecutive integers is always a multiple of three.

"A POSTER"

You are to make and present a poster showing what you have learned from your study of positive and negative rational numbers. Choose a theme for your poster. Be creative!

Choose four rational numbers. At least two of your numbers should be between –1 and 1, one of which should be written as a decimal and the other should be written as a fraction. Two of the numbers should be positive and two of the numbers should be negative. Make your poster using the rubric below:

Comparing

- Use the >, <, and = to compare your negative numbers.
- Graph all four numbers on a number line.

Absolute value

- Write the absolute value of each number and explain what is meant by absolute value.

Number problems

- Create two addition problems; one using numbers with like signs and the other using numbers with different signs.
- Create two subtraction problems; one using numbers with like signs and the other using numbers with different signs.
- Create two multiplication problems; one using numbers with like signs and the other using numbers with different signs.
- Create two division problems; one using numbers with like signs and the other using numbers with different signs.
- Model three of your problems with different operational signs.

Three real-life problems with solutions

- Write three real-life problems involving rational numbers and solve to show their solutions. Use a different operation in ea
- ch problem.

Properties of real numbers

- Use two of your numbers to illustrate the commutative property of addition
- Use two of your numbers to illustrate the associative property of multiplication.
- Use your numbers to illustrate the distributive property.
- Give the additive inverse of one of your numbers.
- Give the multiplicative inverse of one of your numbers.

Rules and common misconceptions

- List any rules you have found for computing with positive and negative numbers
- Give examples of common misconceptions students have when working with positive and negative numbers