4-10: Learning Goals

- Let’s look for patterns when we divide by a fraction.

4-10-1: Dividing by a Whole Number

Work with a partner. One person should solve the problems labeled "Partner A," and the other should solve those labeled "Partner B." Write an equation for each question. If you get stuck, draw a diagram.

1. Partner A

   a. How many 3s are in 12?

   Division equation:

   b. How many 4s are in 12?

   Division equation:

   c. How many 6s are in 12?

   Division equation:

2. Partner B

   a. What is 12 groups of $\frac{1}{3}$?

   Multiplication equation:

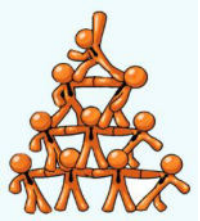
   b. What is 12 groups of $\frac{1}{4}$?

   Multiplication equation:

   c. What is 12 groups of $\frac{1}{6}$?

   Multiplication equation:

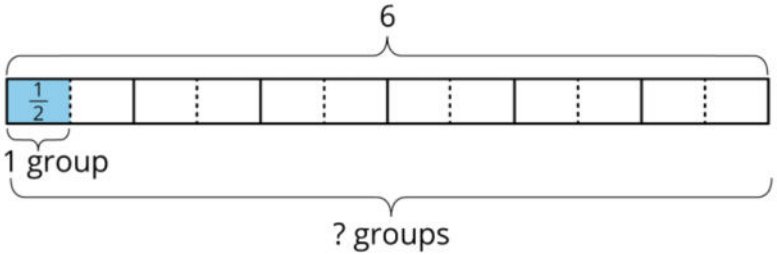
3. What do you notice in the diagrams and equations? Discuss with your partner.
4. Complete this sentence based on your observations: Dividing by a whole number a produces the same result as multiplying by



4-10-2: Dividing by Unit Fractions

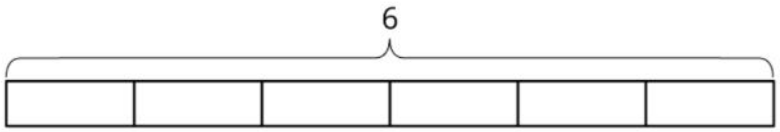
1. To find the value of $6 \div \frac{1}{2}$, Elena thought, "How many $\frac{1}{2}$s are in 6?" and drew a tape diagram. It shows 6 ones with each one partitioned into 2 equal pieces.

$6 \div \frac{1}{2}$



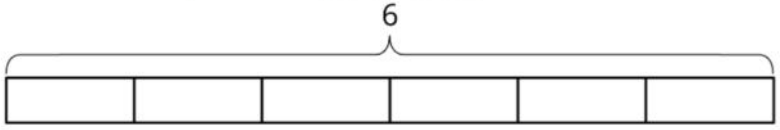
For each division expression, complete the diagram using the same interpretation of division as Elena's. Then, write the value of the expression. Think about how to find that value without counting the pieces in the diagram.

a. $6 \div \frac{1}{3}$



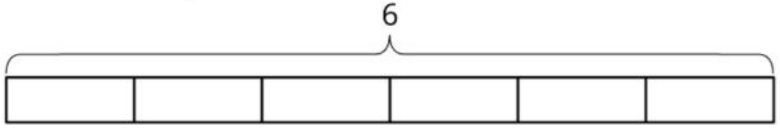
Value of the expression: _____

b. $6 \div \frac{1}{4}$

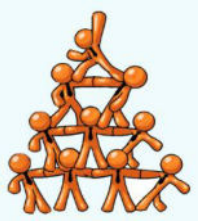


Value of the expression: _____

c. $6 \div \frac{1}{6}$



Value of the expression: _____



4-10-2: Dividing by Unit Fractions

2. Analyze the expressions and your answers. Look for a pattern. How did you find how many $\frac{1}{2}$s, $\frac{1}{3}$s, $\frac{1}{4}$s, or $\frac{1}{6}$s were in 6 without counting? Explain your reasoning.

3. Use your observations from previous questions to find the values of the following expressions. If you get stuck, you can draw diagrams.

   a. $6 \div \frac{1}{8}$

   b. $6 \div \frac{1}{10}$

   c. $6 \div \frac{1}{25}$

   d. $6 \div \frac{1}{b}$

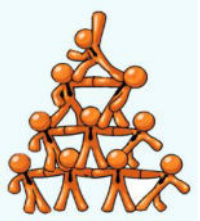
4. Find the value of each expression.

   a. $8 \div \frac{1}{4}$

   b. $12 \div \frac{1}{5}$

   c. $a \div \frac{1}{2}$

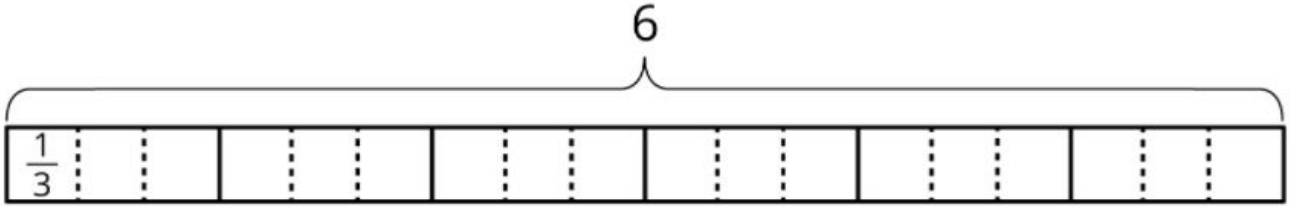
   d. $a \div \frac{1}{b}$



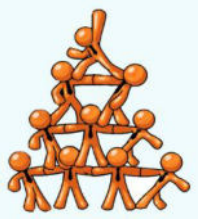
4-10-3: Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena began by drawing her diagram in the same way she did for $6 \div \frac{1}{3}$.

   a. Use her diagram to find out how many $\frac{2}{3}$s are in 6. Adjust and label the diagram as needed.



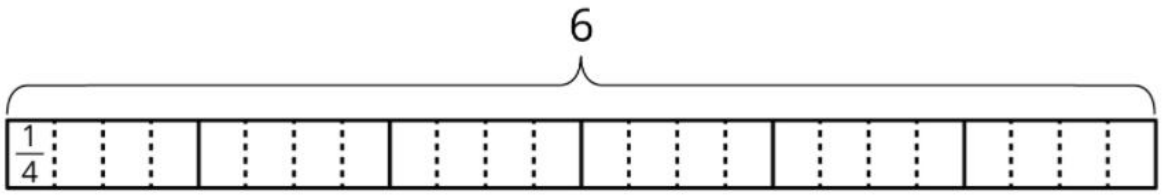
   b. She says, "To find $6 \div \frac{2}{3}$, I can just take the value of $6 \div \frac{1}{3}$ then either multiply it by $\frac{1}{2}$ or divide it by 2." Do you agree with her? Explain why or why not.



4-10-3: Dividing by Non-unit Fractions

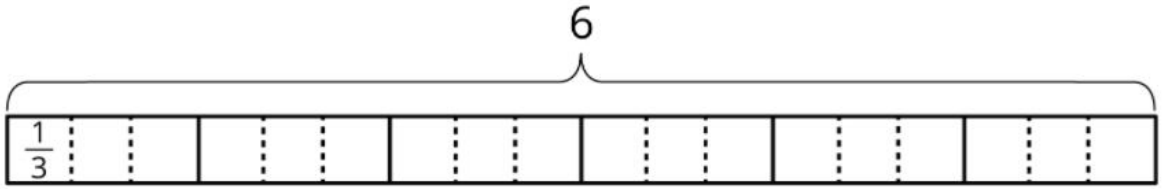
2. For each division expression, complete the diagram using the same interpretation of division that Elena did. Then, write the value of the expression. Think about how you could find the value of each expression without counting the equal pieces in your diagram.

$6 \div \frac{3}{4}$



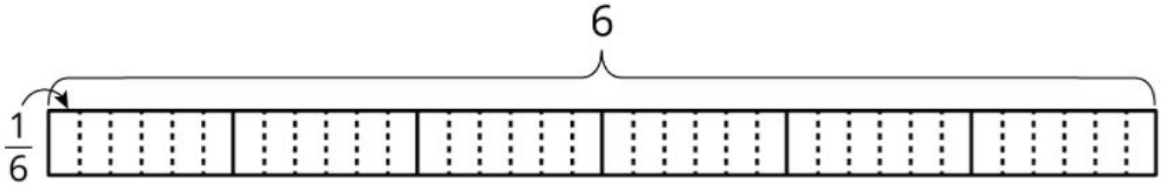
Value of the expression:_____

$6 \div \frac{4}{3}$

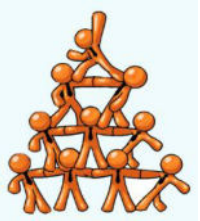


Value of the expression:_____

$6 \div \frac{4}{6}$



Value of the expression:_____



4-10-3: Dividing by Non-unit Fractions

3. Elena studied her diagrams and noticed that she always took the same two steps to represent division by a fraction on a tape diagram. She said:

   "My first step was to partition each 1 whole into as many parts as the number in the denominator. So if the expression is $6 \div \frac{3}{4}$, I would partition each 1 whole into 4 parts. Now I have 4 times as many parts.

   My second step was to put a certain number of those parts into one group, and that number is the numerator of the divisor. So if the fraction is $\frac{3}{4}$, I would put 3 of the $\frac{1}{4}$s into one group. I could then tell how many $\frac{3}{4}$s are in 6."

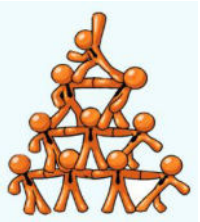
   Which expression represents how many $\frac{3}{4}$s Elena would have after these two steps? Be prepared to explain your reasoning.

   a. $6 \div 4 \cdot 3$

   b. $6 \div 4 \div 3$

   c. $6 \cdot 4 \div 3$

   d. $6 \cdot 4 \cdot 3$



4. Use your work from the previous questions to find the values of the following expressions. Draw diagrams if you are stuck.

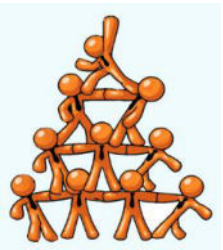
   a. $6 \div \frac{2}{7}$

   b. $6 \div \frac{3}{10}$

   c. $6 \div \frac{6}{25}$

4-10: Lesson Synthesis

- What did we notice about the result of dividing a number by a unit fraction? Can you explain with an example?
- What observations did we make when dividing a number by a non-unit fraction? Can you explain with an example?
- Suppose we are finding $5 \div \frac{7}{25}$. How might these observations help us find this quotient?

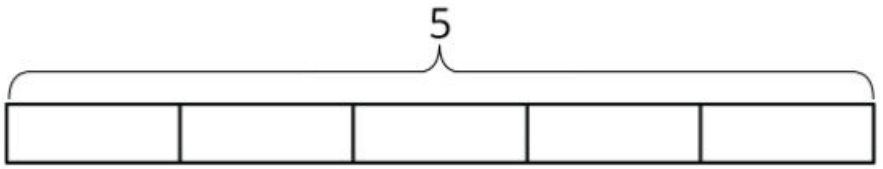


4-10: Learning Targets

- I can divide a number by a unit fraction $\frac{1}{b}$ by reasoning with the denominator, which is a whole number.
- I can divide a number by a non-unit fraction $\frac{a}{b}$ by reasoning with the numerator and denominator, which are whole numbers.

4-10-4: Dividing by ⅓ and ⅗

1. Explain or show how you could find $5 \div \frac{1}{3}$ by using the value of $5 \cdot 3$. If needed, use this diagram to support your reasoning.



2. Find $12 \div \frac{3}{5}$. Only use a diagram if necessary. Show your reasoning.