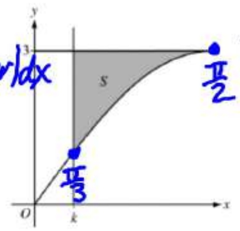
AP Exam Review

Sunday, 5/6/18 2:00 - 5:00

$A = \int_a^b (\text{upper} - \text{lower})\,dx$



$\frac{\pi}{2}$

$\frac{\pi}{3}$

$3 = 3\sin x$

$\sin x = 1$

$x = \frac{\pi}{2}.$

4. Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.

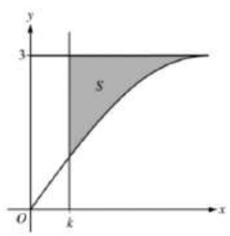
(a) Find the area of S when $k = \frac{\pi}{3}$.

$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3 - 3\sin x)\,dx$$

(a) Area $= \int_{\pi/3}^{\pi/2} (3 - 3\sin x)\,dx = \left[3x + 3\cos x\right]_{\pi/3}^{\pi/2}$

$= 3\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{1}{2}\right)\right] = \frac{\pi}{2} - \frac{3}{2}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$



4. Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.

(b) The area of S is a function of k. Find the rate of change of the area of S with respect to k when $k = \frac{\pi}{6}$.

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(b) Let $A(k)$ be the area of S.

$$A(k) = \int_k^{\pi/2} (3 - 3\sin x)\,dx$$

$$\frac{d}{dk} = 0 - (3 - 3\sin k)(1)$$

$$A'(k) = -3 + 3\sin k$$

$$A'(k) = -3 + 3\sin x \Big|_{\frac{\pi}{6}}$$

$$A'\left(\frac{\pi}{6}\right) = -3 + 3\sin\left(\frac{\pi}{6}\right) = -\frac{3}{2}$$

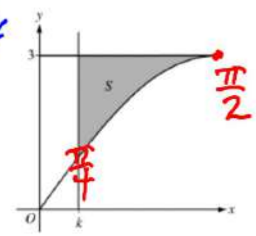
$$= -3 + \frac{3}{2}$$

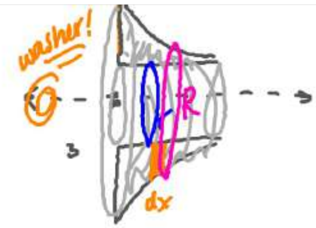
3 : { 1 : expression for area; 1 : expression for $A'(k)$; 1 : answer }

$$V = \pi\int_a^b (R^2 - r^2)\,dx$$

$R =$

$r =$





4. Let S be the shaded region in the first quadrant bounded above by the horizontal line $y = 3$, below by the graph of $y = 3\sin x$, and on the left by the vertical line $x = k$, where $0 < k < \frac{\pi}{2}$, as shown in the figure above.

$r = 5 - 3 = 2$

$R = 5 - (3\sin x) = (5 - 3\sin x)$

(c) Region S is revolved about the horizontal line $y = 5$ to form a solid. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid when $k = \frac{\pi}{4}$.

$$V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left((5 - 3\sin x)^2 - 2^2\right) dx$$

(c) Volume $= \pi\int_{\pi/4}^{\pi/2} \left[(5 - 3\sin x)^2 - (5-3)^2\right] dx$

$= \pi\int_{\pi/4}^{\pi/2} \left[(5 - 3\sin x)^2 - 4\right] dx$

3 : { 2 : integrand; 1 : limits and constant }

Question 6

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

Let f be the function defined above.

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

(c) Find the value of $\int_0^2 f(x)\,dx$.

(a) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \left(10 - 2x - x^2\right) = 7$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \left(3 + 4e^{x-1}\right) = 7$

Therefore, $\lim\limits_{x \to 1} f(x) = 7$.

Since $\lim\limits_{x \to 1} f(x) = f(1)$, f is continuous at $x = 1$.

$2 : \begin{cases} 1 : \text{considers one-sided limits} \\ 1 : \text{answer with explanation} \end{cases}$

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

Let f be the function defined above.

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

drop = [illegible]

$f'(x)$ (c) Find the value of $\int_0^2 f(x)\,dx$.

vert. tang? sharp pt?

$$\boxed{f'(x) = \begin{cases} -2-2x & x < 1 \\ 4e^{x-1} & x > 1 \end{cases}} \quad \begin{matrix} \Big|_{x=1} = -4 \\ \Big|_{x=1} = 4 \end{matrix} \Bigg\} \; f(x) \text{ is not diff. at } x = 1.$$

(b) For $x < 1$, $f'(x) = -2 - 2x$.

$f'(x) = 0 \Rightarrow x = -1$

For $x > 1$, $f'(x) = 4e^{x-1} \neq 0$.

At $x = 1$, $f'(x)$ is not defined.

CV: $x = -1, 1$

f': $+$ $-$ $+$ on -2, -1, 1, 2

Rel max (at -1), Rel min (at 1)

x	$f(x)$
-2	10
-1	11
1	7 abs min.
2	$3 + 4e$ abs max

$4 : \begin{cases} 1 : \frac{d}{dx}\left(10 - 2x - x^2\right) \text{ and } \frac{d}{dx}\left(3 + 4e^{x-1}\right) \\ 1 : \text{identifies } x = -1 \text{ and } x = 1 \text{ as critical points} \\ 1 : \text{evaluates } f \text{ at endpoints} \\ 1 : \text{answers with analysis} \end{cases}$

Question 6

$$f(x) = \begin{cases} 10 - 2x - x^2 & \text{for } x \le 1 \\ 3 + 4e^{x-1} & \text{for } x > 1 \end{cases}$$

Let f be the function defined above.

(a) Is f continuous at $x = 1$? Why or why not?

(b) Find the absolute minimum value and the absolute maximum value of f on the closed interval $-2 \le x \le 2$. Show the analysis that leads to your conclusion.

(c) Find the value of $\int_0^2 f(x)\,dx$.

(c) $\int_0^2 f(x)\,dx = \int_0^1 \left(10 - 2x - x^2\right) dx + \int_1^2 \left(3 + 4e^{x-1}\right) dx$

$= \left[10x - x^2 - \frac{1}{3}x^3\right]_0^1 + \left[3x + 4e^{x-1}\right]_1^2$

$= \left(10 - 1 - \frac{1}{3}\right) + \left[(6 + 4e) - (3 + 4)\right]$

$= \frac{23}{3} + 4e$

$3 : \begin{cases} 1 : \text{sum of integrals} \\ 1 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

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Question 5

For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

$(0, 3°)$ $\frac{1}{2}$ $x=0$

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

$M_{tan} = \frac{1}{2}$ pt: $(0, 3)$

(a) $\left.\frac{dW}{dt}\right|_{(t,W)=(0,3)} = \frac{3\cos 0}{2(3)} = \frac{1}{2}$ $\quad y - 3 = \frac{1}{2}x$ or

An equation for the tangent line is $y = \frac{1}{2}t + 3$.

$W(0.4) \approx \frac{1}{2}(0.4) + 3 = 3.2°\text{C}$

$2 : \begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

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(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

sep. of variables?!!

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24-hour interval.

(b) $2W\,dW = 3\cos t\,dt$ *

$\int 2W\,dW = \int 3\cos t\,dt$

$W^2 = 3\sin t + C$

$3^2 = 3\sin 0 + C \Rightarrow C = 9$

$W^2 = 3\sin t + 9$

Since $W(0) = 3$, $W = \sqrt{3\sin t + 9}$ for $0 \le t \le 24$

5 : { 1 : separation of variables; 2 : antiderivatives; 1 : constant of integration and uses initial condition; 1 : solves for W }

Note: max 3/5 [1-2-0-0] if no constant of integration

Note: 0/5 if no separation of variables

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$$\int_0^{24} 0.001\left(20 - \sqrt{3\sin t + 9}\right)dt$$

OR

(c) $0.001\int_0^{24} (20 - W(t))\,dt$ dollars

2 : { 1 : integrand; 1 : limits and constant }

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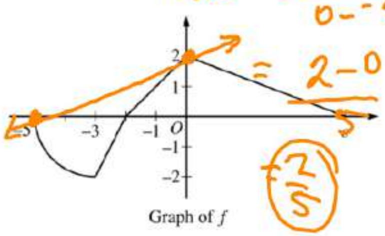
Question 3

$m_{sec} = \frac{f(0)-f(-5)}{0--5} = 2-0$

$\frac{2}{5}$

The function f is defined on the interval $-5 \le x \le c$, where $c > 0$ and $f(c) = 0$. The graph of f, which consists of three line segments and a quarter of a circle with center $(-3, 0)$ and radius 2, is shown in the figure above.



Graph of f

(a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

(b) For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

(a) The average rate of change of f over the interval $[-5, 0]$ is $\frac{f(0)-f(-5)}{0-(-5)} = \frac{2}{5}$.	1 : answer

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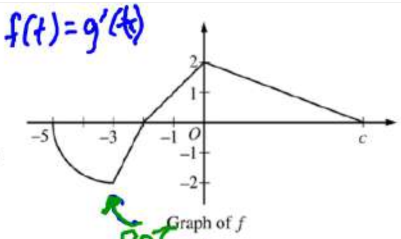
$f(t) = g'(t)$



Graph of f

POI

(a) Find the average rate of change of f over the interval $[-5, 0]$. Show the computations that lead to your answer.

(b) For $-5 \le x \le c$, let g be the function defined by

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$g(x) = \int_{-1}^{x} f(t)\,dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

$g'(x) = f(x)$ $\qquad g''(t) = f'(t)$

(c) Find the value of c for which the average value of f over the interval $-5 \le x \le c$ is $\frac{1}{2}$.

(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

(b) $g'(x) = f(x)$ The graph of g has a point of inflection at $x = -3$ because $g' = f$ changes from decreasing to increasing at this point. The graph of g has a point of inflection at $x = 0$ because $g' = f$ changes from increasing to decreasing at this point.	$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -3 \text{ and } x = 0 \\ 1 : \text{justification} \end{cases}$

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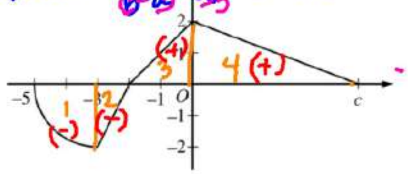
Avg val $= \frac{1}{b-a} \int_a^b f(x)\,dx$



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$\to A_1 + A_2 + A_3 + A_4$

$-\frac{1}{4}\pi(2)^2 + -\frac{1}{2}(1)(2) + \frac{1}{2}(2)(2) + \frac{1}{2}(c)(2)$

(c) $\frac{1}{c+5}\int_{-5}^{c} f(x)\,dx = \frac{1}{2}$

$\frac{1}{c+5}(-\pi + (-1) + 2 + c) = \frac{1}{2}$

$c = 3 + 2\pi$

3 : { 1 : integral, 1 : equation, 1 : answer }

$\frac{1-\pi+c}{c+5} = \frac{1}{2}$

$2 - 2\pi + 2c = c + 5$

$c = 3 + 2\pi$

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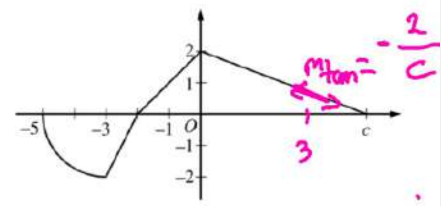

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(d) Assume $c > 3$. The function h is defined by $h(x) = f\left(\frac{x}{2}\right)$. Find $h'(6)$ in terms of c.

(d) $h'(x) = \frac{1}{2} f'\left(\frac{x}{2}\right)\Big|_{x=6} \to h'(6) = \frac{1}{2}\underbrace{f'(3)}$

$h'(6) = \frac{1}{2} f'(3) = \frac{1}{2} \cdot \frac{-2}{c} = -\frac{1}{c}$ $\quad = \frac{1}{2} \cdot \frac{-2}{c}$

2 : { 1 : $h'(x)$, 1 : answer }