Volume of Pyramids and Cones

Warm Up

1. Find the volume of a rectangular prism that is 4 in. tall, 16 in. wide, and 48 in deep.

3072 in^3

2. A cylinder has a height of 4.2 m and a diameter of 0.6 m. To the nearest tenth of a cubic meter, what is the volume of the cylinder? Use 3.14 for π.

1.2 m^3

3. A triangular prism's base is an equilateral triangle. The sides of the equilateral triangle are 4 ft, and the height of the prism is 8 ft. To the nearest cubic foot, what is the volume of the prism?

55.4 ft^3

Volume of Pyramids and Cones

Essential Question:

How can you use volume formulas to solve problems?

Standard:

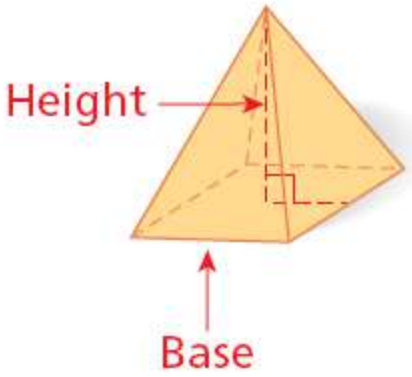
MCC8.G.9: Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Volume of Pyramids and Cones

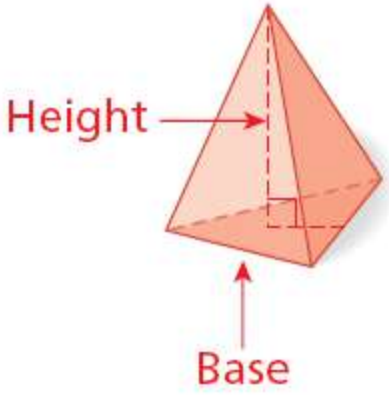
Learn to find the volume of pyramids and cones.

Volume of Pyramids and Cones

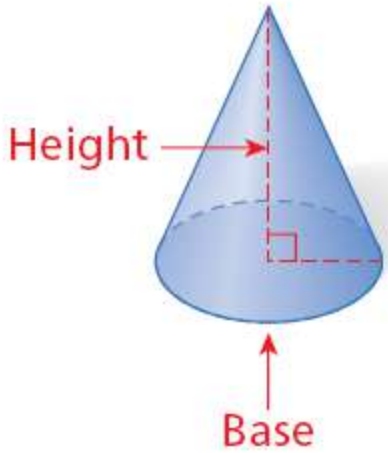
Rectangular pyramid



Triangular pyramid

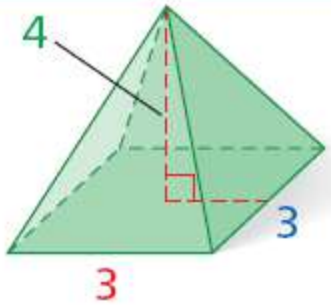
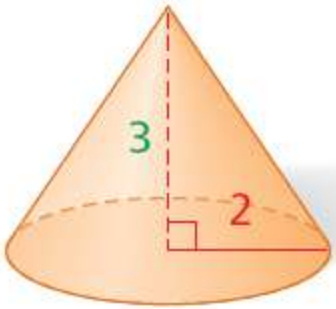


Cone



Volume of Pyramids and Cones

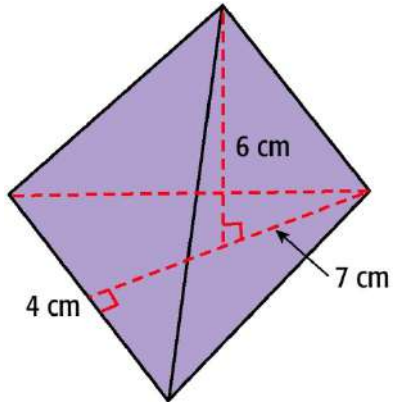
VOLUME OF PYRAMIDS AND CONES

Words	Numbers	Formula
Pyramid: The volume V of a pyramid is one-third of the area of the base B times the height h.	 $B = 3(3)$ $= 9 \text{ units}^2$ $V = \frac{1}{3}(9)(4)$ $= 12 \text{ units}^3$	$V = \frac{1}{3}Bh$
Cone: The volume of a cone is one-third of the area of the circular base B times the height h.	 $B = \pi(2^2)$ $= 4\pi \text{ units}^2$ $V = \frac{1}{3}(4\pi)(3)$ $= 4\pi$ $\approx 12.6 \text{ units}^3$	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$

Volume of Pyramids and Cones

Additional Example 1A: Finding the Volume of Pyramids and Cones

Find the volume of the figure. Use 3.14 for π.



$$B = \frac{1}{2}(4 \cdot 7) = 14 \text{ cm}^2$$

$$V = \frac{1}{3} \cdot 14 \cdot 6$$

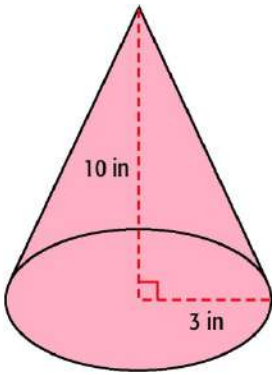
$$V = \frac{1}{3}Bh$$

$$V = 28 \text{ cm}^3$$

Volume of Pyramids and Cones

Additional Example 1B: Finding the Volume of Pyramids and Cones

Find the volume of the figure. Use 3.14 for π.



$B = \pi(3^2) = 9\pi \text{ in}^2$

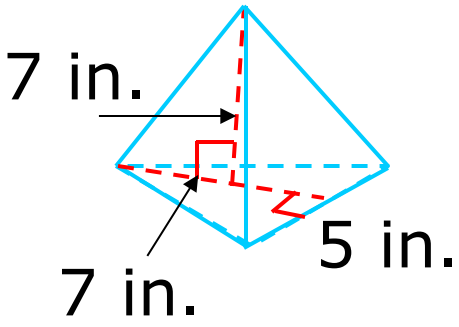
$V = \frac{1}{3} \cdot 9\pi \cdot 10$ $\quad V = \frac{1}{3}Bh$

$V = 30\pi \approx 94.2 \text{ in}^3$ *Use 3.14 for π.*

Volume of Pyramids and Cones

Check It Out: Example 1A

Find the volume of the figure. Use 3.14 for π.



$$B = \frac{1}{2}(5 \cdot 7) = 17.5 \text{ in}^2$$

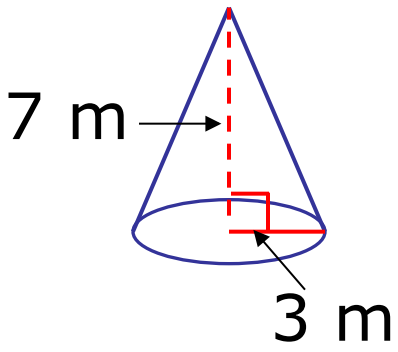
$$V = \frac{1}{3} \cdot 17.5 \cdot 7 \qquad V = \frac{1}{3}Bh$$

$$V \approx 40.8 \text{ in}^3$$

Volume of Pyramids and Cones

Check It Out: Example 1B

Find the volume of the figure. Use 3.14 for π.



$B = \pi(3^2) = 9\pi \text{ m}^2$

$V = \frac{1}{3} \cdot 9\pi \cdot 7$ $\quad V = \frac{1}{3}Bh$

$V = 21\pi \approx 65.9 \text{ m}^3$ *Use 3.14 for π.*

Volume of Pyramids and Cones

Additional Example 2: Exploring the Effects of Changing Dimensions

A cone has a radius of 3 ft. and a height of 4 ft. Explain whether tripling the height would have the same effect on the volume of the cone as tripling the radius.

Original Dimensions	Triple the Height	Triple the Radius
$V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{3}\pi r^2 (3h)$	$V = \frac{1}{3}\pi (3r)^2 h$
$= \frac{1}{3}\pi (3^2)4$	$= \frac{1}{3}\pi (3^2)(3 \cdot 4)$	$= \frac{1}{3}\pi (3 \cdot 3)^2 (4)$
$\approx 37.68\ \text{ft}^3$	$\approx 113.04\ \text{ft}^3$	$\approx 339.12\ \text{ft}^3$

When the height of the cone is tripled, the volume is tripled. When the radius is tripled, the volume becomes 9 times the original volume.

Volume of Pyramids and Cones

Check It Out: Example 2

A cone has a radius of 2 m and a height of 5 m. Explain whether doubling the height would have the same effect on the volume of the cone as doubling the radius.

Original Dimensions	Double the Height	Double the Radius
$V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{3}\pi r^2 (2h)$	$V = \frac{1}{3}\pi (2r)^2 h$
$= \frac{1}{3}\pi(2^2)5$	$= \frac{1}{3}\pi(2^2)(2 \cdot 5)$	$= \frac{1}{3}\pi(2 \cdot 2)^2(5)$
$\approx 20.93\ m^3$	$\approx 41.87\ m^3$	$\approx 83.73\ m^3$

When the height of a cone is doubled, the volume is doubled. When the radius is doubled, the volume is 4 times the original volume.

Volume of Pyramids and Cones

Additional Example 3: *Social Studies Application*

The Pyramid of Kukulcán in Mexico is a square pyramid. Its height is 24 m and its base has 55 m sides. Find the volume of the pyramid.

$B = 55^2 = 3025 \text{ m}^2$ $\quad A = bh$

$V = \frac{1}{3}(3025)(24)$ $\quad V = \frac{1}{3}Bh$

$V = 24{,}200 \text{ m}^3$

Caution!

A lowercase b is used to represent the length of the base of a two-dimensional figure. A capital B is used to represent the area of the base of a solid figure.

Volume of Pyramids and Cones

Check It Out: Example 3

Find the volume of a pyramid with a height of 12 m and a base with 48 m sides.

$B = 48^2 = 2304 \text{ m}^2$

$V = \frac{1}{3}(2304)(12)$

$V = 9216 \text{ m}^3$

$A = bh$

$V = \frac{1}{3}Bh$

Volume of Pyramids and Cones

Additional Example 4: Using a Calculator to Find Volume

Use a calculator to find the volume of a cone to the nearest cubic centimeter if the radius of the base is 15 cm and the height is 64 cm.

Use the *pi* button on your calculator to find the area of the base.

2ND π ^ × 15 X^2 ENTER $B = \pi r^2$

Next, with the area of the base still displayed, find the volume of the cone.

× 64 × (1 ÷ 3) ENTER $V = \frac{1}{3}Bh$

The volume of the cone is approximately 15,080 cm^3.

Volume of Pyramids and Cones

Check It Out: Example 4

Use a calculator to find the volume of a cone to the nearest cubic centimeter if the radius of the base is 14 cm and the height is 16 cm.

Use the *pi* button on your calculator to find the area of the base.

2ND π ^ × 14 X^2 ENTER $B = \pi r^2$

Next, with the area of the base still displayed, find the volume of the cone.

× 16 × (1 ÷ 3) ENTER $V = \frac{1}{3}Bh$

The volume of the cone is approximately 3,282 cm^3.

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Class work/Homework

Worksheet