AP Calculus – Free Response   Post Exam Set #6

Exam Problem #4.) The function f is defined by $f(x)=\sqrt{25-x^2}$ for $-5\le x\le 5$.

(A) Find $f'(x)$.

(B) Write an equation for the line tangent to the graph of f at $x=-3$.

(C) Let g be the function defined by $g(x)=\begin{cases} f(x) & \text{for } -5\le x\le -3 \\ x+7 & \text{for } -3< x\le 5 \end{cases}$. Is g continuous at $x=-3$? Use the definition of continuity to explain your answer.

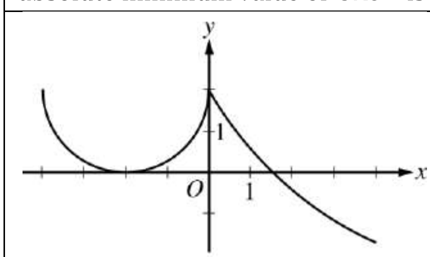
(D) Find the value of $\int_0^5 x\sqrt{25-x^2}\,dx$.

Practice #1 - Let *f* be a function defined by $f(x)=\begin{cases} \sqrt{x+1} & \text{for } 0\le x\le 3 \\ 5-x & \text{for } 3< x\le 5 \end{cases}$. (2003 AB6)

(a) Is *f* continuous at *x* = 3. Explain why or why not?
(b) Find the average value of *f* on the closed interval $0\le x\le 5$.
(c) Suppose the function g is defined by $g(x)=\begin{cases} k\sqrt{x+1} & \text{for } 0\le x\le 3 \\ mx+2 & \text{for } 3< x\le 5 \end{cases}$
Where *k* and *m* are constants. If g is differentiable at $x=3$, what are the values of *k* and *m*?

Practice #2 - Let *f* be a function given by $f(x)=2xe^{2x}$ (1998 AB2)

(a) Is *f* continuous at *x* = 3. Explain why or why not?
(b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
(c) What is the range of f ?
(d) Consider the family of functions defined by $y=bxe^{bx}$, where *b* is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of *b.*



Graph of f'

Practice #3 - The derivative of a function *f* is defined by

$$f'(x)=\begin{cases} g(x) & \text{for } -4\le x\le 0 \\ 5e^{-x/3}-3 & \text{for } 0< x\le 4 \end{cases}$$

The graph of the continuous function f', shown in the figure, has *x*-intercepts at *x* = -2 and $x=3\ln\left(\frac{5}{3}\right)$. The graph of *g* on $-4\le x\le 0$ is a semicircle, and *f* (0) =5. (2009 AB6)

(a) For $-4<x<4$, find all values of *x* at which the graph of *f* has a point of inflection. Justify your answer.
(b) Find *f*(-4) and *f* (4).
(c) For $-4\le x\le 4$, find the value of *x* at which *f* has an absolute maximum.