AP Calculus
Unit 1 Day 7
Activation

$$f(x) = \begin{cases} x^2 & x \le 2 \\ 8-2x & 2 < x < 4 \\ 4 & x \ge 4 \end{cases}$$

Is f continuous @ $x=2$?

Yes $\lim\limits_{x\to 2} f(x) = f(2) = 4$

@ $x=4$?

No $\lim\limits_{x\to 4} f(x)$ DNE

so not continuous.

Evaluate the limit

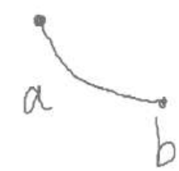
1) $\lim\limits_{x\to 2^-} f(x) = (2)^2 = 4$

2) $\lim\limits_{x\to 2^+} f(x) = 8-2(2) = 4$

3.) $\lim\limits_{x\to 2} f(x) = 4$

4.) $\lim\limits_{x\to 4^-} f(x) = 8-2(4) = 0$

5.) $\lim\limits_{x\to 4^+} f(x) = 4$



6.) $\lim\limits_{x\to 4} f(x) =$ DNE

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Instruction

Today, we will learn about the implications of continuity.

6.) Intermediate Value Theorem

If f is continuous on the closed interval [a,b], $f(a) \neq f(b)$, and k is any number between f(a) and f(b), then there is at least one number c in [a,b] such that $f(c)=k$.

Example...

Show that the function $f(x) = \cos^2 x - 2\sin\frac{x}{4}$ has a zero in the interval [0,2]

$f(0) = (\cos 0)^2 - 2\sin(0/4) = 1 - 0 = 1$

$f(2) = (\cos 2)^2 - 2\sin(2/4) = -0.7856$

-0.786

Since f(x) has domain $(-\infty, \infty)$ it is continuous,

$f(2) < 0 \leq f(0)$, so using IVT there is a c value in (a,b)

such that $f(c) = 0$.

Verify that the IVT applies to the indicated interval and find the value of c guaranteed by it.

$f(x) = \frac{x^2 + x}{x-1}, \; f(c) = 6, \; \left[\frac{5}{2}, 4\right]$ $\qquad D_f = \{x \mid x \neq 1 \;\; x \in \mathbb{R}\}$.

f(x) is continuous over $\left[\frac{5}{2}, 4\right]$

$f\left(\frac{5}{2}\right) = 5.833$ $\quad$ Since $f\left(\frac{5}{2}\right) \leq 6 \leq f(4)$, acc to IVT

$f(4) = 6.667$ $\quad$ there is a c in $\left[\frac{5}{2}, 4\right]$ such that $f(c) = 6$.

$(x-1)6 = \frac{(x^2+x)}{\cancel{(x-1)}} \cdot \cancel{(x-1)}$

$6x - 6 = x^2 + x$

$0 = x^2 - 5x + 6$

$0 = (x-2)(x-3)$

$x = 2, 3$ $\quad x = 3$

$f(3) = 6$ ✓

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Summary

Continuity -
3 Parts

1) $f(a)$ exists

2) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x)$

3) $\lim_{x \to a} f(x) = f(a)$ $\therefore$ $f(x)$ is continuous at $x=a$.

IVT - What to dc

1) check for continuity over $[a,b]$

2) $f(a) = ?$ $\quad f(b) = ?$

3) $f(a) \leq f(c) \leq f(b)$

$\therefore$ by IVT, $a < c < b$

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Work

Today's Assignment

1.4 page 80-81 #83-86 & 91-94 all

$\underline{f(x) = x^2 - 2x + 1}$

$f(x) = \frac{1}{x^2 - 2x + 1}$

$x \neq 3$

$f(x) = \frac{\boxed{(x-3)}(x+2)}{\boxed{(x-3)}(x-1)} = \frac{x+2}{x-1}$