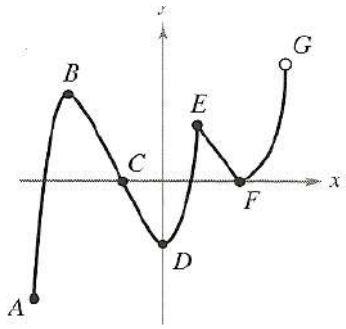
CALCULUS
Practice Handout #5

Work the following on **notebook paper**. You may use your calculator to find $f(x)$ values.

1. For each of the labeled points, state whether the function whose graph is shown has an absolute maximum, absolute minimum, local maximum, local minimum, or neither.




*A* _____ *B* _____

*C* _____ *D* _____

*E* _____ *F* _____

*G*_____

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2. Sketch the graph of a function $f$ that is continuous on [0, 6] and has the given properties: absolute maximum at $x = 0$, absolute minimum at $x = 6$, local minimum at $x = 2$, and local maximum at $x = 4$.

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Find the absolute maximums and absolute minimums of $f$ on the given closed interval by using the Candidates Test, and state where these values occur.

3. $f(x) = 4x^2 - 4x + 1 \quad [0, 2]$

4. $f(x) = \dfrac{x}{x^2 + 2} \quad [-1, 4]$

5. $f(x) = x^{2/3}(20 - x) \quad [-1, 20]$

6. $f(x) = \sin x - \cos x \quad [0, \pi]$

7. $f(x) = |2x - 6| \quad [-1, 5]$

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8. What is the smallest possible slope to $y = x^3 - 3x^2 + 5x - 1$ on $[-2, 3]$?

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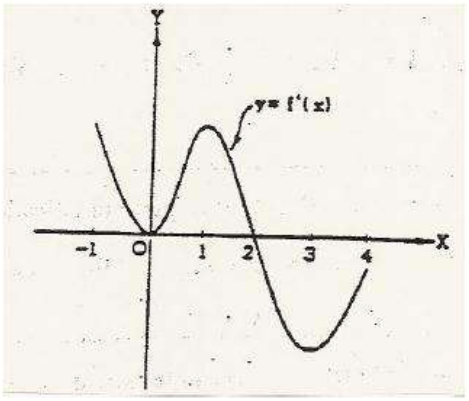
9. A particle moves along a straight line according to the position function $s(t) = t^4 - 4t^3 + 6t^2 - 20$. Find the maximum and minimum velocity on $[0, 3]$. For what values of $t$ do the maximum and minimum occur?

Remember that velocity is the derivative of position.

Part 2

Let $f$ be a function that has domain the closed interval $[-1, 4]$ and range the closed interval $[-1, 2]$. Let $f(-1) = -1$, $f(0) = 0$, and $f(4) = 1$. Also let $f$ has the derivative function $f'$ that is continuous and that has the graph shown in the figure.



Graph of $f'$

(a) Find all values of $x$ for which $f$ assumes a relative maximum. Justify your answer.

(b) Find all values of $x$ for which $f$ assumes its absolute minimum. Justify your answer.

(c) Find the intervals on which $f$ is concave downward.

(d) Give all values of $x$ for which $f$ has a point of inflection.

(e) Sketch the graph of $f$.

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2. A function $f$ is continuous on the closed interval $[-3, 3]$ such that $f(-3) = 4$ and $f(3) = 1$. The functions $f'$ and $f''$ have the properties given in the table below.

| $x$ | $-3 < x < -1$ | $x = -1$ | $-1 < x < 1$ | $x = 1$ | $1 < x < 3$ |
|---|---|---|---|---|---|
| $f'(x)$ | Positive | Fails to exist | Negative | 0 | Negative |
| $f''(x)$ | Positive | Fails to exist | Positive | 0 | Negative |

(a) What are the $x$-coordinates of all absolute maximum and absolute minimum points of $f$ on the interval $[-3, 3]$? Justify your answer.

(b) What are the $x$-coordinates of all points of inflection of $f$ on the interval $[-3, 3]$? Justify your answer.

(c) Sketch a graph that satisfies the given properties of $f$.

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3. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

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4. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of $b$?

(A) $-3$ (B) 0 (C) 1

(D) 3 (E) It cannot be determined from the information given.