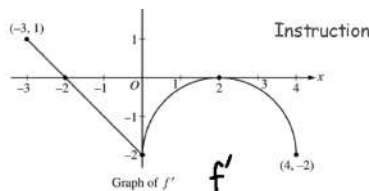


Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0)=3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle as shown.



(a) On what intervals, if any, is  $f$  increasing? Justify your answer.

$f$  is incr on  $(-3, -2)$  b/c  $f' > 0$ .

(b) Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.

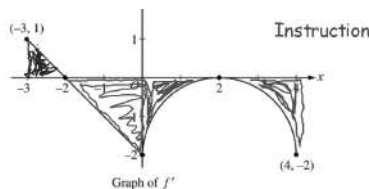
$f$  has POI when  $f'$  changes from incr to dec or vice-versa

This occurs at  $x=0$  and  $x=2$ .

(c) Find an equation for the line tangent to the graph of  $f$  at the point  $(0,3)$ .

(d) Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.

Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0)=3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle as shown.



(c) Find an equation for the line tangent to the graph of  $f$  at the point  $(0,3)$ .

$$f'(0) = -2 \quad (0, 3) \quad y = 3 - 2(x - 0)$$

(d) Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.  $f(0)=3$

$$\int_{-3}^0 f'(x) dx = f(0) - f(-3) \quad f(-3) = f(0) - \int_{-3}^0 f'(x) dx$$

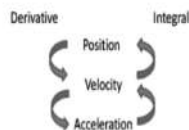
$$= 3 - \left[ \frac{1}{2}(\bar{x})(\bar{y}) + \frac{1}{2}(\bar{x})(\bar{y}) \right] = 3 - (-1.5) = 4.5$$

$$\int_0^4 f'(x) dx = f(4) - f(0)$$

$$f(4) = f(0) + \int_0^4 f'(x) dx = 3 + \left[ 4(-2) + \frac{1}{2}\pi(2)^2 \right] = 3 + (-8 + 2\pi) = 3 - 8 + 2\pi = 2\pi - 5$$

**Facts about motion...**

- 1.) "at rest"  $v(t) = 0$
- 2.) direction change  $v(t) = 0$  and  $v$  changes signs
- 3.) speed  $|v(t)|$ . If  $v(t)$  and  $a(t)$  have the same sign, speed is increasing. If they have different signs, speed is decreasing
- 4.) Moving right or up -  $v(t) > 0$  Moving left or down -  $v(t) < 0$
- 5.) Units - distance / time for velocity.
- 6.) Acceleration is the derivative of velocity. Units for acceleration - distance / time<sup>2</sup>



- 4.) A particle moves along the  $y$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = 1 - \tan^{-1}(e^t)$ . At time  $t=0$ , the particle is at  $y=-1$ . (Note:  $\tan^{-1} x = \arctan x$ ).  $s(0) = -1$
- (A) Find the acceleration of the particle at time  $t=2$ .
- (B) Is the speed of the particle increasing or decreasing at time  $t=2$ ? Give a reason for your answer.

- (C) Find the time  $t \geq 0$  at which the particle reaches its highest point. Justify your answer. (2004 AB3)

a)  $v'(2) = a(2) = -1.329$

b)  $v(2) = -0.4362$  Speed of particle is increasing b/c particle reaches  
acc and vel at  $t=2$  are the same sign.

c)  $t = .44302$   $\int_0^{.44302} v(t) dt = \int_0^{.44302} s'(t) dt = s(.44302) - s(0)$   
 $.04678 = s(.44302) - (-1)$   
 $s(.44302) = -0.9532$

- 4.) A particle moves along the  $y$ -axis so that its velocity  $v$  at time  $t \geq 0$  is given by  $v(t) = 1 - \tan^{-1}(e^t)$ . At time  $t=0$ , the particle is at  $y=-1$ . (Note:  $\tan^{-1} x = \arctan x$ ).
- (A) Find the acceleration of the particle at time  $t=2$ .
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- (C) Find the time  $t \geq 0$  at which the particle reaches its highest point. Justify your answer. (2004 AB3)

- (d) Find the position of the particle at time  $t=2$ . Is the particle moving toward or away from the origin at time  $t=2$ ? Justify your answer.
- $s(.44302) = -0.9532$

$\int_{.44302}^2 v(t) dt = s(2) - s(.44302)$   $v(2) = -0.4362$   
 $-1.40747 = s(2) + (-0.9532)$   
 $-1.36067 = s(2)$