Today's Topic(s) - The "Other" Problem

Mock Exam Free Response #6 - No Calculator

** Pick up today's handout from the podium

Have your problems from set #5 ready to score

Your Target AP Score	5	4	3	2
Your Target on this problem	6	5	4	3

Swap your practice problems - we are going to grade!!!

Please grade in a color different from the worked problems

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The "Other" Problem - ME Free Response #6

Definition

If $y = f(x)$ then the derivative is defined to be
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or
$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$
The derivative at a point x=a where is defined as $f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Continuous Functions

If $f(x)$ is continuous at a then
i.) $f(a)$ exists
ii) $\lim_{x \to a} f(x)$ exists
iii.) $\lim_{x \to a} f(x) = f(a)$

1.) Look at each part independently.

2.) Word association - write the things from your study guide!!!!

3.) If the problem asks for it, attempt to calculate it.

4.) Don't get stuck on a "curve ball" question!

Interpretation of the Derivative
If $y = f(x)$ then,
1.) $m = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$ and the equation of the tangent line at $x = a$ is given by $y = f(a) + m(x - a)$. This formula can also be written as $y = y_1 + m(x - x_1)$ where $m = f'(x_1)$
2.) $f'(a)$ is the **instantaneous rate of change** of $f(x)$ at $x = a$. This is sometimes referred to as the slope of the curve f at $x = a$. **Average rate of change** can be found using the formula for the slope of a line $\left(\frac{\Delta y}{\Delta x}\right)$.

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Exam Problem #4.) The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(A) Find $f'(x)$.

$f(x) = (25 - x^2)^{1/2}$

$f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x)$ $\quad -5 < x < 5$

(a) $f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}$, $-5 < x < 5$ | 2 : $f'(x)$

Exam Problem #4.) The function f is defined by $f(x)=\sqrt{25-x^2}$ for $-5 \le x \le 5$.

(B) Write an equation for the line tangent to the graph of f at $x=-3$.

$f(-3)=\sqrt{25-(-3)^2}=\sqrt{25-9}=\sqrt{16}=4$

$f'(-3)=\frac{1}{2}(25-9)^{-1/2}(6)=\frac{3}{4}$

$f'(x)=\frac{1}{2}(25-x^2)^{-1/2}(-2x)$

$y-4=\frac{3}{4}(x+3)$

(b) $f'(-3)=\frac{3}{\sqrt{25-9}}=\frac{3}{4}$

$f(-3)=\sqrt{25-9}=4$

An equation for the tangent line is $y=4+\frac{3}{4}(x+3)$.

$2:\begin{cases}1: f'(-3)\\ 1: \text{answer}\end{cases}$

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Exam Problem #4.) The function f is defined by $f(x)=\sqrt{25-x^2}$ for $-5 \le x \le 5$.

(C) Let g be the function defined by $g(x)=\begin{cases}f(x) & \text{for} \quad -5\le x\le -3\\ x+7 & \text{for} \quad -3< x\le 5\end{cases}$. Is g continuous at $x=-3$?

Use the definition of continuity to explain your answer.

continuity at $x=a$

① $f(a)$ exists

② $\lim_{x\to a} f(x)$ exists

③ $\lim_{x\to a} f(x)=f(a)$

$g(-3)=f(-3)=\sqrt{25-9}=4$

$\lim_{x\to -3^-} g(x)=4$

$\lim_{x\to -3^+} g(x)=\lim_{x\to -3^+} x+7=4$

$\lim_{x\to -3} g(x)=4$ since LH & RH limits =

since $\lim_{x\to -3} g(x)=g(-3)=4$, g is continuous at $x=-3$.

(c) $\lim_{x\to -3^-} g(x)=\lim_{x\to -3^-} f(x)=\lim_{x\to -3^-}\sqrt{25-x^2}=4$

$\lim_{x\to -3^+} g(x)=\lim_{x\to -3^+}(x+7)=4$

Therefore, $\lim_{x\to -3} g(x)=4$.

$g(-3)=f(-3)=4$

So, $\lim_{x\to -3} g(x)=g(-3)$.

Therefore, g is continuous at $x=-3$.

$2:\begin{cases}1: \text{considers one-sided limits}\\ 1: \text{answer with explanation}\end{cases}$

Exam Problem #4.) The function f is defined by $f(x)=\sqrt{25-x^2}$ for $-5\le x\le 5$.

(D) Find the value of $\int_0^5 x\sqrt{25-x^2}\,dx$.

$$-\frac{1}{2}\int_0^5 -2x(25-x^2)^{1/2}\,dx = -\frac{1}{2}\int_{25}^{0} u^{1/2}\,du = \frac{1}{2}\int_0^{25} u^{1/2}\,du$$

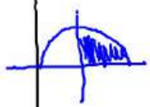
$u = 25-x^2$
$du = -2x\,dx$

$$= -\frac{1}{\cancel{2}}\,\frac{\cancel{2}u^{3/2}}{3}\Big]_{25}^{0}$$

$$= -\frac{1}{3}\left(0^{3/2}-25^{3/2}\right)$$

$$= -\frac{1}{3}(-125) = \frac{125}{3}$$

$$\int_0^5 \sqrt{25-x^2}\,dx$$



$$\frac{1}{4}\pi(5)^2$$

$$\frac{25\pi}{4}$$

(d) Let $u = 25-x^2 \Rightarrow du = -2x\,dx$

$$\int_0^5 x\sqrt{25-x^2}\,dx = -\frac{1}{2}\int_{25}^{0}\sqrt{u}\,du$$
$$= \left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$$
$$= -\frac{1}{3}(0-125) = \frac{125}{3}$$

$3:\begin{cases}2: \text{antiderivative}\\ 1: \text{answer}\end{cases}$

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Complete the remaining practice problems for homework.

One problem per page.

We will score them next time!!!!