AP Calculus – Free Response   Unit 4 – Curve Sketching / Optimization

Instructions – Follow these directions carefully. Work each of the following on notebook paper. You should use at least **1 page** per problem. Your solutions should be in pencil. After completing your solution, use the online file to score your solution with the AP rubric. Make comments or corrections that you find helpful. Your comments should contain information that will help you score 100% in the future. If you do score 100%, you still need to comment. Grading and comments should be in ink. Your grade on this assignment will be determines as follows – Your solutions – 60 points (10 points each – they must be readable), AP Grading – 15 points, Comments – 15 points, Instructions followed – 10 points)

1.) Two particles move along the x-axis. For $0 \le t \le 6$, the position of the particle P at time t is given by $p(t) = 2\cos\left(\dfrac{\pi}{4}t\right)$ while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(A) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.
(B) For $0 \le t \le 6$, find all times t during which two particles travel in opposite directions.
(C) Find the acceleration of particle P at time t=3. Is particle P speeding up, slowing down, or doing neither at time t=3? Explain your reasoning.
(2010 ABB6)

2.) The function g is defined for x>0 with g(1)=2, $g'(x) = \sin\left(x + \dfrac{1}{x}\right)$, and

$$g''(x) = \left(1 - \dfrac{1}{x^2}\right)\cos\left(x + \dfrac{1}{x}\right)?$$

(A) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.
(B) On what subintervals of $0.12, 1$, if any, is the graph of g concave down? Justify your answer.
(C) Write an equation for the line tangent to the graph of g at x=0.3 given that g(0.3)= 1.546.
(D) Does the tangent to the graph of g at x=0.3 lie above or below the graph of g for $0.3 < x < 1$? Why?
(2010 ABB2)

3.) Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for x>0, where k is a positive constant.
(A) Find f'(x) and f''(x).
(B) For what value of the constant k does f have a critical point at x=1? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at x=1. Justify your answer.
(C) For certain values of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.
(2007 AB6)

4.) A particle moves along the y-axis so that it's velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1} e^t$. At time t=0, the particle is at y=-1. (Note: $\tan^{-1} x = \arctan x$).
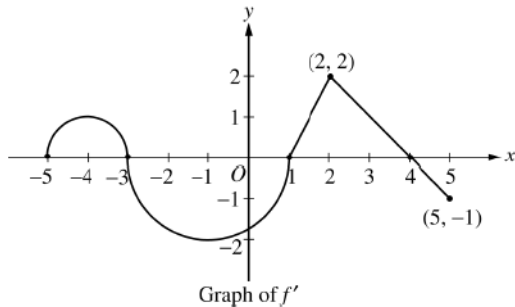
(A) Find the acceleration of the particle at time t=2.
(B) Is the speed of the particle increasing or decreasing at time t=2? Give a reason for

your answer.

(C) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

Graph of f'

5.) Let f be a function on the closed interval $-5 \leq x \leq 5$ with f(1)=3. The graph of f', the derivative of f, consists of 2 semicircles and two line segments as shown.

(A) For $-5 \leq x \leq 5$, find all values of x at which f has a relative maximum. Justify your answer.

(B) For $-5 \leq x \leq 5$, find all values of x at which f has a point of inflection. Justify your answer.

(C) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.