AP Exam Review

Tuesday, 5/8/18 3:30 - 5:30

A A A A A A A A A A A A A A A A A A A A

1. If $f(x) = \left(2x^2 + 5\right)^7$, then $f'(x) =$

$7(2x^2+5)^6 \cdot 4x$

(A) $7(4x)^6$

(B) $7\left(2x^2 + 5\right)^6$

(C) $14x^2\left(2x^2 + 5\right)^6$

(D) $28x\left(2x^2 + 5\right)^6$

$\int \frac{1}{u}\,du = \ln|u| + C$

---

2. $\int \frac{1}{3x + 12}\,dx =$

$\int \frac{1}{3(x+4)}\,dx \qquad u = x+4,\ du = dx$

(A) $-3\ln|x + 4| + C$

(B) $\frac{1}{3}\ln|x + 4| + C$

(C) $\ln|x + 4| + C$

(D) $3\ln|x + 4| + C$

$= \frac{1}{3}\int \frac{1}{x+4}\,dx$

$= \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|x+4| + C$

A A A A A A A A A A A

3. If $f(x) = \dfrac{5-x}{x^3+2}$, then $f'(x) =$

(A) $\dfrac{-4x^3+15x^2-2}{\left(x^3+2\right)^2}$

(B) $\dfrac{-2x^3+15x^2+2}{\left(x^3+2\right)^2}$

(C) $\dfrac{2x^3-15x^2-2}{\left(x^3+2\right)^2}$

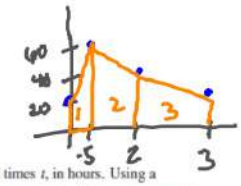
(D) $\dfrac{4x^3-15x^2+2}{\left(x^3+2\right)^2}$

$$\frac{(x^3+2)(-1)-(5-x)(3x^2)}{(x^3+2)^2} = \frac{-x^3-2-(15x^2-3x^3)}{(x^3+2)^2} = \frac{2x^3-15x^2-2}{(x^3+2)^2}$$

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

$A_{trap} = \frac{1}{2}(b_1+b_2)h$

t	0	0.5	2	3
$v(t)$	20	60	40	30



4. The table above gives the velocity $v(t)$, in miles per hour, of a truck at selected times t, in hours. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximate distance, in miles, the truck traveled from time $t = 0$ to $t = 3$?

(A) 140 (B) 130 (C) 125 (D) 120

Total Dist. Traveled $\int_0^3 |v(t)|\,dt \approx \frac{1}{2}\left[(20+60)(.5)+(60+40)(1.5)+(40+30)(1)\right]$

$\approx \frac{1}{2}(40+150+70)$

$\approx \frac{1}{2}(260)$

≈ 130

5. If $f(x) = \sin(x^2+\pi)$, then $f'(\sqrt{2\pi}) =$

(A) $-2\sqrt{2\pi}$ (B) -2 (C) -1 (D) $\cos(2\sqrt{2\pi})$

$f'(x) = \cos(x^2+\pi)\cdot 2x$

$= 2x\cos(x^2+\pi)\Big|_{\sqrt{2\pi}} = 2\sqrt{2\pi}\cos(2\pi+\pi) = -2\sqrt{2\pi}$ ($\cos(2\pi+\pi) = -1$)

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

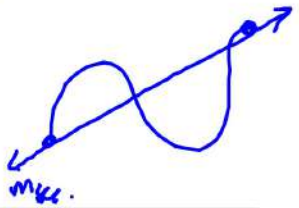
6. If f is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of f on the closed interval $[1, 5]$ is

(A) -21 (B) -13 (C) -12 (D) -9

$\frac{f(5)-f(1)}{5-1} = \frac{(75-125)-(3-1)}{4}$

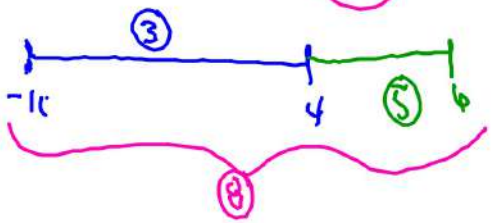
$= \frac{-50-2}{4} = \frac{-52}{4} = -13$

$-\int_{-10}^{4} f(x)\,dx = 3$



7. If $\int_4^{-10} g(x)\,dx = -3$ and $\int_4^6 g(x)\,dx = 5$, then $\int_{-10}^6 g(x)\,dx =$

(A) -8 (B) -2 (C) 2 (D) 8



A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point $(3 \ln 4, 4)$?

(A) $y - 4 = \frac{4}{3}(x - 3\ln 4)$

(B) $y - 4 = 4(x - 3\ln 4)$

(C) $y - 4 = 12(x - 3\ln 4)$

(D) $y - 3\ln 4 = 4(x - 4)$

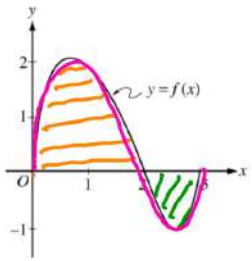
$f'(x) = \frac{1}{3}e^{\frac{x}{3}}\Big|_{3\ln 4}$

$f'(3\ln 4) = \frac{1}{3}e^{\frac{3\ln 4}{3}}$

$= \frac{1}{3}e^{\ln 4}$

$= \frac{1}{3} \cdot 4$

$m_{tan} = \frac{4}{3}$ → $y - 4 = \frac{4}{3}(x - 3\ln 4)$



The graph of a function f is shown above. Which of the following expresses the relationship between $\int_0^2 f(x)\,dx$, $\int_0^3 f(x)\,dx$, and $\int_2^3 f(x)\,dx$?

(A) $\int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx < \int_2^3 f(x)\,dx$

(B) $\int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_2^3 f(x)\,dx$

(C) $\int_2^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx$

(D) $\int_2^3 f(x)\,dx < \int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx$

A A A A A A A A A A A A A A A A A A A A

10. $\int_0^2 \left(x^3+1\right)^{1/2} x^2\,dx =$

(A) $\frac{52}{9}$ (B) 6 (C) $\frac{26}{3}$ (D) $\frac{52}{3}$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$\int_{x=0}^{x=2} u^{1/2}\cdot\frac{1}{3}du$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\Big|_{x=0}^{x=2}$

$\frac{2}{9}\left(x^3+1\right)^{3/2}\Big|_0^2$

$= \frac{2}{9}(27-1)$

$= \frac{2}{9}(26) = \frac{52}{9}$

11. If $x^2 + xy - 3y = 3$, then at the point $(2, 1)$, $\frac{dy}{dx} =$

(A) 5 (B) 4 (C) $\frac{7}{3}$ (D) 2

$\frac{d}{dx}\left[x^2+xy-3y=3\right]$

$2x + \left(x\frac{dy}{dx}+y\right) - 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x-3) = -(2x+y)$

$\frac{dy}{dx} = -\frac{2x+y}{x-3}\Big|_{(2,1)}$

$\frac{5}{-1}$

12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

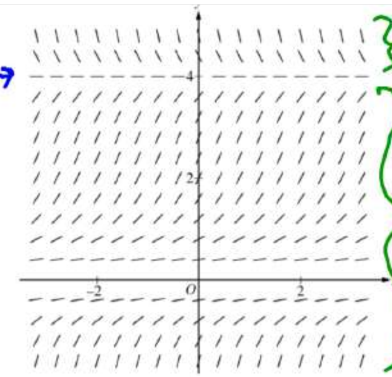
(A) 66 (B) 16 (C) -3 (D) -6

$w'(t) = -2t \,\big|_{t=3}$

$= -2(3)$

$= -6$

zeros →



} neg. slopes $y > 4$.

positive slopes

13. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x(4-y)}{4}$

(B) $\frac{dy}{dx} = \frac{y(4-y)}{4}$

(C) $\frac{dy}{dx} = \frac{xy(4-y)}{4}$

(D) $\frac{dy}{dx} = \frac{y^2(4-y)}{4}$

(+) (−) when $y > 4$

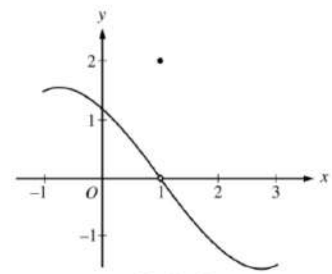
14. The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$

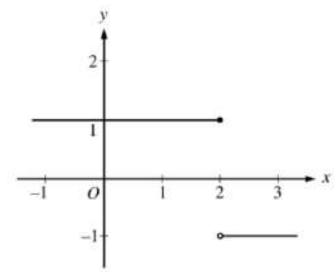
(B) $120e^{t/5}$

(C) $e^{t/5} + 119$

(D) $24t + 120$



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$

(B) $\lim_{x \to 2} g(x)$ does not exist.

(C) $\lim_{x \to 1} (f(x)g(x+1))$ does not exist.

(D) $\lim_{x \to 1} (f(x+1)g(x))$ exists.

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

16. Let f be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of f is both concave up and increasing?

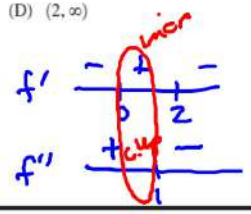
(A) $(0, 1)$ (B) $(1, 2)$ (C) $(0, 2)$ (D) $(2, \infty)$

$f'(x) = 12x - 6x^2$

$6x(2-x)$

$f''(x) = 12 - 12x$

$= 12(1-x)$



17. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

$v(t) = 12e^{-t}\cdot\cos t + \sin t\,(-12e^{-t})$

$12e^{-t}(\cos t - \sin t) = 0$

$12e^{-t} \neq 0$

$\cos t - \sin t = 0$

$\cos t = \sin t$

$\frac{\pi}{4}$ y <- 1st time

$\frac{5\pi}{4}$

A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A A

18. Let F be the function given by $F(x) = \int_3^x (\tan(5t)\sec(5t) - 1)\,dt$. Which of the following is an expression for $F'(x)$?

(A) $\frac{1}{5}\sec(5x) - 1$

(B) $\frac{1}{5}\sec(5x) - x$

(C) $\tan(5x)\sec(5x)$

(D) $\tan(5x)\sec(5x) - 1$

2nd FTC :

$(\tan(5t)\sec(5t) - 1)\Big|_x \cdot 1 - 0.$

19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

$f'(x) = -2\sin x \Big|_{x=\frac{\pi}{2}}$

$= -2\sin\left(\frac{\pi}{2}\right)$

$m_{tan} = -2$

$y - 1 = -2\left(x - \frac{\pi}{2}\right)$

$y - 1 = -2\left(1.5 - \frac{\pi}{2}\right)$

$y - 1 = -3 + \pi \rightarrow y = -2 + \pi$ or $(\pi - 2)$

$f\left(\frac{\pi}{2}\right) = 2\cos\frac{\pi}{2} + 1$

$= 1$

$pt: \left(\frac{\pi}{2}, 1\right)$