Dividing by Unit and Non-Unit Fractions

Lesson # 10

Addressing

6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because 3/4 of 8/9 is 2/3. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?
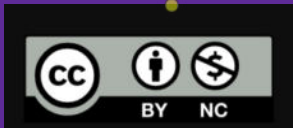
Let's look for patterns when we divide by a fraction.

Today's Goals

- I can divide a number by a non-unit fraction $\frac{a}{b}$ by reasoning with the numerator and denominator, which are whole numbers.

- I can divide a number by a unit fraction $\frac{1}{b}$ by reasoning with the denominator, which is a whole number.

Dividing by a Whole Number

Warm Up 10.1
- Think Pair Share

One person should answer the questions labeled Partner A. The other person should label the questions labeled Partner B.

Write an equation for each question. If you get stuck, draw a diagram.

Problem A

1. Partner A

    a. How many 3s are in 12?

       Division equation:

    b. How many 4s are in 12?

       Division equation:

    c. How many 6s are in 12?

       Division equation:

Problem B

2. Partner B

   a. What is 12 groups of $\frac{1}{3}$?

      Multiplication equation:

   b. What is 12 groups of $\frac{1}{4}$?

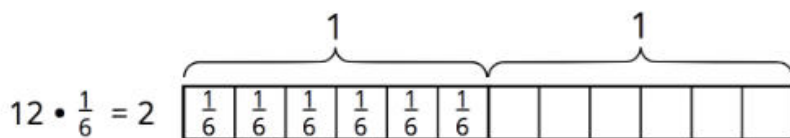   Multiplication equation:

   c. What is 12 groups of $\frac{1}{6}$?
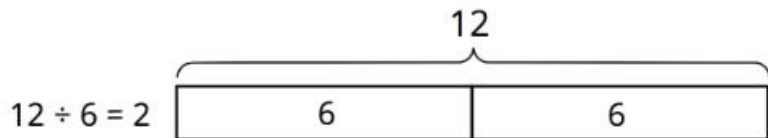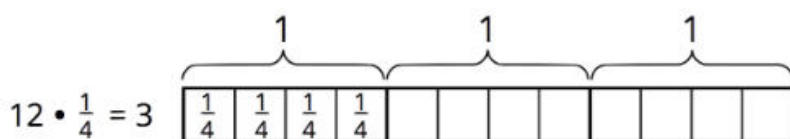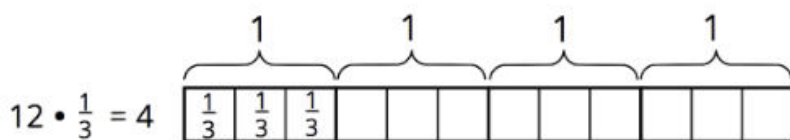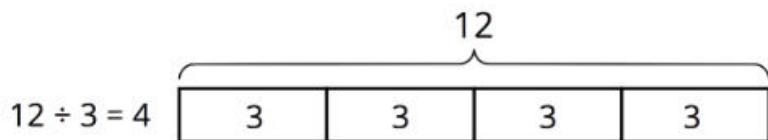
      Multiplication equation:

1. What do you notice in the diagrams and equations? Discuss with your partner.
2. Complete this sentence based on your observations: Dividing by a whole number produces the same result as multiplying by _____ .

Let's Talk About It

$12 \div 3 = 4$

3	3	3	3

$12 \cdot \frac{1}{3} = 4$

$12 \div 4 = 3$

4	4	4

$12 \cdot \frac{1}{4} = 3$

$12 \div 6 = 2$

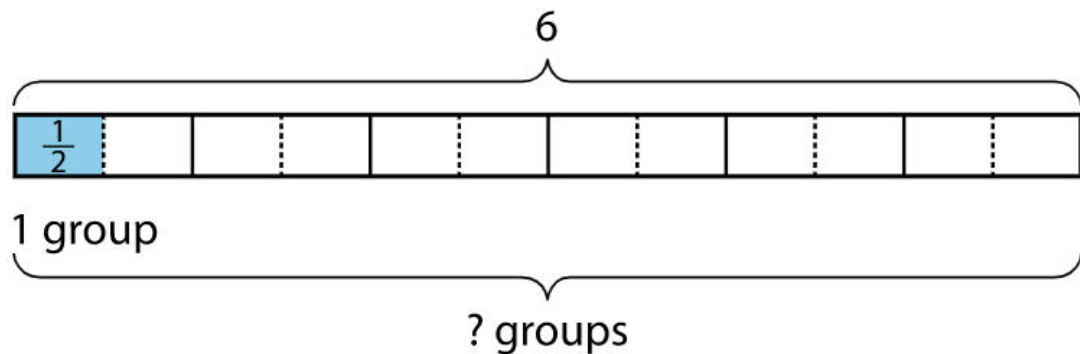|	
6	6

$12 \cdot \frac{1}{6} = 2$

Dividing by Unit Fractions

Activity 10.2
- MLR8: Discussion Supports
- Think Pair Share

1. To find the value of $6 \div \frac{1}{2}$, Elena thought, "How many $\frac{1}{2}$s are in 6?" and drew a tape diagram. It shows 6 ones with each one partitioned into 2 equal pieces.
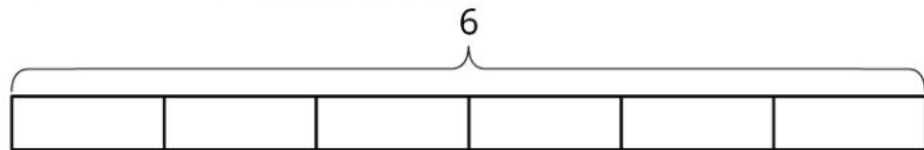
$6 \div \frac{1}{2}$

For each division expression, complete the diagram using the same interpretation of division as Elena's. Then, write the value of the expression. Think about how to find that value without counting the pieces in the diagram.
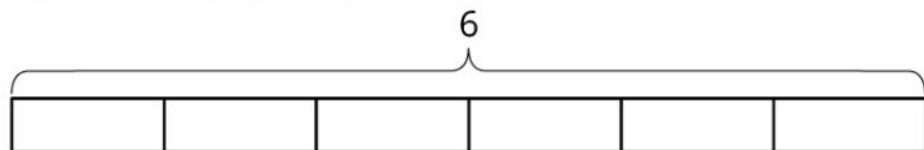
a. $6 \div \frac{1}{3}$

6
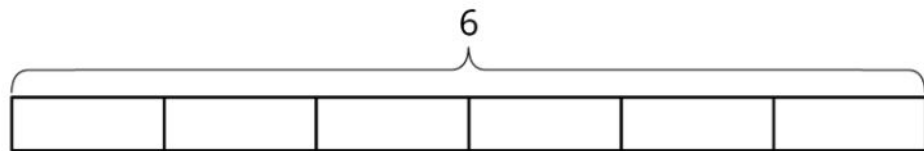
Value of the expression: _____

b. $6 \div \frac{1}{4}$

6

Value of the expression: _____

c. $6 \div \frac{1}{6}$

6

Value of the expression: _____

2. Analyze the expressions and your answers. Look for a pattern. How did you find how many $\frac{1}{2}$s, $\frac{1}{3}$s, $\frac{1}{4}$s, or $\frac{1}{6}$s were in 6 without counting? Explain your reasoning.

3. Use your observations from previous questions to find the values of the following expressions. If you get stuck, you can draw diagrams.

a. $6 \div \frac{1}{8}$

c. $6 \div \frac{1}{25}$

b. $6 \div \frac{1}{10}$

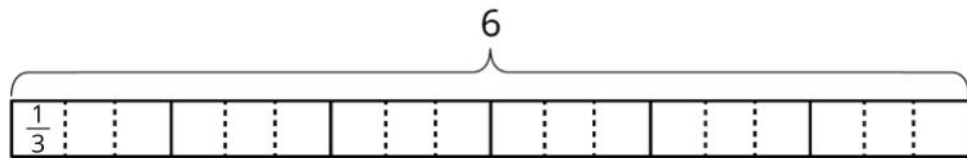d. $6 \div \frac{1}{b}$

Dividing by Non-Unit Fractions

Activity
- MLR3: Clarify, Critique, Correct

1. To find the value of $6 \div \frac{2}{3}$, Elena began by drawing her diagram in the same way she did for $6 \div \frac{1}{3}$.

   a. Use her diagram to find out how many $\frac{2}{3}$s are in 6. Adjust and label the diagram as needed.

   b. She says, "To find $6 \div \frac{2}{3}$, I can just take the value of $6 \div \frac{1}{3}$ then either multiply it by $\frac{1}{2}$ or divide it by 2." Do you agree with her? Explain why or why not.
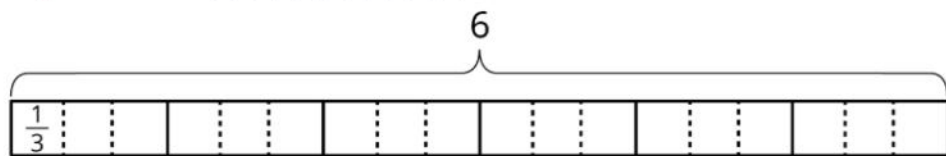
2. For each division expression, complete the diagram using the same interpretation of division that Elena did. Then, write the value of the expression. Think about how you could find the value of each expression without counting the equal pieces in your diagram.

$6 \div \frac{3}{4}$

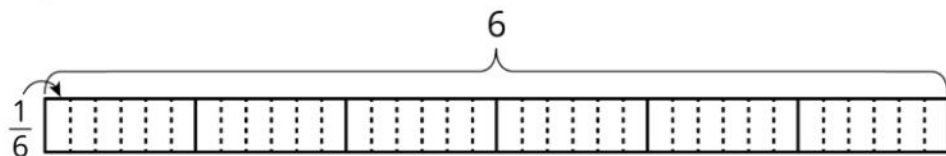6

$\frac{1}{4}$
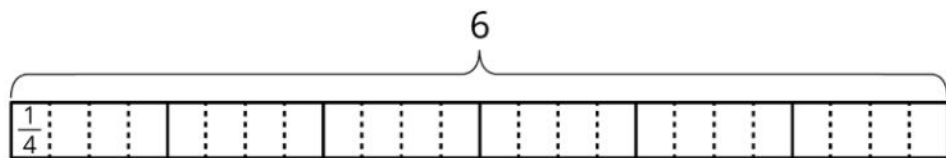
Value of the expression:_____

$6 \div \frac{4}{3}$

6

$\frac{1}{3}$

Value of the expression:_____

$6 \div \frac{4}{6}$

6

$\frac{1}{6}$

Value of the expression:_____

3. Elena studied her diagrams and noticed that she always took the same two steps to represent division by a fraction on a tape diagram. She said:

"My first step was to partition each 1 whole into as many parts as the number in the denominator. So if the expression is $6 \div \frac{3}{4}$, I would partition each 1 whole into 4 parts. Now I have 4 times as many parts.

My second step was to put a certain number of those parts into one group, and that number is the numerator of the divisor. So if the fraction is $\frac{3}{4}$, I would put 3 of the $\frac{1}{4}$s into one group. I could then tell how many $\frac{3}{4}$s are in 6."

Which expression represents how many $\frac{3}{4}$s Elena would have after these two steps? Be prepared to explain your reasoning.

a. $6 \div 4 \cdot 3$

b. $6 \div 4 \div 3$

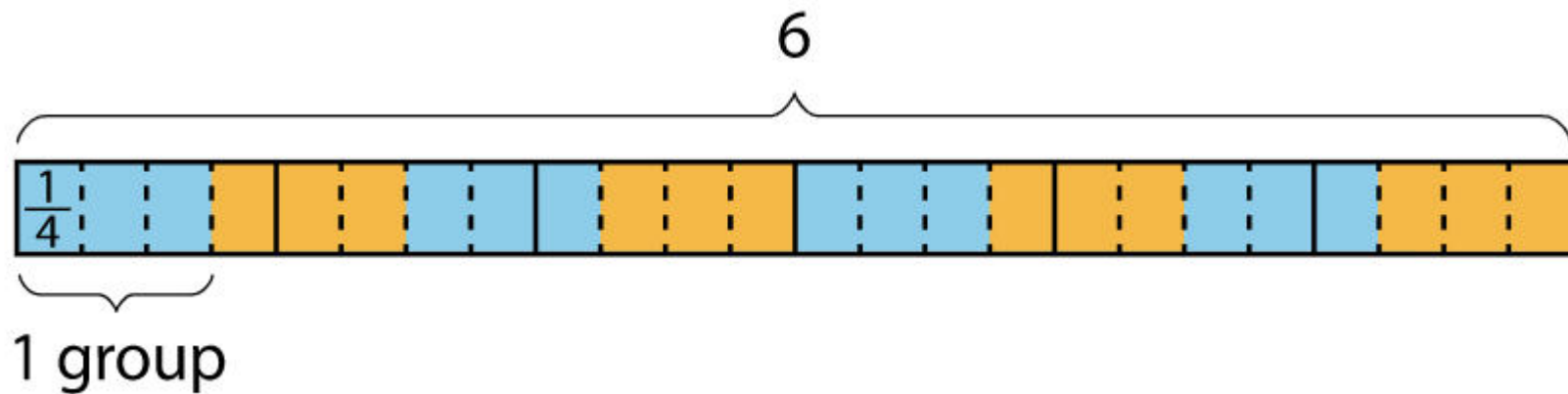c. $6 \cdot 4 \div 3$

d. $6 \cdot 4 \cdot 3$

4. Use your work from the previous questions to find the values of the following expressions. Draw diagrams if you are stuck.

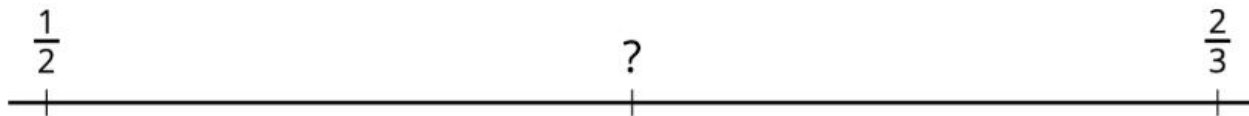a. $6 \div \frac{2}{7}$

b. $6 \div \frac{3}{10}$

c. $6 \div \frac{6}{25}$

Let's Talk About It

Are you ready for more?

Find the missing value.

$$\frac{1}{2} \qquad\qquad\qquad ? \qquad\qquad\qquad \frac{2}{3}$$

Lesson Synthesis

- **What did we notice about the result of dividing a number by a unit fraction?**
- **What observations did we make when dividing a number by a non-unit fraction?**
- **Suppose we are finding $5 \div \frac{7}{25}$. How might these observations help us find the quotient?**

Today's Goals

- I can divide a number by a non-unit fraction $\frac{a}{b}$ by reasoning with the numerator and denominator, which are whole numbers.

- I can divide a number by a unit fraction $\frac{1}{b}$ by reasoning with the denominator, which is a whole number.

Dividing by ⅓ and ⅗

Cool Down 10.4

1. Explain or show how you could find $5 \div \frac{1}{3}$ by using the value of $5 \cdot 3$. If needed, use this diagram to support your reasoning.
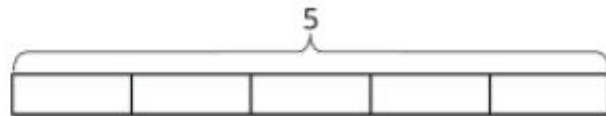
2. Find $12 \div \frac{3}{5}$. Only use a diagram if necessary. Show your reasoning.