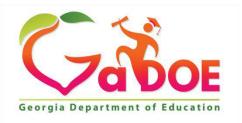


Georgia Standards of Excellence Curriculum Frameworks

Mathematics

GSE Fourth Grade Unit 4: Operations with Fractions



Richard Woods, Georgia's School Superintendent "Educating Georgia's Future"

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Unit 4: Operations with Fractions

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Overview. Thank you!

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OVERVIEW

In this unit students will:

- Identify visual and written representations of fractions
- Understand representations of simple equivalent fractions
- Understand the concept of mixed numbers with common denominators to 12
- Add and subtract fractions with common denominators
- Add and subtract mixed numbers with common denominators
- Convert mixed numbers to improper fractions and improper fractions to mixed fractions
- Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. (for example: model the product of $\frac{3}{4}$ as $3x\frac{1}{4}$).
- Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number.
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- Multiply a whole number by a fraction

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight Standards of Mathematical Practices should be continually addressed as well.

The first unit should establish these routines, allowing students to gradually enhance their understanding of the concept of number and to develop computational proficiency. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the ideas listed under "Big Ideas" be reviewed early in the planning process. A variety of resources should be utilized to supplement. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

For more detailed information about unpacking the content standards, unpacking a task, math routines and rituals, maintenance activities and more, please refer to the Grade Level Overview.

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content

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Standards of this unit. This list is not exhaustive and will hopefully prompt further reflection and discussion.

1. Make sense of problems and persevere in solving them. Students make sense of problems by applying knowledge of making equivalent fractions and multiplying fractions when solving problems "Running Trails" and "How Much Sugar" presented in this unit.

2. Reason abstractly and quantitatively. Students will reason abstractly about the use of repeated addition to multiply fraction by a whole number.

3. Construct viable arguments and critique the reasoning of others. Students construct viable arguments and critique the reasoning of others when articulating a fraction of a set to a peer.

4. Model with mathematics. Students model with mathematics through making equivalent fractions using area models, fractions of a set and adding and subtracting fractions on number lines.

5. Use appropriate tools strategically. Students select and use tools such as two color counters, number line and area models, pattern blocks and tiles to understand fractions.

6. Attend to precision. Students attend to precision when they build fractional proportions of a whole.

7. Look for and make use of structure. Students make use of structure through exploring the commutative property of fractions.

8. Look for and express regularity in repeated reasoning. Students look for and express regularity in repeated reasoning when performing repeated addition of fractions and multiplying fractions.

STANDARDS FOR MATHEMATICAL CONTENT

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

- a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

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- c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

- a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.
- b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)
- c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

<u>Solve problems involving measurement and conversion of measurements from a larger unit</u> to a smaller unit.

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

BIG IDEAS

- Fractions can be represented in multiple ways including visual and written form.
- Fractions can be decomposed in multiple ways into a sum of fractions with the same denominator.
- Fractional amounts can be added and/or subtracted.
- Mixed numbers can be added and/or subtracted.
- Mixed numbers and improper fractions can be used interchangeably because they are equivalent.
- Mixed numbers can be ordered by considering the whole number and the fraction.
- Proper fractions, improper fractions and mixed numbers can be added and/or subtracted.
- Fractions, like whole numbers can be unit intervals on a number line.
- Fractional amounts can be added and/or multiplied.
- If given a whole set, we can determine fractional amounts. If given a fractional amount,

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we can determine the whole set.

- When multiplying fractions by a whole number, it is helpful to relate it to the repeated addition model of multiplying whole numbers.
- A visual model can help solve problems that involve multiplying a fraction by a whole number.
- Equations can be written to represent problems involving the multiplication of a fraction by a whole number.
- Multiplying a fraction by a whole number can also be thought of as a fractional proportion of a whole number. For example, $\frac{1}{4} \ge 8$ can be interpreted as finding one-fourth of eight.

ESSENTIAL QUESTIONS: Choose a few questions based on the needs of your students.

- How are fractions used in problem-solving situations?
- How can equivalent fractions be identified?
- How can a fraction represent parts of a set?
- How can I add and subtract fractions of a given set?
- How can I find equivalent fractions?
- How can I represent fractions in different ways?
- How are improper fractions and mixed numbers alike and different?
- How can you use fractions to solve addition and subtraction problems?
- How do we add fractions with like denominators?
- How do we apply our understanding of fractions in everyday life?
- What do the parts of a fraction tell about its numerator and denominator?
- What happens when I add fractions with like denominators?
- What is a mixed number and how can it be represented?
- What is an improper fraction and how can it be represented?
- What is the relationship between a mixed number and an improper fraction?
- Why does the denominator remain the same when I add fractions with like denominators?
- How can I model the multiplication of a whole number by a fraction?
- How can I multiply a set by a fraction?
- How can I multiply a whole number by a fraction?
- How can I represent a fraction of a set?
- How can I represent multiplication of a whole number?
- How can we model answers to fraction problems?
- How can we write equations to represent our answers when solving word problems?
- How do we determine a fractional value when given the whole number?
- How do we determine the whole amount when given a fractional value of the whole?
- How is multiplication of fractions similar to repeated addition of fraction?
- What does it mean to take a fractional portion of a whole number?
- What strategies can be used for finding products when multiplying a whole number by a fraction?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Identify and give multiple representations for the fractional parts of a whole (area model) or of a set, using halves, thirds, fourths, sixths, eighths, tenths and twelfths.
- Recognize and represent that the denominator determines the number of equally sized pieces that make up a whole.
- Recognize and represent that the numerator determines how many pieces of the whole are being referred to in the fraction.
- Compare fractions with denominators of 2, 3, 4, 6, 10, or 12 using concrete and pictorial models.
- Understand repeated addition is one way to model multiplication, repeated subtraction is one way to model division
- Be able to decompose a whole into fractional parts. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 21/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore, students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

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For more about fluency, see: <u>http://www.youcubed.org/wp-</u> <u>content/uploads/2015/03/FluencyWithoutFear-2015.pdf</u> and <u>https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf</u>

STRATEGIES FOR TEACHING AND LEARNING

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Interdisciplinary and cross-curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition, which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Books such as *Fractions and Decimals Made Easy* (2005) by Rebecca Wingard-Nelson, illustrated by Tom LaBaff, are useful resources to have available for students to read during the instruction of these concepts.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
 - What level of support do my struggling students need in order to be successful with this unit?
 - In what way can I deepen the understanding of those students who are competent in this unit?
 - What real life connections can I make that will help my students utilize the skills practiced in this unit?

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers. Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks. Mathematics Standards glossary:

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary. The terms below are for teacher reference only and are not to be memorized by the students.

- fraction
- denominator
- equivalent sets

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- improper fraction
- increment
- mixed number
- numerator
- proper fraction
- term
- unit fraction
- whole number

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as performance tasks, they also may be used for teaching and learning.

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem-
	solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and
	concepts.
Performance Task	Tasks, which may be a formative or summative assessment, that checks
	for student understanding/misunderstanding and or progress toward the
	standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the
	unit to answer a new or unique situation. Allows students to give
	evidence of their own understanding toward the mastery of the standard
	and requires them to extend their chain of mathematical reasoning.
Formative	Lessons that support teachers in formative assessment which both
Assessment Lesson	reveal and develop students' understanding of key mathematical ideas
(FAL)	and applications. These lessons enable teachers and students to
	monitor in more detail their progress towards the targets of the
	standards.
CTE Classroom	Designed to demonstrate how the Career and Technical Education
Tasks	knowledge and skills can be integrated. The tasks provide teachers with
	realistic applications that combine mathematics and CTE content.
3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3
	distinct parts: an engaging and perplexing Act One, an information and
	solution seeking Act Two, and a solution discussion and solution
	revealing Act Three. More information along with guidelines for 3-Act
	Tasks may be found in the Guide to Three-Act Tasks on
	georgiastandards.org and the K-5 Georgia Mathematics Wiki.

Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)	Task Description
<u>Pizza Party</u>	Scaffolding Task Partner/Small Group Task	Draw fraction representations, add and subtract fractions	MGSE4.NF.3 a,b	Students design pizzas with correct toppings that are reported as fractions on an order form.
Eggsactly	Scaffolding Task Individual/Partner Task	Write number sentences to show addition of fractions.	MGSE4.NF.3 a.	Students use "eggs" to find fractions of a set.
<u>Tile Task</u>	Practice Task Partner/Small Group Task	Subtract and add fractions	MGSE4.NF.3a, b, d	Students will create designs for a tabletop using color tiles and decompose the whole into fractional parts.
Sweet Fraction Bars	Constructing Task Individual/Partner Task	Solve story problems with fractions	MGSE4.NF.3 a, b, d	Students solve problems about candy bars that are divided into fractional pieces using addition and subtraction.
<u>Fraction Cookies</u> <u>Bakery</u>	Constructing Task Individual/Partner Task	Addition with improper and proper fractions	MGSE4.NF.3 a,b,c,d	Similar to Pizza Party, students design cookies with toppings by reading fractional amounts on an order form.

Rolling Fractions	Practice Task Individual/Partner Task	Add and subtract fractions, use mixed numbers and improper fractions	MGSE4.NF.3 a,b,c,d	Students play a game where they create mixed numbers to add and subtract.
<u>Running Trails</u>	3 Act Task Partner/Small Group Task	Adding fractions, Comparing fractions	MGSE4.NF.3a MGSE4.NF.2 MGSE4.MD.2	Students develop and solve problems involving maps and fractions.
<u>The Fraction Story</u> <u>Game</u>	Performance Task Individual/Partner Task	Create story problems with fractions	MGSE4.NF.3 a,b,c,d	Students will develop a game that includes story problems that involve addition and subtraction of fractions.
Fraction Field Events	Performance Task Individual/Partner Task	Solve story problems with mixed numbers	MGSE4.NF.3 a,b,c,d	Students solve a problem about a field event using mixed numbers.
Culminating Task for MGSE4.NF.3: <u>Pizza Parlor</u> (Revisited)	Performance Task Individual/Partner Task	Add and subtract fractions, improper fractions and mixed numbers	MGSE4.NF.3 a,b,c,d	Similar to Fraction Cookie Bakery, students will fill orders by designing pizzas that have been ordered.
<u>A Bowl of Beans</u>	Scaffolding Task Partner/Small Group Task	Multiply a whole number by a fraction, represent fractional products	MGSE4.NF.4 a-c	Students model fractions of a set using multiplication of whole numbers and fractions to find how many are in the set.

Birthday Cake!	Scaffolding Task Individual/Partner Task	Determine the fraction of a set, determine the set when given a fraction	MGSE4.NF.4 a-c	Students use multiplication of fractions by whole numbers to determine the number of candles used on birthday cakes.
Fraction Clues	Constructing Task Individual/Partner Task	Determine the fraction of a set, determine the set when given a fraction, find equivalent fractions	MGSE4.NF.4 a-c	Students create a fraction riddle using color tiles and write clues for the tiles so that other students may try to build it through clues.
<u>Area Models</u>	Scaffolding Task Individual/Partner Task	Represent multiplication of fractions	MGSE4.NF.4 a-c	Students develop riddles using equivalent fractions and solve riddles by multiplying a whole number by a fraction.
How Many CCs	CTE Tasks Individual/Partner Task	Multiply a whole number by a fraction in a real-world context.	MGSE4.NF.4 a-c MGSE4.MD.2	Students apply knowledge of multiplying a whole number by a fraction using measurements given about medication being administered.
Who Put the Tang in Tangram?	Constructing Task Individual/Partner Task	Multiply a fraction by a whole number utilizing repeated addition	MGSE4.NF.3 a-d MGSE4.NF.4 a-c	In this task, students use tangrams to learn about multiplication of a fraction times a whole number as repeated addition when finding the area of tangrams.

Birthday Cookout	Constructing Task Individual/Partner Task	Solve story problems that involve the multiplication of fractions	MGSE4.NF.4 a-c	Students determine the number of people ordering food at a cookout using multiplication of a fraction by a whole number.
A Chance Surgery	CTE Tasks Individual Task	Multiply a whole number by a fraction in a real-world context.	MGSE4.NF.4 a, c MGSE4.MD.2	Students solve a problem involving the field of medicine to apply their knowledge of multiplication of fractions.
Fraction Pie Game	Practice Task Individual/Partner Task	Write number sentences to show multiplication of fractions.	MGSE4.NF.4 a, b	Students will cover 15 wholes on a recording sheet by multiplying a fraction by a whole number to determine how much to shade in on each turn.
How Much Sugar	3 Act Task Individual/Partner Task	Multiply a whole number by a fraction.	MGSE4.NF.4 a-c	Students discover how much sugar is in a 12 pack of Mountain Dew after developing questions and researching what information is needed to find a solution.
Fraction Farm	Performance Task Individual/Partner Task	Solve story problem that involves the multiplication of a whole number by a fraction	MGSE4.NF.4 a-c	Students find the amount of money a fraction of a field is worth using multiplication of whole numbers and fractions.

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If you need further information about this unit please view the webinars at <u>https://www.georgiastandards.org/Archives/Pages/default.aspx</u>

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INTERVENTION TABLE

The Intervention Table below provides links to interventions specific to this unit. The interventions support students and teachers in filling foundational gaps revealed as students work through the unit. All listed interventions are from New Zealand's Numeracy Project.

Cluster of Standards	Name of Intervention	Snapshot of summary or Student I can statement	Materials Master
	Animals	Find unit fractions of sets using addition facts.	
	Fractions in a Whole	Find unit fractions of sets using addition facts.	
	Wafers	Find unit fractions of regions.	
	Fractional Blocks	Find fractions of regions.	
Build fractions from unit fractions	Birthday Cakes	Find fractions of a set using multiplication and division.	
by applying and extending previous understandings of operations on whole numbers. MGSE4.NF.3 MGSE4.NF.4	Fractions Times Whole Numbers	Find fractions of whole number amounts using multiplication and division. Solve multiplication and division problems that involve fractions.	
	Whole Numbers Times Fractions	Find fractions of whole number amounts using multiplication and division. Solve multiplication and division problems that involve fractions.	

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FORMATIVE ASSESSMENT LESSONS (FALS)

Formative Assessment Lessons are designed for teachers to use in order to target specific strengths and weaknesses in their students' mathematical thinking in different areas. A Formative Assessment Lesson (FAL) includes a short task that is designed to target mathematical areas specific to a range of tasks from the unit. Teachers should give the task in advance of the delineated tasks and the teacher should use the information from the assessment task to differentiate the material to fit the needs of the students. The initial task should not be graded. It is to be used to guide instruction.

Teachers may use the following Formative Assessment Lessons (FALS) Chart to help them determine the areas of strengths and weaknesses of their students in particular areas within the unit.

Formative Assessments	FALS (Supporting Lesson Included)	Content Addressed	Pacing (Use before and after these tasks)
Adding Fractions		Adding Fractions	Pizza Party Eggactly Tile Task Sweet Fraction Bar
<u>Cynthia's Perfect</u> <u>Punch</u> <u>Peaches</u>		Adding Mixed Numbers Subtracting Mixed Numbers	Fractions Cookies Bakery Rolling Fractions The Fraction Story Game Fraction Field Day Events
<u>Sugar in Six Cans of</u> <u>Soda</u>		Multiplying Fractions	A Bowl of Beans Birthday Cake! Fraction Clues Area Models Fraction Pie Game
Where are the Cookies	Х	Fractional parts of wholes Fractional comparisons Fractional parts of other fractions	Birthday Cookout Fraction Farm

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SCAFFOLDING TASK: Pizza Party

Back to Task Table

Students will draw fraction representations and use those representations to add and subtract fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

The students should have had multiple opportunities with paper-folding fractions. To create eighths, students can fold a model in half, then in fourths, and finally into eighths. Each student's story problems may be unique. To assess student work, look for an illustration, made with the pizza slices, that matches the events in the story, an accurate number sentence for the story, and clear explanations. Student explanations should provide evidence that they understood why the denominator is 8. The standard explicitly says students should write their fractions as the sum of $\frac{1}{b}$. Guide students toward this goal, having them write number sentences that reflect this. For example, if someone ate $\frac{3}{8}$ of a pizza then they actually ate one slice, then another, then a third slice or $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$. You could simply joke around with kids about how no one really stuffs three slices in their mouth at once!

ESSENTIAL QUESTIONS

- What happens when I add fractions with like denominators?
- How do we add fractions with like denominators?

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MATERIALS

- "Pizza Party" student recording sheet
- "Pizza Party, Pizza Dough" student sheet (each sheet has enough circles for two students)
- Colored pencils or crayons
- Glue stick

GROUPING

Individual Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Using fraction models divided into eighths (pizzas), students create addition and subtraction story problems.

Comments

One way to introduce the task is by describing a family tradition of having pizza and a movie every Friday evening. Explain that the family makes two pizzas for dinner and rents a movie for the family to watch. There is always one cheese pizza and one pepperoni pizza. Each pizza is cut into eight equal slices.

Discuss with the students some possible addition problems that could be done with the pieces of pizza. For example, if the mom ate two slices of cheese pizza and one slice of pepperoni pizza,

how much pizza did she eat? Discuss the whole is cut into 8 equal pieces, so $\overline{\mathbf{8}}$ cheese + $\overline{\mathbf{8}}$

pepperoni = $\overline{\mathbf{8}}$ of a pizza. Have a student record the number sentence on the board, reminding students about the correct fraction notation.

As a subtraction problem, one example would be discussing the amount of cheese pizza left after the mom took two pieces. 8/8 - 2/8 = 6/8. Ask students how they might illustrate subtraction with the pizza slices. (Students may suggest crossing out the pieces removed or circling the pieces that are being subtracted.)

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Sometimes students find it difficult to understand that the whole can be any shape. Therefore, it may be helpful to provide square pizzas for students to work with in addition to the circle-shaped pizzas used in this task.

Time does not permit all students to share their work with the class. However, students may be afforded the opportunity to share their work in a small group and then one student from each group may share with the whole group. Students can also share their work with a partner and two or three students can be selected to share their work with the class. Teachers need to be thoughtful about who will share during the closing of a lesson. The student(s) whose work is shared needs to add to the class discussion or take the class discussion in a specific direction. A teacher needs to think about what type of conversation will help clarify possible student misconceptions and solidify student understanding of the concepts imbedded in the task.

Task Directions

Students will follow the directions below from the "Pizza Party" student recording sheet.

You will be writing two story problems, modeling the problems using pizzas that you create. Fold this paper in half to create two sections on the back to record your stories.

- 1. Create two pizzas.
 - a. Cut out two circles of paper (pizza dough) and color them to look like your two favorite types of pizza.
 - b. Fold the pizzas into eighths.
- 2. Fold this paper in half to create two sections on the back to record your pizza stories.
- 3. Write an addition story problem on the back of this paper.
 - a. Cut out the correct number of pizza slices for your story.
 - b. Glue down the pizza slices to illustrate your story.
 - c. Explain how you solved the problem using words and numbers.
- 4. Write a subtraction story problem on the back of this paper.
 - a. Cut out the correct number of pizza slices for your story.
 - b. Glue down the pizza slices to illustrate your story.
 - c. Explain how you solved the problem using words and numbers.

Be prepared to share your story, illustration, and solution with the class.

FORMATIVE ASSESSMENT QUESTIONS

- In your addition story, how many pieces of pizza do you have in all? How many slices of pizza in one whole? How do you write that as a fraction?
- In your subtraction story, how many pieces of pizza do you have left? How many slices of pizza in one whole? How do you write that as a fraction?
- Why does the denominator stay the same with addition and subtraction?
- Tell me the story that goes with your picture and number sentence.

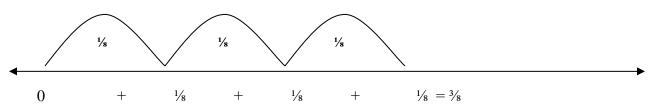
DIFFERENTIATION

Extension

- Have students consider the whole to be both pizzas, for a total of 16 slices equaling one whole.
- What would happen if the pizza restaurant made a mistake and cut one of the pizzas into fourths? How does it make finding the answer to an addition or subtraction sentence more difficult if the denominators of your fractions are not the same? Have students write problems where one pizza is cut into fourths, the other is cut into eighths.

Intervention

- Allow students to tell their story and model it with their pieces in a small group before gluing and labeling it on paper.
- If students have more experience with fraction tiles that are already labeled with unit fractions then they can manipulate them to determine their sums and differences.
- Students may also benefit from having the blank circles to place their decorated pizzas on top of. This visual may help them see what part is missing or serve as a reminder as to what the whole represents. Students can think of the blank circles as the pizza pan.
- Some students may benefit from adding or subtracting their pizzas slices on a number line. For example



Intervention Table

TECHNOLOGY

- <u>http://www.ixl.com/math/grade-4/add-and-subtract-fractions-with-like-denominators</u> This game can be used to practice the concept.
- <u>http://illuminations.nctm.org/activity.aspx?id=4148</u> This interactive lesson, titled "Fraction Game" involves decomposing, adding and subtracting fractions.
- <u>https://www.conceptuamath.com/app/tool/addition-with-common-denominators</u> *This tool provides students with the opportunity to model the addition of like fractions.
- <u>http://www.dreambox.com/teachertools</u> *Using both a concrete bar model and equations, this tool enables students to represent fraction addition and think strategically about which fractional pieces are best to solve a given problem.

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Name ____

_____ Date ____

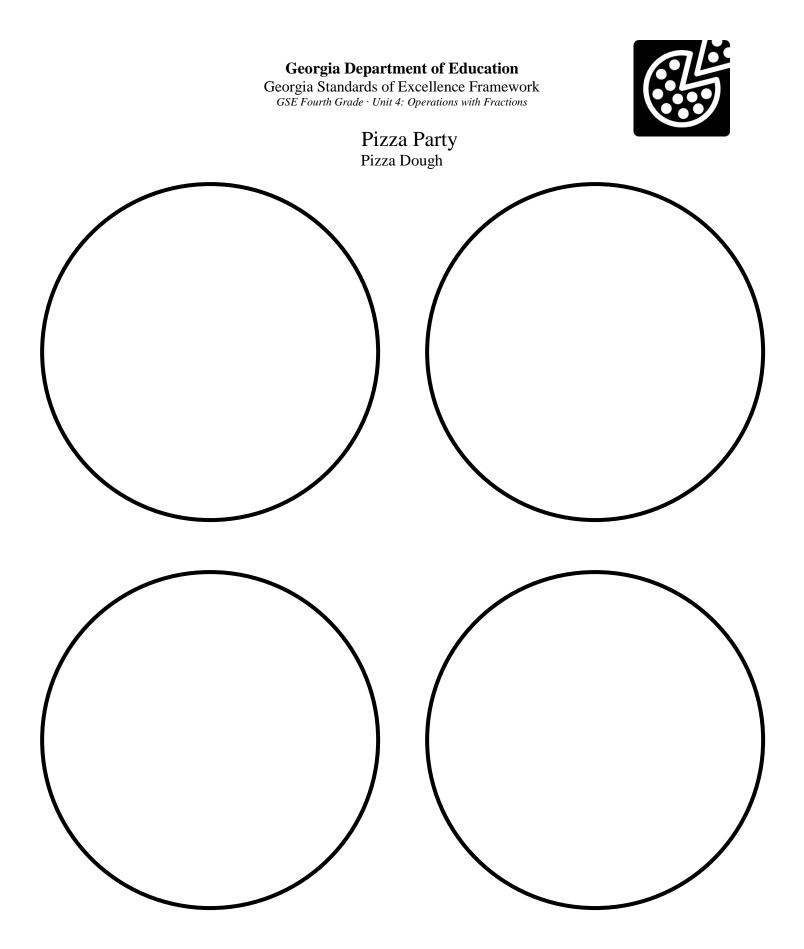
Pizza Party

You will be writing two story problems, modeling the problems using pizzas that you create. Fold this paper in half to create two sections on the back to record your stories.

- 1. Create two pizzas.
 - a. Cut out two circles of pizza dough and color them to look like your two favorite types of pizza.
 - b. Fold the pizzas into eighths.
- 2. Fold this paper in half to create two sections on the back to record your pizza stories.
- 3. Write an addition story problem on the back of this paper.
 - a. Cut out the correct number of pizza slices for your story.
 - b. Glue down the pizza slices to illustrate your story.
 - c. Explain how you solved the problem using words and numbers.
- 4. Write a subtraction story problem on the back of this paper.
 - a. Cut out the correct number of pizza slices for your story.
 - b. Glue down the pizza slices to illustrate your story.
 - c. Explain how you solved the problem using words and numbers.

Be prepared to share your story, illustration, and solution with the class.





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SCAFFOLDING TASK: Eggsactly

Back to Task Table

Students will write number sentences to show addition of fractions using a set of 12 and 18 counters.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should understand how to name fractions and that the denominator represents the number of equal-sized pieces while the numerator represents the number of pieces being considered. Students should also have some understanding of how to divide twelve into subsets of 1, 2, 3, 4, and 6.

Using a dozen, and later eighteen, eggs as a whole allows students to add and subtract values from the dozen eggs and then search for equivalent fractions. Students should be encouraged to see that twelve eggs or eighteen eggs can be grouped in other ways than 12 or 18, for example they could be grouped in two sets of 6 or three sets of 6 respectively.

For example, the red circles in the model below could represent two eggs used in a recipe. The student should be able to do several mathematical steps with this one representation. Students should be able to see the subset as both $\frac{2}{12}$ and $\frac{1}{6}$. Also, students are required to write a number sentence to represent that two eggs are removed from the carton: $\frac{12}{12} - \frac{1}{12} - \frac{1}{12} = \frac{10}{12}$ or $\frac{12}{12} - \frac{2}{12} = \frac{10}{12}$.

Students may also be able to see equivalent number sentences like $\frac{6}{6} - \frac{1}{6} = \frac{5}{6}$.

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Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with an area model
- record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut yellow and red circles)
- write an equation which shows the equivalent fractions

ESSENTIAL QUESTIONS

- How can I add and subtract fractions of a given set?
- How can equivalent fractions be identified?

MATERIALS

- 18 two sided counters
- "Eggsactly" recording sheet
- Crayons or colored pencils

GROUPING

Individual/Partner Task

NUMBER TALKS

Number Talks can also be done using ideas from the "Which Doesn't Belong" website. (http://wodb.ca/shapes.html) The website provides squares that are divided into four sections. Each section has a mathematical idea. Students look at the four ideas presented and decide which idea doesn't belong. For example:

$\frac{1}{2}$	5 3	
2	2	
10	5	
NUMBER 7		

from Erick Lee

The "Which Doesn't Belong" square can be displayed for students on the board and treated as a Number Talk. After displaying the square above, give students a couple of minutes to look at it and develop some ideas about which number doesn't belong. Students may give the thumbs up signal when they have a solution and can continue to look for other solutions that may be

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possible as time allows. The teacher can call on students to give solutions and defend their thinking about the solution they have selected. In the example shown above, students may say:

- Five thirds doesn't belong because it is an improper fraction. It is more than one. All the other fractions are proper fractions, which means they are less than one.
- Five thirds doesn't belong because it cannot be made into an equivalent fraction with 10 equal pieces. All of the others can be made into a fraction that has 10 equal pieces. One half is equal to five tenths, two tenths is already a fraction with ten equal pieces and two fifths is equivalent to four tenths.
- One half doesn't belong because it is a unit fraction. All the other fractions can be decomposed into the sum of a unit fraction.
- Two tenths doesn't belong because it has a composite denominator. All the other fractions have prime denominators.

The class conversation that results is a very rich in mathematical vocabulary and content that will help students grow as mathematicians.

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

It is crucial that students understand that fractions represent part of a whole as well as part of a set. In sets, the whole is the total number of objects or the denominator and subsets of the whole make up the fraction parts or the numerator. As students work through this activity, they are first asked to see the "whole" as twelve eggs and then to see the "whole" as eighteen eggs. They can explore the sets with two sided counters, but ultimately, they will need to represent them with drawings.

Comments

Sets of two-sided counters should be available to the students. The students can use these to represent the "eggs" in the assignment. Students will need sets of 12 and 18 counters and they can then flip them over to show some as red and some as yellow to represent fractional parts of a dozen.

If available, students can glue die-cut yellow and red circles. Alternately, students can manipulate the fraction sets online and easily print and then label their work. One site is NCTM Illuminations "Fraction Models" (<u>http://illuminations.nctm.org/ActivityDetail.aspx?ID=11</u>). Make sure students use "sets" to represent their fractions.

This task could be introduced by bringing in a dozen eggs, either real or plastic. Students have some prior knowledge of this and will be comfortable with seeing the box as now representative of a whole.

Task Directions

Students will follow directions below from the "Twelve Eggsactly" and the "Eighteen Eggsactly" student recording sheet.

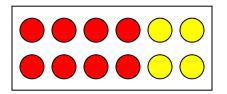
- Obtain a set of two sided counters.
- Use the two-sided counters to act out each recipe in the lesson.

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- Identify each fraction of eggs being used.
- Write a number sentence for each recipe
- Identify any equivalent fractions and write number sentences using these equivalent fractions.

Example: If a recipe calls for 8 eggs a student would have to represent his using two sided counters, red for those eggs being used and yellow for those that still remain



- 1. What fraction of the entire set is 8 eggs? $\frac{8}{12}$
- 2. Can you represent this fraction another way? $\frac{4}{6}$ or $\frac{2}{3}$
- 3. How many eggs still remain? 4 eggs
- 4. What fraction of the set still remains? $\frac{4}{12}$
- 5. Can you represent this fraction another way? $\frac{2}{6} or \frac{1}{3}$

6. Write a fraction sentence to show how many eggs were removed and how many still remain: $\frac{12}{12} - \frac{8}{12} = \frac{4}{12} \text{ or } \frac{6}{6} - \frac{4}{6} = \frac{2}{6} \text{ etc.}$

FORMATIVE ASSESSMENT QUESTIONS

- How are you keeping your work organized?
- Have you found all of the possible equivalent fractions? How do you know?
- How do you know these two fractions are equivalent?
- How many different illustrations can be created to show equivalent fractions? How do you know?
- Is there any other way you could write your number sentence?

DIFFERENTIATION

Extension

• Once students have completed the task above, this lesson can be extended to other sets, such as the "Eighteen Eggsactly" lesson which uses a set of 18 eggs to represent one whole.

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- Students will need to model the "Eighteen Eggsactly" lesson using two sided counters again and this will provide meaningful practice adding and subtracting fraction with the same denominator. It will also challenge them to find more equivalent fractions than the "Twelve Eggsactly" lesson can provide.
- Further lessons could be developed such as a "Twenty-four Eggsactly" task etc.
- The task could also be extended to include more than one set, for example 2 or 3 dozen eggs would develop a stronger understanding of mixed numbers of a set.

Intervention

- Allow students to first gain a lot of experience with smaller sets of "eggs". Using the two-sided counters create sets of 6 or 8 "eggs" to explore adding, subtracting and finding equivalent fractions.
- Some students could benefit from more concrete manipulatives such as egg cartons and plastic eggs.
- Students can manipulate the fractions online and represent them as a set and easily print and then label their work. One site for fraction sets is NCTM Illuminations "Fraction Models." <u>http://illuminations.nctm.org/ActivityDetail.aspx?ID=11</u>

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L338</u> Eggsactly Equivalent: This lesson can be used to reinforce the concept, additional practice or for remediation purposes.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L336</u> Eggsactly with a Dozen Eggs: This lesson allows students to explore parts of a set. It can be used for additional practice or for remediation purposes.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L337</u> Eggsactly with Eighteen Eggs: This lesson allows students to explore parts of a set. It can be used for additional practice or for remediation purposes.

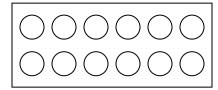
Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name _____

_____Date _____

Twelve Eggsactly

<u>PART 1:</u> Your brother needs help baking cookies for the school bake sale. One recipe he has calls for six eggs. Remove six eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



1. What fraction of the entire set is 6 eggs?

2. Write an equivalent fraction for the amount in question 1.

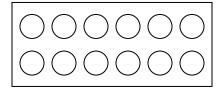
3. How many eggs still remain?

4. What fraction of the set still remains?

5. Write an equivalent fraction for the amount in question 4.

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

<u>PART 2:</u> Your brother needs help baking brownies for the school bake sale. One recipe he has calls for eight eggs. Remove eight eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



- 1. What fraction of the entire set is 8 eggs? _____
- 2. Write an equivalent fraction for the amount in question 1.

3. How many eggs still remain?

4. What fraction of the set still remains?

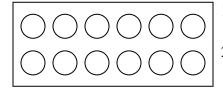
5. Write an equivalent fraction for the amount in question 4. _____

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

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PART 3: Your sister also needs help baking cupcakes for the school bake sale. One recipe she has calls for $\frac{1}{4}$ of a dozen. Remove $\frac{1}{4}$ of the eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



1. What fraction of the entire set is $\frac{1}{4}$ of the eggs? _____

2. Write an equivalent fraction for the amount in question 1.

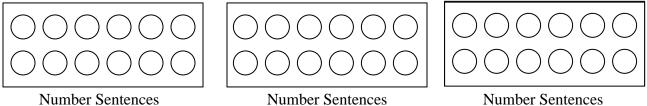
3. How many eggs still remain?

4. What fraction of the set still remains?

5. Write an equivalent fraction for the amount in question 4.

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

<u>PART 4</u>: Use the cartons below to show all the different ways to represent $\frac{1}{3}$ of a dozen eggs. Then write a number sentence for each model to show how many eggs were removed and how many still remain.



Number Sentences

Number Sentences

PART 5: How many cartons of eggs did your mother have to buy in order for your brother to make cookies and brownies and for your sister to make cupcakes? Use pictures, numbers and words to show your answer.

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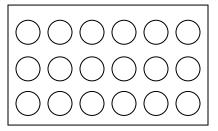
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Name _____

_____ Date ___

Eighteen Eggsactly

<u>PART 1:</u> Your brother needs help baking cookies for the school bake sale. One recipe he has calls for six eggs. Remove six eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



1. What fraction of the entire set is 6 eggs?

2. Write an equivalent fraction for the amount in question 1.

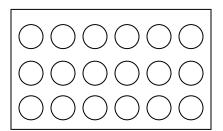
3. How many eggs still remain?

4. What fraction of the set still remains?

5. Write an equivalent fraction for the amount in question 4.

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

<u>PART 2:</u> Your brother needs help baking brownies for the school bake sale. One recipe he has calls for eight eggs. Remove eight eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



1. What fraction of the entire set is 8 eggs?

2. Write an equivalent fraction for the amount in question 1.

3. How many eggs still remain?

4. What fraction of the set still remains?

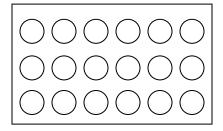
5. Write an equivalent fraction for the amount in question 4.

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

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<u>PART 3</u>: Your sister also needs help baking cupcakes for the school bake sale. One recipe she has calls for $\frac{5}{6}$ of a dozen. Remove $\frac{5}{6}$ of the eggs from the carton below. To show you have removed eggs color them red. Shade in the remaining eggs yellow.



1. What fraction of the entire set is $\frac{5}{6}$ of the eggs?

2. Write an equivalent fraction for the amount in question 1.

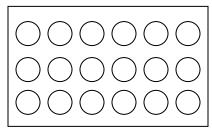
3. How many eggs still remain?

4. What fraction of the set still remains?

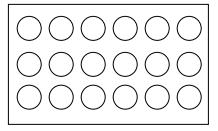
5. Write an equivalent fraction for the amount in question 4.

6. Write a fraction sentence to show how many eggs were removed and how many still remain.

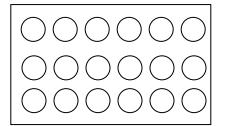
<u>PART 4</u>: Use the cartons below to show all the different ways to represent $\frac{1}{3}$ of a carton of eggs. Then write a number sentence for each model to show how many eggs were removed and how many still remain.



Number Sentences



Number Sentences



Number Sentences

<u>PART 5</u>: How many cartons of eggs did your mother have to buy in order for your brother to make cookies and brownies and for your sister to make cupcakes? Use pictures, numbers and words to show your answer.

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CONSTRUCTING TASK: Tile Task

Back to Task Table

Students will add and subtract fractions with common denominators.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students will have experienced seeing fractions as both a part of a whole and as part of a set. This activity will have students see fractions as portions of an area. By having students create multiple designs with the same criteria they will be forced to verify their results repeatedly, as well as show the cost of each design.

Students will also be able to copy their colored tile designs on to grid paper, however they may need their colored tiles to be rearranged to help them determine their fractional worth. For example a student could make the design below but have a difficult time determining what fraction of each color he or she used. Tiles can easily be rearranged to aid the students' fractional understanding.



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By allowing students to make several designs they will be forced into verifying their answers as well as thinking critically about what looks artistically pleasing while keeping the cost of each tile in mind.

Before asking students to work on this task, be sure students are able to identify the number of equal pieces needed to cover one whole, be comfortable with different size "wholes" such as 12 in a dozen, show equivalent fractions with an area model and as fractions of a set, write an equation which shows the equivalent fractions, and write an equation that shows addition of fractions with like denominators.

ESSENTIAL QUESTIONS

- What is a fraction and how can it be represented?
- How can a fraction represent parts of a set?
- How can I represent fractions in different ways?
- How can I find equivalent fractions?
- How can I add and subtract fractions of a given set?

MATERIALS

- Colored tiles
- Tile Task recording sheet
- Crayons or colored pencils

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimals,* and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students are asked to design tiled coffee tables for a local furniture store. It allows for a lot of creativity, but the tiles cost different amounts so some designs are not profitable. Students will need to design several colorful coffee tables. However, some tiles cost more than others.

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Comments

A great way to introduce this activity is to bring in some small ceramic tiles and discuss their uniform size. Colored tiles should be available to the students. The students can use these to represent the ceramic tiles. Students will be asked to make arrays and model an area model for fractions.

If available, students can glue die-cut squares of blue, yellow, red and green. Alternately, students can manipulate the color tiles online and easily print and then label their work.

Task Directions

Students will follow directions below from the Fraction Clues task sheet.

- Obtain a set of colored tiles.
- Work with a partner to make several designs and record it on their activity sheet.
- Keep record of fractional values as well as cost.
- Determine which of their designs is the most cost effective and artistic.

FORMATIVE ASSESSMENT QUESTIONS

- Tell me about your design.
- What tiles are you using most frequently?
- What fraction of the total are your blue tiles? Red tiles? Etc.
- Could you make this design a different way? If so, would it be cheaper or more expensive?

DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson can be extended to have students make a slightly larger coffee table that is perhaps four by eight or even four by nine tiles in area.
- Students could be asked to determine the perimeter of their coffee tables if they were to use standard four-inch square ceramic tiles.
- Students could be asked to determine the cost of putting molding around the tiles given a certain cost per foot.

Intervention

- If necessary students could begin this activity with a smaller set of tiles.
- Also, if students are struggling they could attempt the task with only three colors instead of using all four colored tiles.

Intervention Table

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TECHNOLOGY

• <u>http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.ht</u> <u>m</u> Sheppard Software: Fruit Splat - This game can be used to practice adding like denominators using levels 1a, 2a, 1b and 2b.

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Name	
------	--

Date

Tile Task

Part 1

Sammy's Small Furniture Store is selling tiled coffee tables. The tables have four inch tiles on them in an assortment of colors; yellow, red, green and blue. The store is selling coffee tables that are 4 tiles wide and 6 tiles long. Sammy needs your help to design some coffee tables. He wants each table top to have some of each color, and of course he wants it to look great. However, some tiles cost more than others. For example, the yellow tiles are very expensive. Help Sammy out by designing 3 table tops of your own. Make sure to include ALL the colors, and pay attention to the price!

Your job has several parts:

1. Use colored tiles and the grid paper below to design at least three coffee tables.

2. For each design, find the fraction each color represents of the entire design and record the fractions in Table 1.

3. Find the fraction that represents the total number of tiles used for each design and record the fraction in Table 1.

4. Next, answer the questions in Table 2 to compare and contrast your designs.

5. Determine the total cost of each design.

6. Write a letter to Sammy's Small Furniture Store explaining to them which of the three designs you think they should select to sell in their store. Explain your reasoning.

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Design 1

	1	1	1	

Design	Fraction of Yellow Tiles \$3	Fraction of Red Tiles \$1	Fraction of Blue Tiles \$2	Fraction of Green Tiles \$1	Sum of Fractions used in the Design
1					
2					
3					

Design	Find the difference between the fraction of yellow tiles and fraction of red tiles used in the design.	Find the sum of the fraction of red tiles and the fraction of green tiles used in the design.	What fraction of the tiles is NOT blue?
1			
2			
3			

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Design 2

Design	Fraction of Yellow Tiles \$3	Fraction of Red Tiles \$1	Fraction of Blue Tiles \$2	Fraction of Green Tiles \$1	Sum of Fractions used in the Design
1					
2					
3					

Design	Find the difference between the fraction of yellow tiles and fraction of red tiles used in the design.	Find the sum of the fraction of red tiles and the fraction of green tiles used in the design.	What fraction of the tiles is NOT blue?
1			
2			
3			

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Design 3

Design	Fraction of Yellow Tiles \$3	Fraction of Red Tiles \$1	Fraction of Blue Tiles \$2	Fraction of Green Tiles \$1	Sum of Fractions used in the Design
1					
2					
3					

Design	Find the difference between the fraction of yellow tiles and fraction of red tiles used in the design.	Find the sum of the fraction of red tiles and the fraction of green tiles used in the design.	What fraction of the tiles is NOT blue?
1			
2			
3			

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<u>CONSTRUCTING TASK:</u> Sweet Fraction Bars

Back to Task Table

Students will solve story problems involving fractions with a denominator of ten.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have experience working with adding and subtracting fractions and using a number line, an area model, and a set model. Also, students should be able to record the operation using fractional notation. The tasks "Eggsactly" and "Tile Task" should have provided students a chance to decompose fractions using a set model and an area model. With this task students will only use the denominator of 10.

ESSENTIAL QUESTIONS

- How can you use fractions to solve addition and subtraction problems?
- What happens to the denominator when I add fractions with like denominators?

MATERIALS

- "Sweet Fraction Bars, Story Problems" student recording sheet
- "Sweet Fraction Bars, Ten-Frames" student recording sheet

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GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)*

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will be given problems to solve involving a candy bar divided into ten equal sections.

Comments

Students may use a ten-frame or a number line to solve these problems. Alternatively, students may choose to use math pictures to solve the problems. Allow students to choose a model that makes sense to them. As students work, look for strategies that students use that may be beneficial to other students. Allow students who used these helpful strategies to share their thinking during the summary part of the lesson.

Task Directions

Students will follow the directions below from the "Sweet Fraction Bars" student recording sheet below.

A Sweet Fraction Bar is a chocolate candy bar that is divided into ten equal sections. Solve the following problems.

- 1. Hannah had 10 of a Sweet Fraction Bar. She gave 10 of the candy bar to Carlos. How much of the candy bar does she have left?
 3
- 2. Sarah has $\overline{10}$ of a Sweet Fraction Bar. Brianna has $\overline{10}$ of the same candy bar. 2

Also, Mika has $\overline{10}$ of the same candy bar. What fraction of a Sweet Fraction Bar do the girls have altogether? **4 3**

- 3. Marissa gave Paulo 10 of a Sweet Fraction Bar. Michael gave Paulo 10 of a Sweet Fraction bar. What fraction of a Sweet Fraction Bar does Paulo have now?
 9
- 4. Caleb had $\overline{10}$ of a Sweet Fraction Bar. He gave Mika $\overline{10}$ of the candy bar. What fraction of the candy bar does he have left?

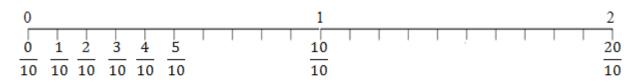
FORMATIVE ASSESSMENT QUESTIONS

- Does the story involve combining or taking away? How do you know?
- How many tenths of a candy bar do you have in all? How many tenths of a candy bar do you have left? How do you know?
- Can you show what happened in the story on a number line? Using the ten-frames? Using a set of counters? In a math picture?
- How many tenths would you need to equal a whole candy bar? How many tenths do you have?
- Is there any other way you could write that fraction?

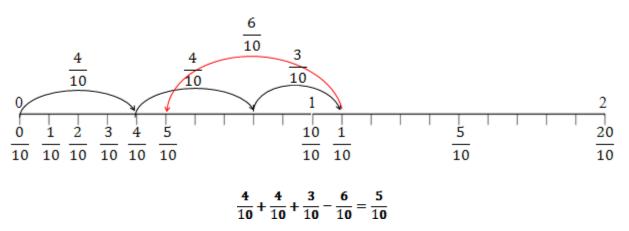
DIFFERENTIATION

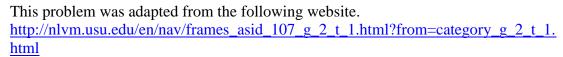
Extension

- Ask students to create an addition and a subtraction problem. Have students solve the problems on the back of the paper.
- Give students four numbers and a target number (e.g. ³/₁₀, ⁴/₁₀, ⁶/₁₀ and the target number ⁵/₁₀) Ask students to use the number line below to show an addition/subtraction sequence that would result in the target number.



For this example, students could show the following.





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Intervention

- Allow students to act out the problems with a partner or in a small group. Students may cut the ten-frames and use the fractional pieces when acting out the stories.
- Many students would benefit from a number line and/or fraction tiles that are already labeled with the unit fraction ¹/₁₀.

Intervention Table

TECHNOLOGY

- <u>http://www.ixl.com/math/grade-4/add-and-subtract-fractions-with-like-denominators</u> This game can be used to practice the concept.
- <u>http://www.sheppardsoftware.com/mathgames/fractions/mathman_add_subtract_fraction</u> <u>s.htm</u> Mathman: Add and Subtract Fractions - This interactive arcade game involves addition and subtraction of fractions with like denominators.
- <u>https://www.conceptuamath.com/app/tool/subtraction-with-common-denominators</u> *This tool provides students with the opportunity to model the addition and subtraction of like fractions.
- <u>http://www.dreambox.com/teachertools</u> *Using both a concrete bar model and equations, this tool enables students to represent fraction addition and think strategically about which fractional pieces are best to solve a given problem. *Using the "removal" or "take away" representation, this tool helps your students learn to subtract fractions.

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Name _____

Date _____

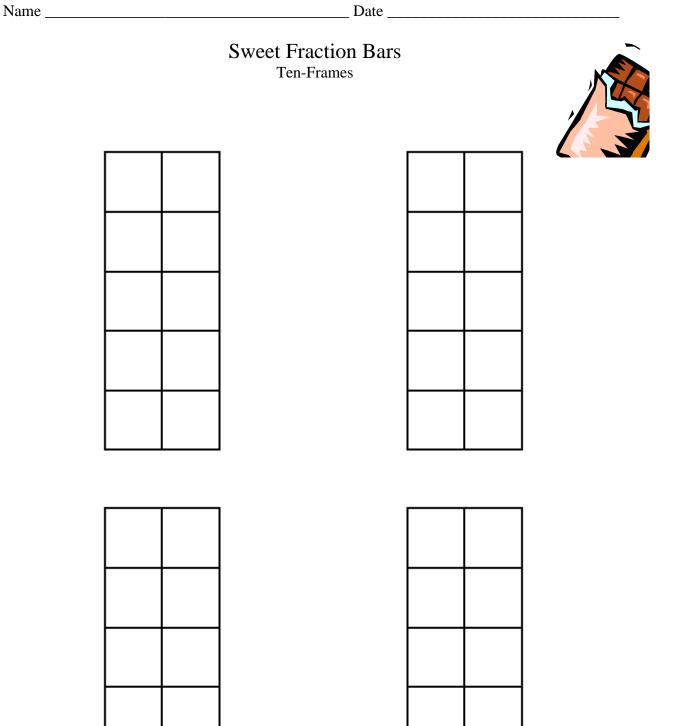
Sweet Fraction Bars Story Problems

A Sweet Fraction Bar is a chocolate candy bar that is divided into ten equal sections. Solve the following problems.



7	5
1. Hannah had $\overline{10}$ of a Sweet Fraction Bar.	2.Sarah has $\frac{5}{10}$ of a Sweet Fraction bar.
She gave 10 of the candy bar to Carlos. What fraction of the candy bar does she have left?	Brianna has $\overline{10}$ of the same candy bar. Also, Mika has $\overline{10}$ of the same candy bar. What fraction of a Sweet Fraction Bar do the girls have altogether?
4	9
3. Marissa gave Paulo $\overline{10}$ of a Sweet Fraction 3	6
Bar. Michael gave Paulo 10 of a Sweet Fraction bar. What fraction of a Sweet Fraction Bar does Paulo have now?	gave Mika 10 of the candy bar. What fraction of the candy bar does he have left?

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CONSTRUCTING TASK: Fraction Cookies Bakery

Back to Task Table

Revisited - Adapted from "Fraction Cookies" Grade 2, Unit 5

Students will perform addition with improper and proper fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

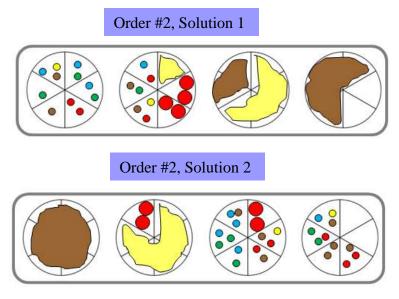
- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have an understanding about how to represent a fraction using an area model, such as a circle.

When creating the cookie confirmations, students are able to use the associative property to add different fractions first if that makes more sense. For example, a solution for order # 2 is shown below. In this case, a student added the fractions in the order they appear on the order form.

If a student added the fractions in the following order, $\frac{6}{6} + \frac{5}{6} + \frac{2}{6} + \frac{9}{6}^{1}$ the solution could be as shown in the second example. Be sure students understand that either solution is correct because the correct fraction of each type of topping is represented.



ESSENTIAL QUESTIONS

- What is a fraction and how can it be represented?
- What is an improper fraction and how can it be represented?
- What is a mixed number and how can it be represented?
- What is the relationship between a mixed number and an improper fraction?
- How can improper fractions and mixed numbers be used interchangeably?
- How do we add fractions?
- How do we apply our understanding of fractions in everyday life?

MATERIALS

- "Fraction Cookie Bakery, Order Form" student recording sheet
- "Fraction Cookie Bakery, Order Confirmation Form" student recording sheet
- colored pencils, crayons, or markers

GROUPING

Individual/Partner Task

NUMBER TALKS

By now number talks should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. In addition, Catherine Fosnot has developed "strings" of numbers and fractions that could be

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included in a number talk to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

This task provides students with their first opportunity to explore addition with improper and proper fractions. Students will add proper and improper fractions to create cookie order confirmation notices. They will be required to write each sum as an improper fraction and a mixed number.

Comments

This task may be introduced by reading *The Hershey's Milk Chocolate Fractions Book* by Jerry Pallotta, focusing on the addition that is modeled in the book. Continue by explaining the task and modeling the example problem as shown below.

<u>O das</u>		Toppings							Oi	rder Tot	als
Order Number	M & Ms	Walnuts	Chocolate Chips	Raspberries	Peanut Butter	V anilla Icing	Chocolate Icing	Sprinkles	Improper fraction	Mixed Number	Number of Whole Cookies
Example	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{2}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$3\frac{1}{2}$	4

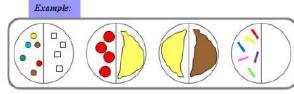
Cookie Orders

As students are working, ask them how they created the confirmations for each order. Ask questions that will cause students to think about how many fractional parts make a whole, different ways they can group toppings to create a whole cookie, and how they know what the sum is, written as an improper fraction and as a mixed number. Some sample questions are given in the "FORMATIVE ASSESSMENT QUESTIONS" section below.

This task provides students with an opportunity to explore sums of improper and proper fractions using models. Therefore, students SHOULD NOT use an algorithm to change improper fractions to mixed numbers. Instead, students will be using their models to determine the sums.

During the lesson summary, be sure students are aware that an improper fraction can be written $\frac{7}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{1}{7}$

as a mixed number because $\overline{2} = \overline{2} + \overline{2} + \overline{2} + \overline{2}$ as shown in the cookie model the students created (see example below). This way, students will develop an understanding of what an improper



fraction represents. Students can use this same understanding when writing a mixed number as an improper fraction.

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They should recognize that:

$$3 \frac{1}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} = \frac{7}{2}.$$

Because each $\frac{2}{2}$ is equal to one whole, there needs to be three $\frac{2}{2}$ to represent the 3 wholes in the mixed number. When the fractions are added the sum is $\frac{7}{2}$.

		Toppings								rder Tot	als
Order Number	M & Ms	Walnuts	Chocolate Chips	Raspherries	Peanut Butter	Vanilla Icing	Chocolate Icing	Sprinkles	Improper fraction	Mixed Number	Number of Whole Cookies
Example	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	$\frac{2}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$3\frac{1}{2}$	4
#1	$\frac{4}{5}$		3 5		$\frac{2}{5}$			3 5	$\frac{12}{5}$	$2\frac{2}{5}$	3
#2	$\frac{9}{6}$			$\frac{2}{6}$		5 6	$\frac{6}{6}$		$\frac{22}{6}$	$3\frac{4}{6}$	4
#3	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$				$\frac{1}{4}$		$\frac{10}{4}$	$2\frac{2}{4}$	3
#4	$\frac{7}{3}$			$\frac{4}{3}$		$\frac{7}{3}$			$\frac{18}{3}$	6	6
#5		$\frac{4}{2}$	$\frac{2}{2}$	$\frac{5}{2}$					$\frac{11}{2}$	$5\frac{1}{2}$	6
#6			$\frac{4}{3}$		$\frac{1}{3}$	$\frac{3}{3}$		$\frac{7}{3}$	$\frac{15}{3}$	5	5
#7		6 8		4 8		$\frac{12}{8}$			$\frac{22}{8}$	2 6/8	3
#8	$\frac{3}{4}$		$\frac{9}{4}$		$\frac{5}{4}$			$\frac{2}{4}$	$\frac{19}{4}$	$4\frac{3}{4}$	5

Task Directions

Students will follow the directions below from the "Fraction Cookies Bakery, Order Form" student recording sheet and "Fraction Cookies Bakery, Order Confirmation Form" student recording sheet below.

"Fraction Cookies Bakery, Order Form" student recording sheet

You own a bakery that specializes in fraction cookies. Customers place orders from all over the country for your unique cookies. You recently received the orders shown below. Before making the cookies to fill the order, you need to confirm each order by sending a confirmation notice to each customer. (If the toppings ordered do not cover an entire cookie, customers want the remaining portion of the cookie to be left plain.) Using

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the circle templates below, show how you would create each cookie order with the correct fractional amounts of toppings.

"Fraction Cookies Bakery, Order Confirmation Form" student recording sheet

Customers expect you to use the fewest number of cookies possible to complete each order. No part of a cookie should be without a topping except for one. You may split a topping between two cookies as shown below (the vanilla icing was shared between two cookies rather than covering both halves of one cookie with vanilla icing).

FORMATIVE ASSESSMENT QUESTIONS

- How do you know you have recorded the order correctly?
- How many sections do you need to cover a whole cookie? How do you know?
- How did you determine the improper fraction?
- How did you determine the mixed number?
- How did you determine how much of a cookie would be plain?

DIFFERENTIATION

Extension

- Challenge students with one or more of the orders on the "Fraction Cookie Bakery, Order Form Version 2" student recording sheet. Be sure students USE MODELS ONLY to solve these problems.
- Ask students to create orders of their own, then switch with a partner to create the confirmations for those orders. Students can be given a blank confirmation sheet or they can create their own fraction models.

Intervention

- Some students may need more examples modeled before they are able to complete this task on their own. Provide an opportunity for further small group instruction before students are asked to complete this task.
- Allow students to use pre-made circle fraction pieces to create the cookies. It might be necessary to combine several sets of pieces in order to make multiple cookies.

Intervention Table

TECHNOLOGY

- <u>http://www.ixl.com/math/grade-4/add-and-subtract-fractions-with-like-denominators</u> This game can be used to practice the concept.
- <u>http://www.sheppardsoftware.com/mathgames/fractions/mathman_add_subtract_fraction</u> <u>s.htm</u> Mathman: Add and Subtract Fractions - This interactive arcade game involves addition and subtraction of fractions with like denominators.
- <u>https://www.conceptuamath.com/app/tool/subtraction-with-common-denominators</u> *This tool provides students with the opportunity to model the addition and subtraction of like fractions.

Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name

Date

Fraction Cookies Bakery

Order Form



You own a bakery that specializes in fraction cookies. Customers place orders from all over the country for your unique cookies. You recently received the orders shown below. Before making the cookies to fill the order, you need to confirm each order by sending a confirmation notice to each customer. Using the circle templates below, show how you would create each cookie order with the correct fractional amounts of toppings.

Order Number	Toppings								Order Totals		
	M & M's	Walnuts	Chocolate Chips	Raspberries	Peanut Butter	Vanilla Icing	Chocolate Icing	Sprinkles	Improper fraction	Mixed Number	Number of Whole Cookies
Example	$\frac{1}{2}$	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{2}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$3\frac{1}{2}$	4
#1	4 5		3 5		2 5			3 5			
#2	9 6			$\frac{2}{6}$		5 6	6 6				
#3	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{4}{4}$				$\frac{1}{4}$				
#4	7 3			4 3		$\frac{7}{3}$					
#5		$\frac{4}{2}$	$\frac{2}{2}$	5 2							
#6			4 3		1 3	3 3		$\frac{7}{3}$			
#7		6 8		4 8		$\frac{12}{8}$					
#8	$\frac{3}{4}$		9 4		5 4			$\frac{2}{4}$			

Cookie Orders

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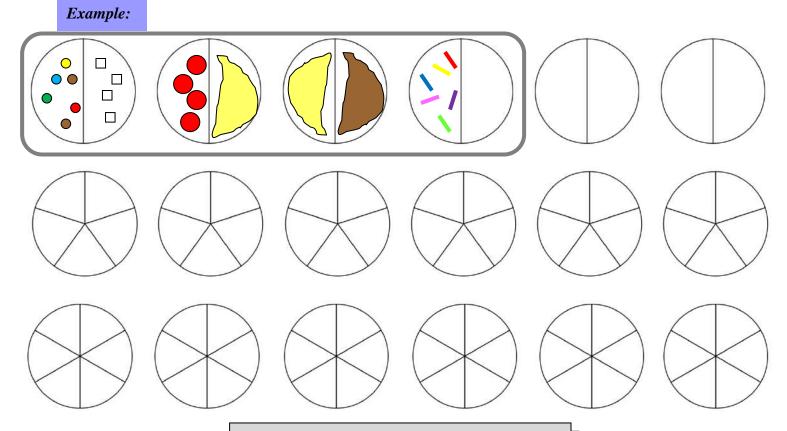
Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name ____

Date

Fraction Cookie Bakery Order Confirmation Form

Customers expect you to use the fewest number of cookies possible to complete each order. No part of a cookie should be without a topping except for one. You may split a topping between two cookies as shown below (the vanilla icing was shared between two cookies rather than covering both halves of one cookie with vanilla icing).

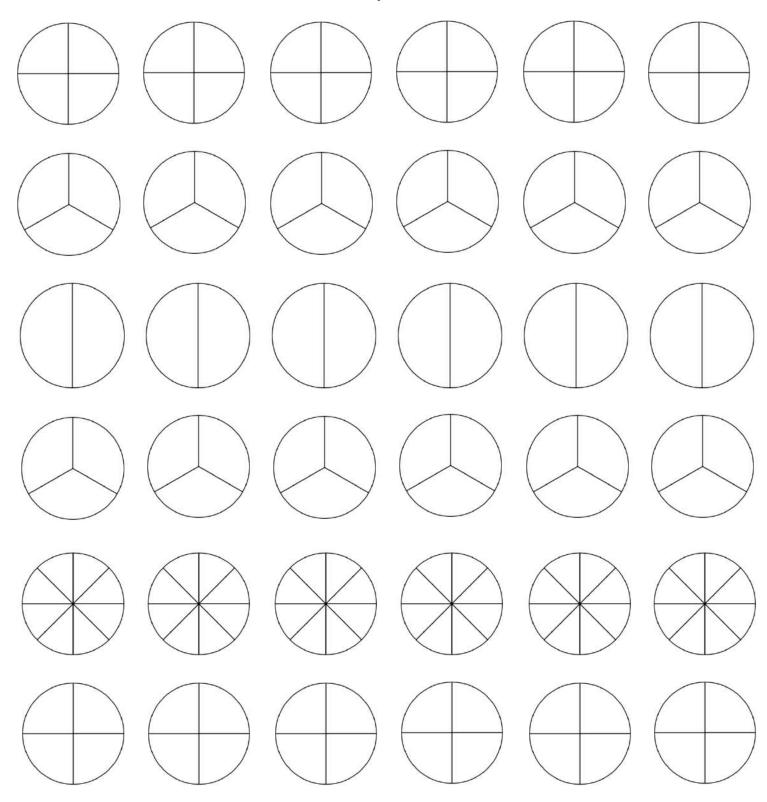


Cookie Order Codes

M & M's Walnuts Chocolate Chips Raspberries Peanut Butter Mint Icing Vanilla Icing Chocolate Icing Sprinkles Colorful Circles Squares Brown Triangles Red Circles Light Brown Light Green Yellow Dark Brown Colorful Specs

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Fraction Cookie Bakery: Order Confirmations, Page 2



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Name ____

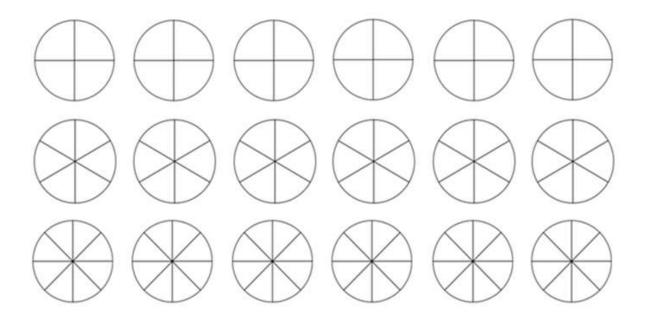
_____ Date _____

Fraction Cookies Bakery Order Form – Version 2

You recently received the orders shown below. Confirm each order below. Using the circle templates below, show how you would create each cookie order with the correct fractional amounts of toppings.

Customers expect you to use the fewest number of cookies possible to complete each order. No part of a cookie should be without a topping except for one. You may split a topping between two cookies if necessary.

Order Number		Toppings								Order Totals		
	M & Ms	Walnuts	Chocolate Chips	Raspberries	Peanut Butter	Vanilla Icing	Chocolate Icing	Sprinkles	Improper fraction	Mixed Number	Number of Whole Cookies	
#1	$\frac{1}{4}$		$\frac{1}{2}$		$\frac{1}{4}$			3 4			P	
#2	9 6	а. И.		$\frac{1}{3}$	4. 47:	$\frac{1}{6}$	$\frac{2}{3}$	e.			2	
#3	3 8	$\frac{1}{4}$	$\frac{4}{4}$		0	0	58	2			2	



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PRACTICE TASK: Rolling Fractions

Back to Task Table

Students will add and subtract fractions using mixed numbers and improper fractions

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

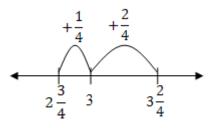
Students should have prior experiences and/or instruction with subtracting fractions with like denominators. In this task students will discover how to convert improper fractions to mixed numbers. Students should have prior experience representing one whole as a fraction with the same numerator and denominator.

Based on students' understanding of subtraction and of fractions, students may develop several strategies for subtracting mixed numbers. Some examples are shown below. Remember, these are strategies that students may use; students do not need to know how to subtract mixed numbers using all of these strategies, they are provided for teacher information only.

How might students solve the problem $3\frac{2}{4} - 2\frac{3}{4}$?

Some students may subtract the whole numbers first, 3-2=1 and then think about the fractions $\frac{2}{4} - \frac{3}{4}$. To subtract they may think of $\frac{3}{4}$ as $\frac{2}{4} + \frac{1}{4}$. If they subtract $\frac{2}{4}$ first, they will be left with $1 = \frac{4}{4}$, students can subtract $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$. Other students may choose to count up from $2\frac{3}{4}$ to $3\frac{2}{4}$. Adding $\frac{1}{4}$ to $2\frac{3}{4}$ makes 3. Adding $\frac{2}{4}$ to 3 makes $3\frac{2}{4}$. Because $\frac{1}{4}$ and $\frac{2}{4}$ were added to count up to $3\frac{2}{4}$ the

difference is $\frac{1}{4} + \frac{2}{4}$ or $\frac{3}{4}$. This method is shown on the open number line below.



Finally, some students may start by regrouping. The mixed number $\frac{3}{4}$ can be

rewritten as $2 + \frac{4}{4} + \frac{2}{4}$

Some students may take that one step further and regroup as shown •

 $2 + \frac{4}{4} + \frac{2}{4} = 2 + \frac{6}{4}$. Then students can subtract $2\frac{6}{4} - 2\frac{3}{4} = \frac{3}{4}$ by subtracting 2-2=0 and $\frac{6}{4}-\frac{3}{4}=\frac{3}{4}$ Some students may subtract $2\frac{2}{4} + \frac{4}{4} - 2\frac{3}{4} = \frac{3}{4}$ by first subtracting 2 - 2 = 0and then subtracting $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$. The difference $\frac{1}{4}$ is added to the other fraction $\overline{4}$ giving $\overline{4}$.

These examples demonstrate several ways students could apply their understanding of subtraction and fractions to subtraction and mixed numbers. It is important to remember that all students do not need to know how to subtract using all of the procedures above. They may use

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the same strategy each time or they may become proficient in more than one strategy that allows them to choose the best strategy for the problem they are trying to solve. **ESSENTIAL QUESTIONS**

- How do we subtract mixed numbers and improper fractions?
- What is an improper fraction and how can it be represented?
- What is a mixed number and how can it be represented?

MATERIALS

- "Rolling Fractions, Directions" student sheet
- "Rolling Fractions, Game Sheet" student recording sheet (2 pages; copy page 2 on the back of page 1)
- Four six-sided dice two in one color, two in a different color (If available, 4 ten-sided dice (0-9) could be used per team)

GROUPING

Partner/Small Group Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students play a game that requires them to subtract and compare mixed numbers with like denominators. This task gives students lots of opportunities for practice with subtraction of fractions with like denominators. It also gives students the opportunity to investigate mixed numbers and improper fractions.

Task Directions

Students will follow the directions below from the "Rolling Fractions" student recording sheet.

Players: 2 or more Materials: "Rolling Fractions, Directions" student sheet "Rolling Fractions, Game Sheet" student recording sheet 4 dice (2 dice one color, 2 dice different color) Pencil

Directions:

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- 1. With your partner, choose a denominator of 2, 3 or 4. You and your partner <u>must</u> have the same denominator.
- 2. Roll a set of four dice. Use the chosen denominator and the dice of one color to write the first mixed number; use the dice of the other color to write the second mixed number. Find the difference of the two fractions created.
- 3. If possible, write your difference as an improper fraction.

Example:

If you roll the dice as shown below, you could create the following subtraction problem:



The 5 and the 3 can be placed as the whole number or the numerator of the fraction for one mixed number, the 4 and 1 can be placed as the whole number or the numerator of the fraction for the other mixed number.

5 ³/₄ - 4 ¹/₄ = 1
$$\frac{2}{4} = \frac{4}{4} + \frac{2}{4} = \frac{6}{4}$$

You would fill in the table as shown below. Each player should have their own recording sheet. (Please do not teach students to draw a giant 1 around 4/4 or any other fraction representation of a whole number. They should make sense of equivalence mathematically, not by using a visual trick.)

Rolling Fractions									
Round	l ^{≈t} Mixed Number	2 nd Mixed Number	Difference	Improper Fraction	Score				
Example	$5\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{2}{4}$	$\begin{bmatrix} \frac{4}{4} \\ \frac{4}{4} \end{bmatrix} + \frac{2}{4} = \frac{6}{4}$	1				

(Please do not teach students to draw a giant 1 around 4/4 or any other fraction representation of a whole number. They should make sense of equivalence mathematically, not by using a visual trick.)

- 4. Your partner should do the same (see steps 2 &3). Now compare your fractions.
- 5. The partner with the largest fraction receives 1 point. Continue the game until one player has 10 points.
- 6. The player that earns 10 points first is the winner.

Comments

There are several situations that add depth to this game, but when students are first learning this game, those situations should be avoided. If the dice show numbers that don't allow students to create a mixed number where the fraction part is a proper fraction, then tell students to roll that pair of dice again. For example, if the denominator chosen is 4, and the two dice show a 5 and a

6, allow students to roll again rather than creating a mixed number such as $5\frac{1}{4}$ or $6\frac{1}{4}$. After students have played the game several times, remove the limitation of mixed numbers with proper fractions. This will allow rich conversations regarding the meaning of improper fractions as part of a mixed number.

If students are given the opportunity to create any mixed number, the following discussion gives some examples of the rich conversations that could develop. Present the following problem to the students to determine their understanding of subtraction with mixed numbers and of fractions in general.

It's your turn. You roll the dice shown below. You and your partner have agreed the denominator for this round is 4. What subtraction problem with two mixed numbers can you write to create the largest possible difference?



In this situation, students may create the following straightforward subtraction problem. But that doesn't give the largest difference.

$$5\frac{3}{4} - 4\frac{1}{4} = 1\frac{2}{4}$$

Some students may write the following mixed numbers.

$$5\frac{3}{4}-1\frac{4}{4}$$

- Students, who recognize that $\overline{4}$ is equivalent to 1, may rewrite the problem as
 - $5\frac{3}{4}-2=3\frac{3}{4}$ and solve the problem by subtracting the whole numbers.
- Others may rewrite $5\frac{3}{4}$ as $\left(4+\frac{4}{4}\right)+\frac{3}{4}$ and then subtract $1\frac{4}{4}$ by subtracting the whole

numbers, 4 - 1 = 3 and then subtracting the fractions $\frac{1}{4} - \frac{1}{4} = \frac{1}{4}$, leaving $\frac{1}{4}$. Therefore,

their answer is
$$3 + \frac{3}{4}$$
 or $3\frac{3}{4}$.

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• Some students may rewrite $5\frac{3}{4}$ as $\left(4+\frac{4}{4}\right)+\frac{3}{4}$ and then add $\frac{4}{4}+\frac{3}{4}=\frac{7}{4}$. The problem

would then be $4\frac{7}{4} - 1\frac{4}{4} = 3\frac{3}{4}$.

Some students may write the following mixed numbers.

$$4\frac{1}{4} - 3\frac{5}{4}$$

To solve this, students may struggle with subtracting $\overline{4}$ from $\overline{4}$. Help them to make sense of what this means, but allow them to determine how best to solve the problem.

• Students may recognize that $3\frac{5}{4}$ is the same as $3 + \frac{4}{4} + \frac{1}{4}$ and because $\frac{4}{4} = 1$, students may rewrite $3\frac{5}{4}$ as $4\frac{1}{4}$. However, when the problem is rewritten, it is $4\frac{1}{4} - 4\frac{1}{4} = 0$ which does not give the greatest difference!

• Some students may solve this problem by subtracting $\frac{1}{4}$ from $4\frac{1}{4}$ leaving $3\frac{4}{4}$ to subtract

from 4
$$\left(i.e. \ 4\frac{1}{4} - \left(3\frac{4}{4} + \frac{1}{4}\right) = 4\frac{1}{4} - \left(\frac{1}{4} + 3\frac{4}{4}\right) = \left(4\frac{1}{4} - \frac{1}{4}\right) - 3\frac{4}{4} = 4 - 3\frac{4}{4}\right)$$
. Students may

then subtract the whole numbers (4-3 = 1) and be left with $1 - \overline{4}$. If they recognize that $\overline{4}$ is equivalent to 1 then they can subtract 1 - 1 = 0.

All of the examples above highlight the idea that students will make sense of subtraction of mixed numbers based on their understanding of subtraction and numbers. It is important to allow students to solve mixed-number problems in a way that makes sense to them. Rather than teaching addition and subtraction of mixed numbers separately from fractions, allow students to solve mixed-number problems by building on their understanding of fractions. (See Van de Walle & Lovin, 2006, p. 166)

While playing the game, be sure students are aware that they need to subtract the smaller mixed number from the larger mixed number. Therefore, one pair of dice (one color) **does not** always have to be the minuend, and the other always the subtrahend. Instead, either pair of dice can be used for the minuend. (While students should understand the term "difference" in the context of subtraction, they are not expected to know the terms minuend and subtrahend. These terms are related as follows: *minuend* – *subtrahend* = *difference*. This information is for teachers only, students are not expected to use the terms minuend and subtrahend.)

Some variations for this game are listed below.

• Keep the same denominator throughout the game and declare the winner to be the person who wins the most rounds after a given amount of time.

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- This game can be played as an addition game. Instead of subtracting the two mixed numbers, students could add them and then compare their answers in the same way as the subtraction version.
- Allow students to make a rule for what will happen if it is not possible to create a mixed number without an improper fraction. What if the yellow dice showed 5 and 6 and the denominator was 4? Would the player have to forfeit their turn; would the player need to roll those two dice again; or would the students have to agree to allow improper fractions as part of a mixed number? Determining this rule provides for some rich conversations regarding the make-up of mixed numbers and what each part means.
- Rather than having the player with the higher fraction win, students could use a spinner that shows two sections, high and low. This way, it is unknown if the higher or lower fraction wins until it is determined by the spinner at the end of each round.
- Once students are familiar with the game, they do not need the record sheet. They can easily record the game and keep score on a blank sheet of paper.

FORMATIVE ASSESSMENT QUESTIONS

- Can your difference be written as an improper fraction? How do you know?
- How do you subtract the two fractions?
- What does a model of this subtraction problem look like when fraction pieces or a sketch of a model is used?
- How does the denominator affect the value of your fractions?

DIFFERENTIATION

Extension

• Ask students to create a real-life context for one of the subtraction problems from the game. Using the context created, students will model the problem and solution and then explain their model and solution.

Intervention

- If it proves beneficial, allow students to use fraction pieces to model the subtraction and to find the answer.
- Some students may benefit from a number line in order to visualize the fractions and utilize an "counting up" strategy

Intervention Table

TECHNOLOGY

• <u>http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.ht</u> <u>m</u> Sheppard Software: Fruit Splat - This game can be used to practice adding like denominators using levels 1a, 2a, 1b and 2b.

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- <u>http://www.uen.org/Lessonplan/preview.cgi?LPid=21526</u> Delightfully Different Fractions: This lesson uses virtual pattern blocks to practice the concept. It can be used as an introduction to this task or as an extension of the task.
- <u>https://www.conceptuamath.com/app/tool/subtraction-with-common-denominators</u> *This tool provides students with the opportunity to model the addition and subtraction of like fractions.
- <u>http://www.dreambox.com/teachertools</u> *Using both a concrete bar model and equations, this tool enables students to represent fraction addition and think strategically about which fractional pieces are best to solve a given problem. *Using the "removal" or "take away" representation, this tool helps your students learn to subtract fractions.

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Name ____

_ Date _

Rolling Fractions

Players: 2 or more

Materials: "Rolling Fractions, Directions" student sheet

"Rolling Fractions, Game Sheet" student recording sheet (one per student) 4 dice (2 dice one color, 2 dice different color) Pencil

Directions:

- 1. For each round, with your partner, choose a denominator of 2, 3 or 4. You and your partner <u>must</u> have the same denominator.
- 2. Roll a set of dice. Use the chosen denominator and the dice of one color to write the first mixed number; use the dice of the other color to write the second mixed number. Find the difference of the two fractions created. Record.

Example:

If you roll the dice as shown below, you could create the following subtraction problem:



The 5 and the 3 can be placed as the whole number or the numerator of the fraction for one mixed number, the 4 and 1 can be placed as the whole number or the numerator of the fraction for the other mixed number.

5 ³/₄ - 4 ¹/₄ = 1
$$\frac{2}{4} = \frac{4}{4} + \frac{2}{4} = \frac{6}{4}$$

You would fill in the table as shown below. Each player should have their own recording sheet.

Koning Fractions										
Round	l ^{≈t} Mixed Number	2 nd Mixed Number	Difference	Improper Fraction	Score					
Example	$5\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{2}{4}$	$\left[\frac{4}{4}\right] + \frac{2}{4} = \frac{6}{4}$	1					

Rolling Fractions

3. If possible, write your difference as an improper fraction. Your partner should do the same.

- 4. Compare your improper fraction to your partner's.
- 5. The player with the largest fraction receives 1 point.
- 6. Continue the game until one player has 10 points.
- 7. The player that reaches 10 points first is the winner.

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Name _____ _____ Date _____ Rolling Fractions Game Sheet 1st Mixed 2nd Mixed Improper Round Difference Score Fraction Number Number $5\frac{3}{4}$ $\frac{1}{4}$ $1\frac{2}{4}$ $\frac{4}{4} + \frac{2}{4} = \frac{6}{4}$ Example 1 1 2 3 4 5 6 7 8 9 10

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Rolling Fractions Game Sheet (Back)

Round	First Fraction	Second Fraction	Difference	Mixed Number	Score
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

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<u>3-ACT TASK:</u> Running Trails

Back to Task Table

Adapted from Illustrative Mathematics Running Laps

TASK CONTENT: In this task, students will problem solve by applying knowledge of fractions. Approximate time - 1 class period.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

MGSE4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at <u>http://blog.mrmeyer.com/category/3acts/</u>. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the *Guide to Three-Act Tasks* on georgiastandards.org and the K-5 Georgia Mathematics Wiki.

Constructing the idea that fractions are based on relationships and that the size or amount of the whole matters is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked

with the concept of fair share or partitioning in 2^{nd} and 3^{rd} grade, with standards that refer to same-sized shares or equal shares.

COMMON MISCONCEPTIONS

Some common misconceptions, found in *Math Misconceptions*, that children may experience include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes circles, squares, and rectangles they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as 2/4 of the unit instead of 2/6 or 1/3 (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

ESSENTIAL QUESTIONS

- How can equivalent fractions be identified?
- How can fractions be ordered on a number line?

MATERIALS

- 3 Act Task Student Recording Sheet
- Act 1 image
- Fraction strips

GROUPING

• Partners/small group

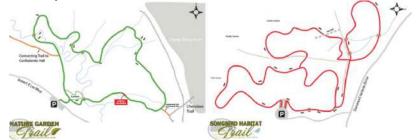
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will consider the scenario and then tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. Their curiosities will be recorded as questions on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Task Directions

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing the Act I picture.

1. Show the Act I picture to students.



Dawn ran 3 laps around the green trail. Mike ran 1 lap around the red trail.

- 2. Pass out the 3 Act recording sheet.
- 3. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first before sharing aloud and then record these questions on the recording sheet (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions students may ask and wish to answer:

How many miles did Dawn run? How many miles did Mike run? Which runner ran the longest distance? How much farther did Dawn run than Mike? or How much farther did Mike run than Dawn?

4. As the facilitator, you can select which question you would like every student to answer, have students vote on which question the class will answer or allow the students to pick which question they would like to answer. Once the question is selected ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they establish a range in which the solution should occur. Instruct students to record their estimates on a number line.

Act II – Student Exploration - Provide additional information as students work toward solutions to their questions.

- 1. Ask students to determine what additional information they will need to solve their questions. The teacher provides that information only when students ask for it.:
 - The green trail is ³/₄ of a mile around.

- The red trail is 1 mile around.
- 2. Ask students to work in small groups or with a partner to answer the questions they created in Act I. The teacher provides guidance as needed during this phase by asking questions such as:
 - Can you explain what you've done so far?
 - What strategies are you using?
 - What assumptions are you making?
 - What tools or models may help you?
 - Why is that true?
 - Does that make sense?

Act III – Whole Group - Share student solutions and strategies as well as the Act III solution.

- 1. Ask students to present their solutions and strategies.
- 2. Compare solutions.
- 3. Lead discussion to compare these, asking questions such as:
 - How reasonable was your estimate?
 - Which strategy was most efficient?
 - Can you think of another method that might have worked?
 - What might you do differently next time?

Comments

Act IV is an extension question or situation of the above problem. An Act IV can be implemented with students who demonstrate understanding of the concepts covered in Acts II and III. The following questions and/or situations can be used as an Act IV:

- If Mike runs his trail in 8 minutes, how long would it take him to run the green trail?
- How many laps would Dawn and Mike each have to run on their trails to run the same distance?

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

• Have students double the size of each trail prior to answering the question.

Intervention

• Allow students to use fraction strips to model the total distance of each trail.

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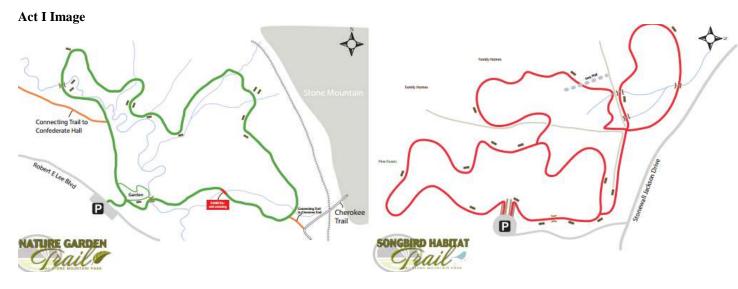
• Require students to first answer the question using the scenario, Dawn ran 1 lap around the green trail. Mike ran 1 lap around the red trail.

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L541</u> More Fun with Fraction Strips: This lesson has students work with fraction strips to compare and order fractions using relationships. It can be used for additional practice, remediation or extending understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L347</u> Exploring the Value of the Whole: This lesson has students consider the size or value of the same fraction when different wholes compared. It can be used for remediation purposes, additional practice, and/or extending understanding of the concept.

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Dawn ran 3 laps around the green trail. Mike ran 1 lap around the red trail.

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Name:

Adapted from Andrew Stadel

Task Title: ACT 1

What questions come to your mind?

Main question:

On an empty number line, record an estimate that is too low, just right and an estimate that is too high. Explain your estimates.

<u>ACT 2</u>

What information would you like to know or need to solve the MAIN question?

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Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Record the actual answer on the number line above containing the three previous estimates.

ACT 4 (use this space when necessary)



<u>CONSTRUCTING TASK</u>: The Fraction Story Game

Back to Task Table

Students will create story problems with fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students may not understand what you mean by "common classroom materials." While many classrooms have standard dice that can be used, give alternative examples such as, a penny can be flipped to determine how many spaces the players get to move (heads = 2 spaces, tails =1 space). For game pieces, extra marker caps, manipulatives, or coins can be used.

Begin by having students review lessons or activities that have been done during the fraction unit that they think were important. Record their thoughts on chart paper or the board. You may want to post a list of the elements of the standard covered during the unit and brainstorm tasks and activities you did that addressed each element.

Since a good game should have at least 20-30 questions, you may want the children to work with a partner or in small groups to create enough questions.

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ESSENTIAL QUESTION

• How are fractions used in problem-solving situations?

MATERIALS

Materials Required Per Group

- "The Fraction Story Game, Directions" student sheet
- "The Fraction Story Game, Game board" student sheet
- Colored pencils or crayons
- Index cards (about 60)
- Common classroom materials recycled items for game pieces (about 6)

GROUPING

Small Group Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents,* 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students create a game while reviewing all the different aspects of fractions they have studied.

Comments

- This task represents the level of depth, rigor, and complexity expected of all fourth grade students to demonstrate evidence of learning.
- Students should have had multiple opportunities to write story problems by this time in the school year.
- Questions should match a standard/element.
- Creating questions to match elements of the standard taught is a wonderful way to review. It is a strategy that can be used from elementary school through college and is very effective.
- Index cards may be used for the problem cards. Insist that the students write legibly. All problem cards should have the solutions on the back.
- Solutions should be accompanied by an explanation/illustration.

- Game boards, playing pieces, and cards can be stored in large Ziploc bags or manila folders.
- The cards students create for their games can be used in a variety of ways. The problem cards can be used to create a Jeopardy type game which can be played as a review of the unit. Also, the problem cards can be reproduced and used as a set of review questions before the unit assessment.

Task Directions

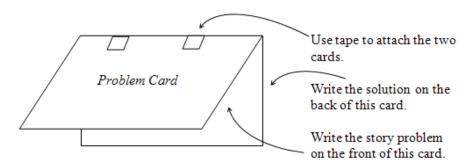
Prior to showing the task, ask students what they have learned during this unit. Make a list on the board or a piece of chart paper of the ideas students brainstorm during the discussion. Explain to students that these are the elements that they will use to create story problems for "The Fraction Story Game" task they are going to complete.

Students will follow the directions below from "The Fraction Story Game, Directions" student sheet.

Your task is to create a fraction story game using what you learned about adding, subtracting and decomposing proper fractions, improper fractions and mixed numbers. Use the fraction game board on "The Fraction Story Game, Game Board" student sheet to create a game that other students will want to play.

Directions:

- Look at the list of elements of the standard that you studied in class. The problem cards you create must match those elements.
- You will need to make approximately 30 problem cards for your game. Most of the cards should be written in story problem form.
- Be sure you have some problem cards for each of the elements of the standard addressed in this unit. Make sure you use both decimal fractions and common fractions in your problem cards.
- Each problem card must have the correct answer on the back. Cover each problem card with a blank index card so players cannot see the problems before their turn. See sample below.
- Write the rules for your game.



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Things to remember:

- You can only use common classroom materials.
- You may decorate your game board in a way that makes the game interesting and fun to play.
- Be sure to play your game with a partner to be sure it works.

FORMATIVE ASSESSMENT QUESTIONS

- What are the skills you learned during this unit?
- What kind of problem can you create for _____ (one of the elements of the standard)?
- How do you know this is the correct solution for your problem?

DIFFERENTIATION

Extension

• Students can create their own game board format with penalties, rewards, and more complex rules.

Intervention

- Allow students to work in a small group so each student will need to make only one card per element of the standard.
- For some of the elements of the standard, give the students the problem and require them to create the solution to the problem.
- Students with a significant problem with manual dexterity may need to type their problems, then cut and paste then onto the index cards.

Intervention Table

TECHNOLOGY

- <u>https://learnzillion.com/lesson_plans/1794</u> Three Jars Decompose Mixed Numbers: This lesson plan from LearnZillion is a tutorial that includes videos of how to decompose a mixed number.
- <u>http://www.uen.org/Lessonplan/preview.cgi?LPid=21526</u> This lesson uses virtual pattern blocks to practice the concept. It can be used as an introduction to this task or as an extension of the task.
- <u>https://www.conceptuamath.com/app/tool/subtraction-with-common-denominators</u> *This tool provides students with the opportunity to model the addition and subtraction of like fractions.
- <u>http://www.dreambox.com/teachertools</u> *Using both a concrete bar model and equations, this tool enables students to represent fraction addition and think strategically about which fractional pieces are best to solve a given problem. *Using the "removal" or "take away" representation, this tool helps your students learn to subtract fractions.

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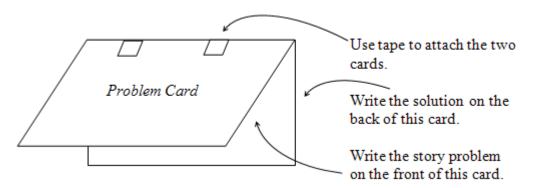
The Fraction Story Game Directions



Your task is to create a fraction story game using what you learned about common fractions and decimal fractions. Use the fraction game board on "The Fraction Story Game, Game Board" student sheet to create a game that other students will want to play.

Directions:

- Look at the list of elements for the standard that you studied in class. The problem cards you create must match the elements of the standard.
- You will need to make approximately 30 problem cards for your game. Most of the cards should be written in story problem form.
- Be sure you have some problem cards for each of the elements of the standard addressed in this unit. Make sure you use both decimal fractions and common fractions in your problem cards.
- Each problem card must have the correct answer on the back. Cover each problem card with a blank index card so players cannot see the problems before their turn. See sample below.



• Write the rules for your game.

Things to remember:

- You can only use common classroom materials.
- You may decorate your game board in a way that makes the game interesting and fun to play.
- Be sure to play your game with a partner to be sure it works.

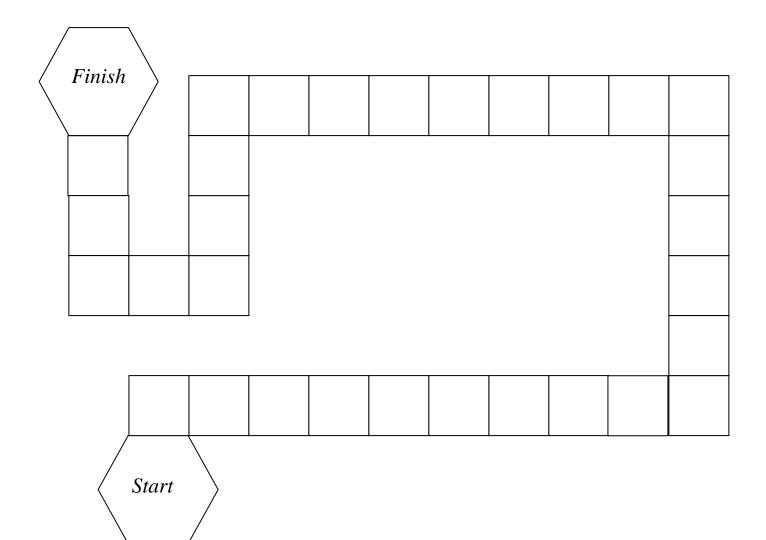
Georgia Department of Education Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name _____ Date _____

The Fraction Story Game

Game Board





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CONSTRUCTING TASK: Fraction Field Events

Students will solve story problems with involving mixed numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students have been adding and subtracting fractions with like denominators prior to this task within this unit. Students are now extending that knowledge to include adding and subtracting mixed numbers.

ESSENTIAL QUESTIONS

• How do we add/subtract fractions and mixed numbers?

MATERIALS

"Fraction Field Event" student recording sheet

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GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents,* 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Task Directions

1.

Students will follow the directions below from the "Fraction Field Event" student recording sheet.

Carter Elementary School is having a field day! One of the events is the long jump. Participants in this event take a running start and then jump as far as they can. The winner is determined by adding the distances jumped in three trials. The greatest total wins. Using the jump measures below, determine the winner of this year's girls' and boys' long jump. Show all of your work on a separate sheet of paper.

Name	1 st Jump	2 nd Jump	3 rd Jump	Total Score
Kim	$7\frac{3}{12}$ feet	$6\frac{11}{12}$ feet	$6\frac{5}{12}$ feet	
Amanda	$5\frac{9}{12}$ feet	$6\frac{1}{12}$ feet	$6\frac{4}{12}$ feet	
Malaika	$7\frac{9}{12}$ feet	$6\frac{2}{12}$ feet	$7\frac{11}{12}$ feet	
Mary	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	
Freida	$7\frac{10}{12}$ feet	$7\frac{10}{12}$ feet	$8\frac{2}{12}$ feet	

- 2. Who had the greatest total score for the girls' long jump?
- 3. Frieda wants to find how long her 2nd jump needed to be in order to win the event. In order to score greater than the winner, how far would Frieda need to jump? Explain your thinking using words, numbers, and math pictures as needed.

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4.	Name	1⁵t Jump	2 nd Jump	3 rd Jump	Total Score
	Carlos	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{6}{12}$ feet	
	Emmett	$7\frac{7}{12}$ feet	$6\frac{10}{12}$ feet	$8\frac{4}{12}$ feet	
	Bob	$8\frac{9}{12}$ feet	$9\frac{2}{12}$ feet	$8\frac{11}{12}$ feet	
	Thomas	$6\frac{7}{12}$ feet	$8\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	
	Gene	$7\frac{10}{12}$ feet	$7\frac{3}{12}$ feet	$8\frac{5}{12}$ feet	

5. Who had the greatest total score for the boys' long jump?

1.

6. Carlos wants to find how long his 2nd jump needed to be in order to win the event. In order to score greater than the winner, how far did Carlos need to jump? Explain your thinking using words, numbers, and math pictures as needed.

The boys' and girls' long jump event total scores are shown below. Mary was the winner for the girls with a total score of $24\frac{3}{12}$, Bob was the winner for the boys with a total score of $26\frac{10}{12}$.

Name	1 st Jump	2 nd Jump	3 rd Jump	Total Score
Kim	$7\frac{3}{12}$ feet	$6\frac{11}{12}$ feet	$6\frac{5}{12}$ feet	$20\frac{7}{12}$ feet
Amanda	$5\frac{9}{12}$ feet	$6\frac{1}{12}$ feet	$6\frac{4}{12}$ feet	$18\frac{2}{12}$ feet
Malaika	$7\frac{9}{12}$ feet	$6\frac{2}{12}$ feet	$7\frac{11}{12}$ feet	$21\frac{10}{12}$ feet
Mary	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	$24\frac{3}{12}$ feet
Freida	$7\frac{10}{12}$ feet	$7\frac{10}{12}$ feet	$8\frac{2}{12}$ feet	$23\frac{10}{12}$ feet

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4.	Name	1⁵t Jump	2 nd Jump	3 rd Jump	Total Score
	Carlos	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{6}{12}$ feet	$24\frac{6}{12}$ feet
	Emmett	$7\frac{7}{12}$ feet	$6\frac{10}{12}$ feet	$8\frac{4}{12}$ feet	$22\frac{9}{12}$ feet
	Bob	$8\frac{9}{12}$ feet	$9\frac{2}{12}$ feet	$8\frac{11}{12}$ feet	$26\frac{10}{12}$ feet
	Thomas	$6\frac{7}{12}$ feet	$8\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	$23\frac{9}{12}$ feet
	Gene	$7\frac{10}{12}$ feet	$7\frac{3}{12}$ feet	$8\frac{5}{12}$ feet	$23\frac{6}{12}$ feet

Comments

To find the distance Carlos would need to jump on his second jump to win the event, students would first need to determine how far he was from first place. To compare Carlos' and Bob's

scores, subtract $26\frac{10}{12} - 24\frac{6}{12} = 2\frac{4}{12}$.

• This can be done easily by subtracting the whole numbers, 26 - 24 = 2 and subtracting

the fractions $\frac{10}{12} - \frac{6}{12} = \frac{4}{12}$.

• To use a counting up strategy, students may add $\frac{6}{12}$ to $24\frac{6}{12}$ to get

 $24\frac{6}{12} + \frac{6}{12} = 24\frac{12}{12} = 25$. To get to $26\frac{10}{12}$ students would need to add $1\frac{10}{12}$ to the 25 giving $25 + 1\frac{10}{12} = 26\frac{10}{12}$. Since students added $\frac{6}{12}$ and $1\frac{10}{12}$ to count up to $26\frac{10}{12}$ the difference of $26\frac{10}{12} - 24\frac{6}{12}$ is the sum of $\frac{6}{12}$ and $1\frac{10}{12}$ which is $2\frac{4}{12}$ (i.e. $\frac{6}{12} + 1\frac{10}{12} = 1\frac{16}{12} = 1 + \frac{12}{12} + \frac{4}{12} = 2\frac{4}{12}$).

The difference of the two scores needs to be added to Carlos' second score of $7\frac{11}{12}$. Therefore,

students should add $7\frac{11}{12} + 2\frac{4}{12} = 9\frac{15}{12} = 9 + \frac{12}{12} + \frac{3}{12} = 10\frac{3}{12}$. Carlos needed a second jump of $10\frac{3}{12}$ to tie with Bob. To win, he needed to jump a distance greater than $10\frac{3}{12}$.

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To find the distance Frieda would need to jump on her second jump to win the event, students would first need to determine how far she was from first place. To compare Mary's and Frieda's scores, subtract $24\frac{3}{12} - 23\frac{10}{12} = \frac{5}{12}$.

- To use a counting up strategy, students may add $\frac{2}{12}$ to $23\frac{10}{12}$ to get
 - $23\frac{10}{12} + \frac{2}{12} = 23\frac{12}{12} = 24$. To get to $24\frac{3}{12}$ students would need to add $\frac{3}{12}$ to the 24 giving $24 + \frac{3}{12} = 24\frac{3}{12}$. Since students added $\frac{2}{12}$ and $\frac{3}{12}$ to count up to $24\frac{3}{12}$ the difference of $24\frac{3}{12} - 23\frac{10}{12}$ is the sum of $\frac{2}{12}$ and $\frac{3}{12}$ which is $\frac{5}{12}$.

• To use a regrouping strategy, students may rewrite $24\frac{3}{12}$ as follows:

$$24\frac{3}{12} = 23 + \frac{12}{12} + \frac{3}{12} = 23\frac{15}{12}$$
. Then students can subtract $23\frac{10}{12}$ from $23\frac{15}{12}$ leaving $\frac{5}{12}$.
 $23\frac{15}{12} - 23\frac{10}{12} = \frac{5}{12}$.

The difference of the two scores needs to be added to Frieda's second score of $7\frac{12}{12}$. Therefore,

students should add $7\frac{10}{12} + \frac{5}{12} = 7\frac{15}{12} = 7 + \frac{12}{12} + \frac{3}{12} = 8\frac{3}{12}$. Frieda needed a second jump of 3

 $8\frac{3}{12}$ to the with Mary. To win, she needed to jump a distance greater than $8\frac{-12}{12}$.

FORMATIVE ASSESSMENT QUESTIONS

- How did you find the sum of these mixed numbers?
- Is the fraction a proper or improper fraction? How do you know?
- If the fraction part of the mixed number is improper, what should you do?

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- What do you know about the fraction $\overline{12}$?
- How can you rewrite that mixed number?
- What strategies helped you add and subtract mixed numbers?

DIFFERENTIATION

Extension

• Challenge students to write and solve a problem based on the jumping distances provided. Then ask students to give the problem to a partner to solve.

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- Students can add up all of the boys jumps to determine a total length, then do the same for the girl jumps and compare the two.
- Whenever applicable students can write equivalent fractions for their sums and differences.
- Students can be encouraged to participate in field events and add their scores to find the total score as required in this task.

Intervention

• Allow students to use two or three rulers or a yardstick to work with the fractions in this

task. Students can count the number of inches $\begin{pmatrix} 1 \\ 12 \end{pmatrix}$ along the ruler to find the sum of the fractional part of the mixed number. This also allows students to recognize that 1 whole 12

(the length of one ruler) is equivalent to $\overline{12}$.

- Continue using number lines as well for students that showed aptitude with this strategy.
- Whenever possible have the students actually perform a long jump and compete with one another. These activities serve to provide students with both prior knowledge and a greater meaning and motivation to complete the task.

Intervention Table

TECHNOLOGY

• <u>http://www.uen.org/Lessonplan/preview.cgi?LPid=21526</u> Delightfully Different Fractions: This lesson uses virtual pattern blocks to practice the concept. It can be used as an introduction to this task or as an extension of the task.

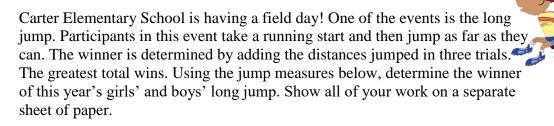
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Name ____

1.

_ Date _

Fraction Field Event



Name	1 st Jump	2 nd Jump	3 rd Jump	Total Score
Kim	$7\frac{3}{12}$ feet	$6\frac{11}{12}$ feet	$6\frac{5}{12}$ feet	
Amanda	$5\frac{9}{12}$ feet	$6\frac{1}{12}$ feet	$6\frac{4}{12}$ feet	
Malaika	$7\frac{9}{12}$ feet	$6\frac{2}{12}$ feet	$7\frac{11}{12}$ feet	
Mary	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{3}{12}$ feet	
Freida	$7\frac{10}{12}$ feet	$7\frac{10}{12}$ feet	$8\frac{2}{12}$ feet	

- 2. Who had the greatest total score for the girls' long jump?
- 3. Freida wants to find how long her 2nd jump needed to be in order to win the event. In order to score greater than the winner, how far would Freida need to jump? Explain your thinking using words, numbers, and math pictures as needed.

Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name	1 st Jump	2 nd Jump	3 rd Jump	Total Score
Carlos	$8\frac{1}{12}$ feet	$7\frac{11}{12}$ feet	$8\frac{6}{12}$ feet	
Emmett	$7\frac{7}{12}$ feet	6 10 12 feet	$8\frac{4}{12}$ feet	
Bob	8 <mark>9</mark> 12 feet	$9\frac{2}{12}$ feet	8 ¹¹ / ₁₂ feet	
Thomas	$6\frac{7}{12}$ feet	8 <mark>11</mark> 12 feet	$8\frac{3}{12}$ feet	
Gene	$7\frac{10}{12}$ feet	$7\frac{3}{12}$ feet	8 <mark>5</mark> 12 feet	

5. Who had the greatest total score for the boys' long jump?

6. Carlos wants to find how long his 2nd jump needed to be in order to win the event. In order to score greater than the winner, how far did Carlos need to jump? Explain your thinking using words, numbers, and math pictures as needed.

4.

<u>CULMINATING TASK</u>: Pizza Parlor (Revisited)

Back to Task Table

In this culminating task students are asked to add and subtract fractions, improper fractions and mixed numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

The solutions for the different order cards are shown below.

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		Toppings														Order Totals				
	Extra Cheese	Beef	Buffalo Chicken	Ham	Pepperoni	Sausage		Anchovies	Green Peppers	Jalapeño Peppers	Mushrooms	Onions	Pineapple	Sliced Tomatoes	Improper Fraction	Mixed Number	Fraction of Plain Cheese	Total Pizzas Ordered		
Order #1 Customer Name Mrs. SanChez	$\frac{1}{2}$				$\frac{3}{4}$	$\frac{1}{4}$						$\frac{1}{2}$			$\frac{8}{4}$	2	0 4	2		
Order #2 Customer Name Mr. Adams											$\frac{3}{2}$	$\frac{1}{4}$		$\frac{3}{4}$	$\frac{10}{4}$	$2\frac{2}{4}$	$\frac{2}{4}$	3		
Order #3 Customer Name Sammie				3 8				$\frac{1}{8}$					$\frac{3}{4}$		$\frac{10}{8}$	$1\frac{2}{8}$	6 8	2		
Order #4 Customer Name Ally					$\frac{11}{8}$				$\frac{7}{4}$		$\frac{7}{4}$				$\frac{39}{8}$	$4\frac{7}{8}$	$\frac{1}{8}$	5		
Order #5 Customer Name Reggie		5 3							$\frac{3}{6}$	$\frac{4}{3}$					$\frac{21}{6}$	$3\frac{3}{6}$	$\frac{3}{6}$	4		

		Toppings														Order Totals		
	Extra Cheese	Beef	Buffalo Chicken	Ham	Pepperoni	Sausage		Anchovies	Green Peppers	Jalapeño Peppers	Mushrooms	Onions	Pineapple	Sliced Tomatoes	Improper Fraction	Mixed Number	Fraction of Plain Cheese	Total Pizzas Ordered
Order #1 Customer Name Hilda	$\frac{3}{2}$			5 4									$\frac{1}{4}$		$\frac{12}{4}$	3	0 4	3
Order #2 Customer Name Nimesh						$\frac{1}{2}$						$\frac{15}{10}$	$\frac{6}{10}$		$\frac{26}{10}$	$2\frac{6}{10}$	$\frac{4}{10}$	3
Order #3 Customer Name Norah	5 4		$\frac{3}{4}$			$\frac{3}{4}$									$\frac{11}{4}$	$2\frac{3}{4}$	$\frac{1}{4}$	3
Order #4 Customer Name Ms. Thomas	7 8				$\frac{3}{4}$			$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$					$\frac{31}{4}$	$7\frac{3}{4}$	$\frac{1}{4}$	8
Order #5 Customer Name LatiCia		$\frac{3}{10}$	$\frac{15}{10}$									$\frac{8}{10}$		$\frac{8}{10}$	$\frac{34}{10}$	$3\frac{4}{10}$	$\frac{6}{10}$	4

ESSENTIAL QUESTIONS

- What is an improper fraction and how can it be represented?
- What is a mixed number and how can it be represented?
- What is the relationship between a mixed number and an improper fraction?
- How do we add fractions?

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MATERIALS

- "Pizza Parlor, Order Form" student recording sheet
- "Pizza Parlor, Order Cards" students sheet
- "Pizza Parlor, Pizzas" student recording sheet
- Colored pencils or crayons
- Scissors and glue
- Plain paper (students will glue their work on a sheet of paper for display)

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students use improper fractions and mixed numbers interchangeably and add fractions to complete pizza orders.

Comments

This task is designed to be used after students have done the Fraction Cookies Bakery (see task in this unit). Therefore, students should have worked with improper fractions, mixed numbers, and addition of common fractions. In this task, students will use rectangular models for the pizzas because rectangles are much easier for students to divide equally into fifths, sixths, and tenths.

Introduce this task by telling students that they have been hired at a Pizza Parlor and they will be in charge of creating the pizzas with the correct toppings. Explain to students that the customers are very picky and quite specific when ordering pizzas. Only one topping goes on each part of the pizza and if there aren't enough toppings for a whole pizza, the remaining part will be left plain.

Before students work on this task with a partner or independently, they should solve one problem as a class, providing a model of what is expected.

Kaden called to order pizza for his family. Most of the people in his family like sausage 7

on their pizzas, so he ordered $\overline{4}$ sausage. Kaden is the only one who likes pepperoni, so

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he ordered $\overline{\mathbf{8}}$ pepperoni. His sisters, Hannah and Tamara, like vegetables on their pizza; $\mathbf{3}$ $\mathbf{5}$

so, he also ordered $\overline{4}$ mushroom, and $\overline{8}$ onion.

Ask students the following questions:

- How many pizzas did he order in all?
- Will any part of the pizzas be only cheese?
- How could he write his order as an improper fraction?
- How could he write his order as a mixed number?

		Toppings										Order Totals							
	Extra Cheese	Beef	Buffalo	Chicken	Ham	Pepperoni	Sausage		Anchovies	Green Pennere	Jalapeño Penners	Mushrooms	Onions	Pineapple	Sliced Tomatoes	Improper Fraction	Mixed Number	Fraction of Plain Cheese	T otal Pizzas Or der ed
Order #1 Customer Name Kaden						3 8	$\frac{7}{4}$					$\frac{3}{4}$	5 8			28 8	$3\frac{4}{8}$	$\frac{4}{8}$	4

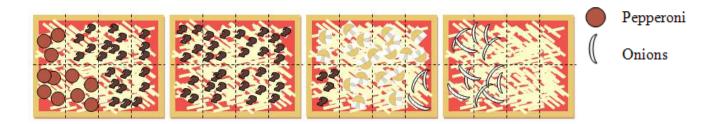
Put four blank, rectangular pizzas on the board. Discuss with the students how Kaden could show the correct toppings on each of the pizzas. First, ask students how the pizzas should be divided. Should they be divided into fourths? Eighths? (It is okay to divide the pizzas into fourths, but students would need to recognize that some of the fourths would need to be divided

in half to create eighths as required.) Next, ask students how to cover $\overline{4}$ pizzas with sausage if each pizza is divided into eighths. Looking at the picture, students should recognize that to cover 7

 $\overline{4}$, a total of 14 eighths would need sausage. With the onions, a total of 10 eighths would need to 4

be covered with onion. When finished placing the toppings, students should see that $\overline{\mathbf{8}}$ or $\overline{\mathbf{4}}$ of a

pizza is left plain. Discuss how this could also be represented as $\overline{2}$ of the pizza has no additional topping. Sausage



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Using the example above as a model, allow students to work with a partner or on their own to complete the task. After students have created their pizzas, have a few students share their solution for one pizza order with the class. Allow other students to ask questions and make comments about the pizza models and their work.

Task Directions

Students will follow the directions below from the "Pizza Parlor, Pizzas" student task sheet.

Use the pizzas below to make the customer orders. Use colored pencils or crayons to create the pizzas ordered. Once you have completed an order, cut out the pizzas and the order card and glue them to a piece of paper to display your work. Add words and numbers as needed to understand your work. Remember, customers expect you to use the fewest number of pizzas possible to complete each order. No part of a pizza should be without a topping except for one.

Also, students will follow the directions below from the "Pizza Parlor, Order Form" student recording sheet.

Choose five of the pizza orders from the "Pizza Parlor, Order Cards" student sheet and complete the order form below.

FORMATIVE ASSESSMENT QUESTIONS

- What task have you done that will help you with this "Pizza Parlor" task?
- What order are you working on? How can you make sure you make the fewest pizzas and still fill the order?
- Into how many equal parts did you divide your pizzas? Why?
- How will you represent this improper fraction on your pizzas?
- How many pieces will have that topping? How do you know?
- How do you know this fraction of a pizza will be left plain?
- How do you know there were this many pizzas ordered?

DIFFERENTIATION

Extension

• Ask students to create orders of their own, then switch with a partner to create the confirmations for those orders. Students can be given a blank confirmation sheet or they can create their own fraction models.

Intervention

• Some students may need more examples modeled before they are able to complete this task on their own. Provide an opportunity for further small group instruction before students are asked to complete this task.

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- Allow students to use pre-made fraction pieces to create the pizzas. It might be necessary to combine several sets of pieces in order to make multiple pizzas.
- Students could also use two colored counters, pattern blocks, etc. to represent toppings and physically manipulate and build their pizzas.

Intervention Table

TECHNOLOGY

- <u>http://www.ixl.com/math/grade-3/add-and-subtract-fractions-with-like-denominators</u> This game allows students to add or subtract fractions with like denominators. It can be used for additional practice in an independent center or an online assessment.
- <u>http://www.uen.org/Lessonplan/preview.cgi?LPid=21526</u> Delightfully Different Fractions: This lesson uses virtual pattern blocks to practice the concept. It can be used as an introduction to this task or as an extension of the task

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Name _____

Date

Pizza Parlor Order Form



Choose five of the pizza orders from the "Pizza Parlor, Order Cards" student sheet and complete the order form below.

		Toppings											Order Totals						
	Extra Cheese	Beef	Buffalo Chicken	Ham	Pepperoni	Sausage		Anchovies	Green Penners	Jalapeño Penners	Mushrooms	Onions	Pineapple	Sliced Tomatoes		Improper Fraction	Mixed Number	Fraction of Plain Cheese	Total Pizzas Ordered
Order #1 Customer Name																			
Order #2 Customer Name																			
Order #3 Customer Name																			
Order #4 Customer Name																			
Order #5 Customer Name																			

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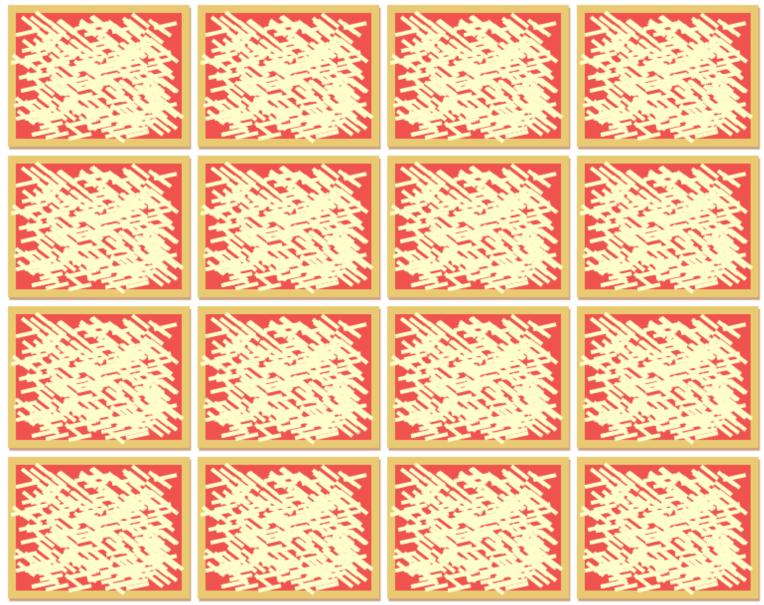
Pizza Parlor Order	r Cards
Mrs. Sanchez called to order pizza with $\frac{3}{4}$ pepperoni, $\frac{1}{2}$ extra cheese, $\frac{1}{2}$ onions, and $\frac{1}{4}$ sausage.	Mr. Adams came to pick up his pizza order. He wanted pizza with $\frac{3}{2}$ mushrooms, $\frac{1}{4}$ onions, and $\frac{3}{4}$ sliced tomatoes.
Sammie ordered pizza over the phone. He ordered $\frac{3}{4}$ pineapple and $\frac{3}{8}$ ham, and $\frac{1}{8}$ anchovies.	Ally ordered pizza for a party with her friends. She ordered $\frac{7}{4}$ green peppers, $\frac{11}{8}$ pepperoni, and $\frac{7}{4}$ mushrooms.
Reggie ordered some pizza to share with his friends. He ordered $\frac{4}{3}$ jalapeño peppers, $\frac{3}{6}$ green peppers, and $\frac{5}{3}$ beef.	Hilda called to order pizza. She wanted $\frac{3}{2}$ extra cheese, $\frac{1}{4}$ pineapple, and $\frac{5}{4}$ ham.
Mr. Nimesh ordered pizza. He ordered $\frac{15}{10}$ onions, $\frac{1}{2}$ sausage, and $\frac{6}{10}$ pineapples.	Norah ordered pizza to share with her family. She ordered $\frac{5}{4}$ extra cheese, $\frac{3}{4}$ buffalo chicken and $\frac{3}{4}$ sausage.
Ms. Thomas ordered pizza for her students. She ordered $\frac{3}{4}$ jalapeño peppers, $\frac{3}{4}$ green peppers, $\frac{3}{4}$ pepperoni, $\frac{7}{8}$ extra cheese, and $\frac{3}{4}$ anchovies.	Laticia called to order pizza. She wanted $\frac{3}{10}$ beef, $\frac{8}{10}$ onions, $\frac{15}{10}$ buffalo chicken, and $\frac{8}{10}$ sliced tomatoes.

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Name _____ Date _____

Pizza Parlor Pizzas

Use the pizzas below to make the customer orders. Use colored pencils or crayons to create the pizzas ordered. Once you have completed an order, cut out the pizzas and the order card and glue them to a piece of paper to display your work. Add words and numbers as needed to understand your work. Remember, customers expect you to use the fewest number of pizzas possible to complete each order. No part of a pizza should be without a topping except for one.



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SCAFFOLDING TASK: A Bowl of Beans

Back to Task Table

Adapted from Riddle Math: Using Student-Written Riddles to Build Mathematical Power (2001) by Carl M. Sherrill

In this task students will begin to multiply a whole number by a fraction and represent fractional products.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Before assigning the task, students should have several experiences with finding a fraction of a set. Also, students should have some degree of fluency with multiplication facts with a product of 24 or less.

Student solutions to the riddles should include a picture of the beans and an explanation using words and numbers. Some possible solutions are shown below:

Riddle #1

- I have 8 beans.
- $\frac{1}{4}$ of my beans are red.
- The rest are pinto beans.
- Show my set of beans.

"I divided 8 beans onto 4 plates because I needed to find $\overline{4}$ of the beans. There were 2 beans on each plate. I made the other six beans red beans. The answer to Riddle #1 is a set of 8 beans, 2 red beans and 6 pinto beans."

1

Riddle #2

- $\overline{\mathbf{3}}$ of my beans are red.
- I have 9 beans.

1

- $\overline{\mathbf{3}}$ of my beans are black.
- Show my set of beans.

"I divided 9 beans into 3 groups because I needed to find $\overline{\mathbf{3}}$ of the beans. There were 3 beans in each group. I know that 3 beans are

1

 $\overline{\mathbf{3}}$ of the total beans, so $\overline{\mathbf{3}}$ of the total beans must be 2 x 3 or 6 beans. So, I made six beans black beans. The answer to Riddle #2 is a set of 9 beans, 3 red beans and 6 black beans."

"I divided 9 beans onto three plates because I needed to find $\frac{1}{3}$ of the beans. There were 3 beans on each plate. I know that $\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$ so the rest of the beans must be $\frac{2}{3}$ of the total beans. So, I made rest of the beans black beans. The answer to Riddle #2 is a set of 9 beans, 3 red beans and 6 black beans."

Riddle #3

- $\overline{\mathbf{2}}$ of my beans are black-eyed peas.
- $\overline{\mathbf{4}}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.
- Show my set of beans.

"I shared 12 beans in 2 groups because I needed to find $\frac{1}{2}$ of the beans. There were 6 beans in each group. I know that 6 beans are $\frac{1}{2}$ of the total beans, so 6 beans must be black-eyed peas. Then I shared 12 beans in 4 groups because I need to find $\frac{1}{4}$ of the total beans. There were 3 beans in each group. I know that 3 beans are $\frac{1}{4}$ of the beans, so 3 beans must be pinto beans. The rest of the beans are red beans. If 6 are black-eyed peas and 3 are pinto beans, 6 + 3 = 9 and 12 - 9 = 3, so there are 3 beans left and those are red. The answer to riddle #3 is a set of 12 beans, 6 black-eyed peas, 3 pinto beans, and 3 red beans."

ESSENTIAL QUESTIONS

• How can I be sure fractional parts are equal in size?

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- What do the numbers (terms) in a fraction represent?
- How does the number of equal pieces affect the fraction name?

MATERIALS

• *Clean-Sweep Campers* (2000) by Lucille Recht Penner, or similar book about fractions of a set

Each student will need the following materials:

- "A Bowl of Beans" student recording sheet
- Handful of mixed beans (red kidney beans, pinto beans, black beans, black-eyed peas, etc.)
- Small paper plates
- Colored pencils or crayons

GROUPING

Individual Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students write and solve riddles regarding fractions of a set. Students write number sentences using whole numbers and fractions.

Comments

To introduce this task read *Clean-Sweep Campers* (2000) by Lucille Recht Penner, illustrated by Paige Billin-Frye (A Math Matters book) or a similar book that discusses fractions of a set. Discuss the different solutions the girls devised for cleaning groups and/or discuss possible ways students could be grouped for Sports Day (see page 32).

Other related books that could be used with this task are *Jump, Kangaroo, Jump!: Fractions* (1999) by Stuart J Murphy, illustrated by Kevin O'Malley and *The Wishing Club: A story about fractions* (2007) by Donna Jo Naoli, illustrated by Anna Currey.

It is very important that beans are available for students to use. Students can use any type of bean initially, using the total number of beans given in the riddle. Once they find the fraction of the whole, they can trade the correct number of beans for the correct kind of bean.

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After reviewing student-created riddles for accuracy, allow students to write their riddle on an index card with the solution drawn and explained on the back. Have students share their riddles with others in the class or leave them as an independent activity that is self-checking.

Task Directions

Students will follow the directions below from the "A Bowl of Beans" student recording sheet. You will need a bowl of different kinds of dried beans to solve the riddles below. Draw a picture of your set of beans for each riddle. Explain how you solved each riddle, using words and numbers. Finally, represent your answer with a number sentence (for example: if you have 6 beans and $\frac{1}{3}$ are red then write the number sentence $\frac{6}{3} = 6 \times \frac{1}{3}$ or $6 \times \frac{1}{3} = \frac{6}{3}$)

Riddle #1

- I have 8 beans.
- 1
- 4 of my beans are red.
- The rest are pinto beans.

Show my set of beans.

Riddle #2

1

- $\overline{\mathbf{3}}$ of my beans are red.
- I have 9 beans.
- $\overline{\mathbf{3}}$ of my beans are black.

Show my set of beans.

Riddle #3

1

- $\frac{1}{2}$ of my beans are black-eyed peas.
- $\overline{4}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.

Show my set of beans.

Using beans create your own bean riddle below. Show the answer to your riddle on the back of this paper.

FORMATIVE ASSESSMENT QUESTIONS

- What information in the riddle did you need first?
- Is there any missing information in the riddle?
- If you have _____ (i.e., 9) beans in _____ (i.e., 3) equal groups, how many beans are in each group? How many groups do you need? How do you know?
- If we know **3** of the beans are 3 beans, how many beans would be **3** of the beans? How do you know?
- How did you know how many ____ (i.e., red) beans to use?
- How would this set of beans look if there were 12 beans instead of 9 total beans?

DIFFERENTIATION

Extension

- Ask students to create riddles with at least three types of beans.
- Have students explore equivalent fractions represented in a set (i.e., in a set of 9 beans if 1/3 of the set is red, then 3/9 of the beans are red).

Intervention

• An alternative activity could be used as follows: Riddle #1:

I have 6 beans. One-third $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ of the beans are pinto beans. How many beans are pinto beans?

Riddle #2

I have 8 beans. One-eighth $(\frac{1}{8})$ of the beans are black beans. How many beans are black beans? Riddle #3

I have 12 beans. One-fourth $\left(\frac{1}{4}\right)$ of the beans are red beans. How many beans are red beans?

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.

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- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>https://www.conceptuamath.com/app/tool/multiplying-fractions-tool</u> *This tool provides students with the opportunity to virtually model the multiplication of fractions by whole numbers.

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Name ____

_____ Date

A Bowl of Beans

You will need a bowl of different kinds of dried beans to solve the riddles below. Draw a picture of your set of beans for each riddle. Explain how you solved each riddle using words and numbers. Finally, represent your answer with a number sentence (for example: if you have 6 beans and $\frac{1}{3}$ are red then write the number sentence $\frac{6}{3} = 6 \times \frac{1}{3}$ or $6 \times \frac{1}{3} = \frac{6}{3}$)



Riddle #1

- I have 8 beans.
- $\overline{\overline{4}}$ of my beans are red.
- The rest are pinto beans.

Show my set of beans.

Riddle #2

- $\frac{1}{2}$ c 1
- $\overline{\mathbf{3}}$ of my beans are red.
- I have 9 beans.
- $\overline{\mathbf{3}}$ of my beans are black.

Show my set of beans.

Riddle #3

1

- $\overline{2}$ of my beans are black-eyed peas.
- $\overline{4}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.

Show my set of beans.

Create your own bean riddle below. Show the answer to your riddle on the back of this paper.

CONSTRUCTING TASK: Birthday Cake!

Back to Task Table

Students determine the number of candles used on birthday cakes using multiplication of fractions and whole numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

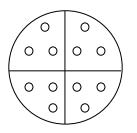
For this activity students will be asked to determine the given number of candles on each piece of birthday cake when given a total number of candles on the cake. All problems assume that the cake pieces will be equal and everyone will always receive the same number of candles. The first part of the assignment provides students with whole to part problems. In other words, the students receive the whole amount in the question but need to produce the part of the whole to determine their answer. In the second part of the task students are given the number of candles on just one piece of cake or one fraction of the cake and have to then determine how many candles were on the entire cake. The second part of the assignment provides students with part to whole problems where they receive a part or fraction in the question but need to produce the whole amount to determine the answer. For example:

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Whole to Part

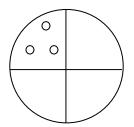
The four people at Tanya's birthday will get one-quarter (one-fourth) of the cake each. Tanya puts 12 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



In the problem above the students need to determine the "part" when given the whole of 12 candles (i.e. 3 candles). This can be determined by using the expression $12 \times \frac{1}{4} \frac{1}{4}$, which can be interpreted as $\frac{1}{4}$ of 12 candles, which is $\frac{12}{4}$, or 3 candles on each piece of cake.

Part to Whole

Ricardo put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has three candles, how many candles did Ricardo put on his cake?



In the problem above the students need to determine the "whole" when given only the part (i.e. $\frac{1}{4}$ of the cake had 3 candles, therefore the whole must be 12 candles). Although not always the case, these types of problems often pose a greater challenge to 4th graders. Students may have trouble seeing how multiplying a fraction by a whole number helps to solve this problem. Students must determine the set when given the fraction and the product. The equation that helps solve this problem is $\frac{1}{4} \ge 3$. Students should think of a whole number that when divided into four equal groups has three in each group. The c, or unknown would be 12, so there would be 12 candles on the cake.

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Before asking students to work on this task, be sure students are able to:

- Use repeated addition to add fractions with the same denominator.
- Be able to decompose fraction, for example $\frac{4}{4} = \frac{1}{2} + \frac{1}{2}$ or $\frac{1}{4} + \frac{3}{4}$.
- Have a strong understanding that the whole can be any number/size and the fractions always depend on taking a portion of this whole.

Many of these could be completed by using simple division or multiplication without fractions at all. By completing tasks such as these students will begin to see a pattern and develop different strategies for multiplying and dividing denominators in order to solve problems that involve fractions.

ESSENTIAL QUESTIONS

- What does it mean to take a fraction, or part of a whole number?
- How do we determine the whole amount when given a fractional value of the whole?

MATERIALS

- Paper plates or large circles either cut out or drawn
- Two sided counters, base ten units, or some other small counter
- Birthday Cake student recording sheet.

GROUPING

Group/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fraction, Decimal, and Percents,* 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will use a pie model to multiply a whole number by a fraction. Students will gain experience solving both part-to-whole and whole-to-part word problems that ask them to multiply a fraction by a whole number or multiply a whole number by a fraction

Comments

Paper plates and counters should be made available for students to act out each of these

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problems. If paper plates are not available, a large circle drawn on an $11 \ge 8.5$ inch paper will work just as well. Students could also color or draw their candles, or use glue and die cuts. This task could be introduced by bringing in a cake and showing students how to distribute the candles in such a way that everyone receiving cake would get the same number of candles.

Task Directions

Students will follow the directions below from the Birthday Cake! task sheet.

- Obtain a set of counters and paper plates.
- Work with a partner or small group to make a fraction cake and record it on your task sheet.
- Be ready to articulate your reasoning.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know how many pieces of cake there are?
- Can you write an equivalent fraction for your answer? (for the example above, $\frac{1}{4} = \frac{3}{12}$)
- Are the candles evenly distributed or fairly distributed?
- In what other situations do we need to share evenly?

DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson could be extended to use larger numbers of candles and larger fractions.
- Students could solve problems where the numerator is a number other than 1, for example, $\frac{5}{2}$ of 30.
- Students could also extend this task by exploring how the task would change if you had 2 or 3 cakes rather than just one whole cake.

Intervention

- Students may need to review division concepts with whole numbers.
- Students can use repeated addition to solve these problems.
- Students may be given cakes already "cut" or drawn in parts to help determine the denominator.
- Initially students can start with a smaller amount of candles. Start students with 4 candles on a cake and then cut the cake into fourths. Next, have students solve the same problem, but this time the cake will have 8 candles total and have pieces that are $\frac{1}{4}$ in size. Students

can independently find the solution to having 12 candles on a cake that is being cut into $\frac{1}{4}$

sized pieces. Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look at Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> Identify Fractional Parts of a Group: This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> Another Look at the Set Model Using Attribute Pieces: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>https://www.conceptuamath.com/app/tool/multiplying-fractions-tool</u> *This tool provides students with the opportunity to virtually model the multiplication of fractions by whole numbers.

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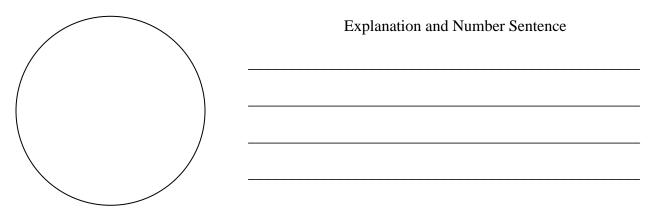
Name ____

_____Date ___

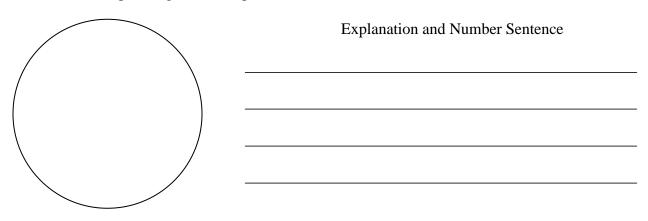
Birthday Cake! Part 1

- Act out the problem using circles and counters.
- Draw your answer using the circle.
- Explain your answer using words.
- Lastly, write a number sentence for each problem

1. The four people at Carla's birthday will get one-quarter (one-fourth) of the cake each. Carla puts 16 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



2. The five people at Estella's birthday will get one-fifth of the cake each. Estella puts 25 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?

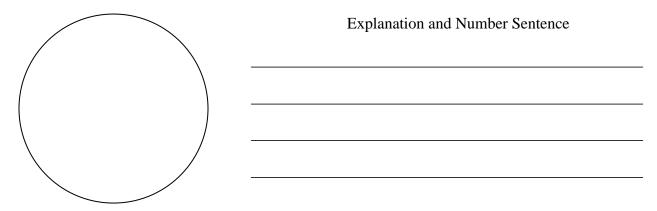


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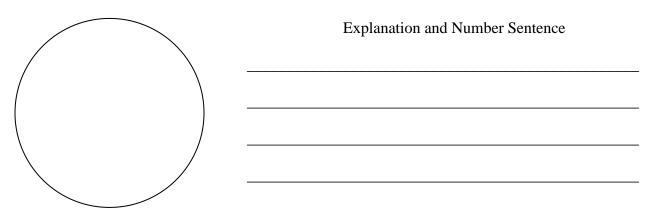


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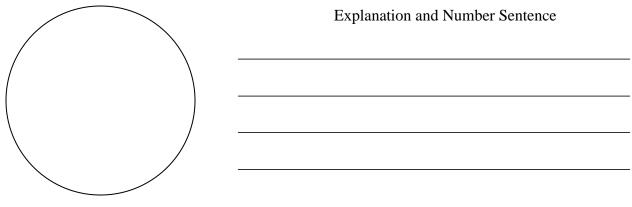
3. Three people are at Emmanuel's birthday party. Emmanuel puts 21 candles on the cake and cuts it into thirds so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?



4. The six people at Zoe's birthday will get one-sixth of the cake each. Zoe puts 18 candles on the cake so that each person gets the same number of candles on their piece of cake. However, one friend doesn't like cake so Zoe ate a second piece. How many candles will each person get on their piece of cake? How many candles will Zoe get?



5. At the party, the cake is cut into quarters (fourths). Twelve candles are put on the cake. Greedy Greg eats three-quarters of the cake. How many candles does he get?



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Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

Name ____

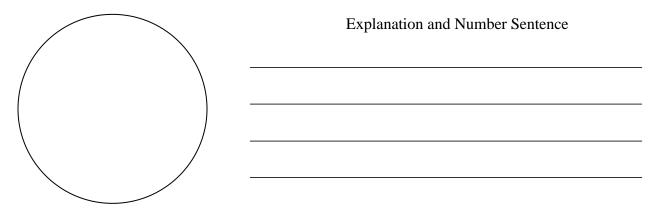
____ Date ___

Birthday Cake! Part 2

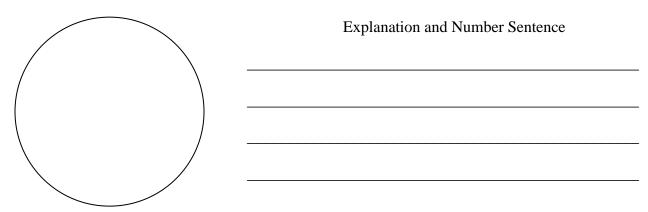
- Act out the problem using circles and counters.
- Draw your answer using the circle.
- Explain your answer using words.
- Lastly, write a number sentence for each problem.



1. Reyna put enough candles on her birthday cake so that everyone would have the same number of candles. She then cut the cake into fifths. If each slice has four candles, how many candles did Reyna put on her cake?



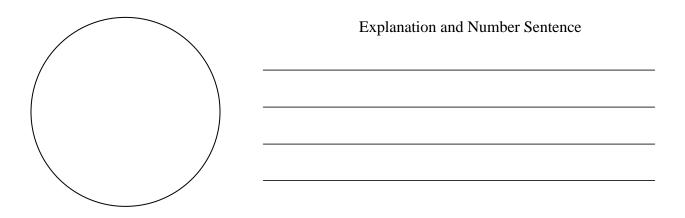
2. Stan put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has six candles, how many candles did Stan put on his cake?



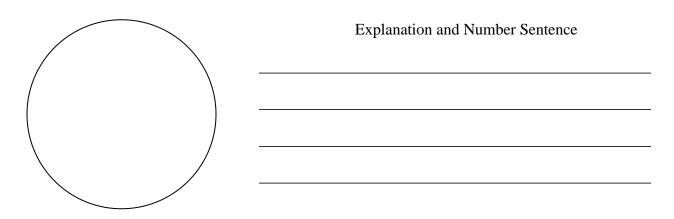
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3. Pedro put enough candles on his birthday cake so that everyone would have the same number of candles. After cutting himself a large slice he noticed that two-thirds of the cake has eight candles on it. How many candles are on the whole cake?

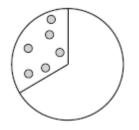


4. Priya and her father made a cake for her birthday and put enough candles on it so that everyone would have the same number of candles. Priya's father cut the cake into fourths and gave Priya the first slice. He then noticed that the three-fourths of the cake that was left had twelve candles on it. How many candles were on the whole cake?

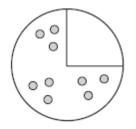


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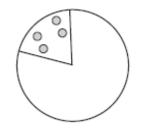
The cake below was cut into thirds. How many candles were on the whole cake?



The cake below was cut into fourths. How many candles were on the whole cake?



The cake below was cut into fifths. How many candles were on the whole cake?



CONSTRUCTING TASK: Fraction Clues

Back to Task Table

Students will make riddles using equivalent fractions and solve riddles by multiplying fractions by whole numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

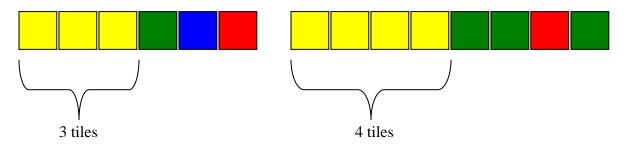
- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students need practice with open-ended activities that allow them to design their own problems and then assess one another. This activity also makes students use mathematical language, verify answers, and work collaboratively with another student. Students are also able to see equivalent fractions in a concrete way. This activity also helps build the "guess and check" strategy as each student tries to build the fraction bar based on the set of clues.

This activity is also valuable because students start to realize that a different number of tiles in a different fraction bar can still be represented by the same fraction. For example:

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In the first bar three yellow tiles represent $\frac{1}{2}$ and in the second bar four tiles represent $\frac{1}{2}$. Students will gain further understanding that the number of tiles being used (numerator) is always dependent on its relationship to the total number of tiles (denominator)

Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with an area model
- record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut squares)
- write an equation which shows the clues and verify their answer.

ESSENTIAL QUESTIONS

- How can I find equivalent fractions?
- How can I multiply a set by a fraction?

MATERIALS

- Colored tiles
- Fraction Clues recording sheet
- Crayons or colored pencils

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)*

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students will use what they have learned about adding and subtracting fractions, using equivalent fractions and multiplying a fraction by a whole number to give another student clues about the fraction strip they created. There is a lot of emphasis on communicating mathematically in this task.

Comments

To introduce this activity, display these two fraction bars made from Color Tiles.



Ask students to find out what unit fraction a tile in the first bar represents and what unit fraction e a tile in the second bar represents. Students should be able to determine that each tile in the first bar represents ¹/₄ of the whole and each tile in the second bar represents ¹/₆ of the whole. Ask students to explain what fractional part each color represents in each fraction bar. Give the following set of fraction clues that describe one of the fraction bars. Stop after each clue and ask children which fraction bar is the solution and how they know.

- The fraction bar is one-half green.
- The fraction bar is one-third red.
- The fraction is one-sixth blue.

Many children will not need all three clues to determine the solution however they should be comfortable arguing and verifying their answers and they may need all three clues to conclude that the solution is the second bar.

Part 2 is a much more challenging version of the task where students create fraction bars with any number of tiles, requiring students to use different denominators, such as 6, 8, 10, and 12. This allows students to develop other strategies for determining the unit fraction and the size of the pieces. For example, a student may be forced to find a common denominator or they may figure out on their own that the largest denominator must refer to the total number of tiles. If available, students can glue die-cut red, yellow, blue and green squares as they complete the work in this task.

Task Directions

Students will follow the directions below from the Fraction Clues activity sheet.

- Obtain a set of colored tiles.
- Work with a partner to make a fraction bar and record it on the activity sheet.
- Write at least 3 clues that describe your fraction bar.
- Exchange only your clues with another group.
- Represent your answer with number sentences (For example: if you have 10 tiles and $\frac{1}{2}$ are red then write the number sentence $\frac{10}{2} = 10 \text{ x} \frac{1}{2}$ which is 5 tiles)
- Attempt to build another group's fraction bar as they attempt to build yours.

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• Discuss results with each other. As students solve their partner's riddles, they should use multiplication of fraction and whole numbers.

FORMATIVE ASSESSMENT QUESTIONS

- What clues did you write to describe your fraction bar?
- Have you found all of the possible equivalent fractions? How do you know?
- Were you able to build the fraction bar based on the clues? If not, why?
- What number sentence can describe the tiles in your bar?

DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson can be extended to have two pairs of students combine their fraction bars to make a larger fraction bar, then continue the activity writing clues for another group to solve.
- Students could also work with larger fraction bars as well as write more clues for determining those fraction bars. Most color tiles only have red, blue, green and yellow tiles, so the activity will never have more than four fractions to represent.
- Often the clue with the largest denominator tells you how many tiles can be used. However, students could be challenged to use only 2 clues and therefore force them into situations where they need to find common denominators. For example, my fractions are ¹/₄ red and ¹/₃ green. They will then need to build several bars that have 12 or 24 tiles.

Intervention

- If necessary students could begin this activity with a smaller set, such as using only four tiles.
- If students are struggling, they could attempt the activity with only three colors instead of using all four colored tiles.

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look At Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> Fractional Parts of a Group: This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> Another Look at the Set Model Using Attribute Pieces: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

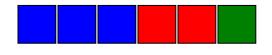
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Name	Date		
	Fraction Clues		

(Part 1)

Make a Color Tile fraction bar and then write a set of clues so that someone else could build it.

- Work with a partner. Choose 6 Color Tiles and arrange them in any way to form a fraction bar.
- Decide what fractional part of the whole bar is represented by each color you used. For example:



Blue: ³/₆ or ¹/₂ Red: ²/₆ or ¹/₃ Green: ¹/₆

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue*.

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

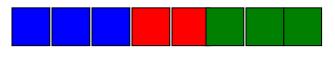
• Represent your answer with a number sentence (for example: if you have 10 tiles and $\frac{1}{2}$ are red, then write the following: Half of $10 = \frac{10}{2} = 10 \div \frac{1}{2} = 5$ tiles).

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Georgia Department of Education Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions Clue 1: _____ Clue 2: _____ _____ Clue 3: _____ Clue 4: _____

- Work with a partner. Choose 8 color tiles and arrange them in any way to form a fraction bar.
- Decide what fractional part of the whole bar is represented by each color you used. For example:



Blue: ³/₈ Red: ²/₈ or ¹/₄ Green: ³/₈

• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue*.

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $\frac{10}{2} = 10 \div 2 = 5$ tiles)

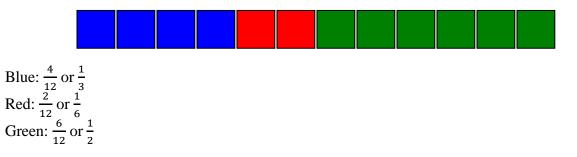
• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Г

Clue 1:			
Clue 2:	Clue 1:		
Clue 2:			
Clue 2:		-	
Clue 2:		-	
Clue 2:			
Clue 3:		-	
Clue 3:			
Clue 3:	Clue 2:		
Clue 3:			
Clue 3:		-	
		-	
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	Clue 3:		
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Clue 4:		-	
Clue 4:		_	
Clue 4:			
	Chue A:		
	Ciue 4		
		-	
		-	
		-	

- Work with a partner. Choose 12 Color Tiles and arrange them in any way to form a fraction bar.
- Decide what fractional part of the whole bar is represented by each color you used. For example:



• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue*.

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

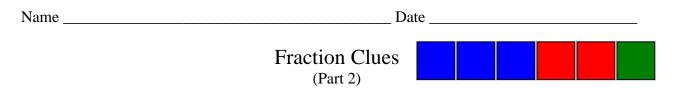
• Represent your answer with a number sentences (for example: if you have 10 tiles and $\frac{1}{2}$ are red then write the number sentence $\frac{10}{2} = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Georgia Department of Education Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions Clue 1: _____ Clue 2: _____ _____ Clue 3: _____ Clue 4: _____

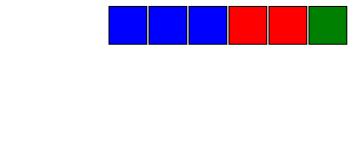
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Make a Color Tile fraction bar and then write a set of clues so that someone else could build it

• Work with a partner. Choose any number of Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:



• Record your fraction bar on grid paper. Beneath the grid paper, write several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue*.

• Exchange lists with another pair. Be careful not to peek at the back of the list! Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentence (for example: if you have 10 tiles and $\frac{1}{2}$ are red then write the number sentence $\frac{10}{2} = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Blue: $\frac{3}{6} \text{ or } \frac{1}{2}$ Red: $\frac{2}{6} \text{ or } \frac{1}{3}$

Green: $\frac{1}{6}$

	Georgia Department of Education	
	Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions	
Clue 1:		
Clue 2:		
Clue 3.		
Clue 4:		
Clue 4		

CONSTRUCTING TASK: Area Models

Back to Task Table

Students will represent multiplication of fractions using an area model.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

On misconception that students may have is that the operation of multiplication produces a larger product and the operation of division produces a smaller quotient. Multiplying whole numbers does produce larger products and dividing whole numbers does produce smaller numbers. However, when students begin using these operations with fractions the exact opposite occurs. This task will illustrate that idea. Below is one example of how the task could be accomplished.

Draw an area model to represent each of the following operations. Use your area model to help you compute the answer to each problem.

 $6 \cdot \frac{2}{3}$

Answers will vary.

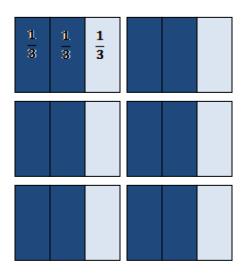
Draw an area model to represent each of the following operations. Use your area model to find the product.

$$6 \cdot \frac{2}{3}$$

Answers will vary.

Possible Solution

A possible solution for $6 \cdot \frac{2}{3}$ is below. This model shows six rectangles with each having $\frac{2}{3}$ of the area shaded. The results show shaded which is equivalent to 4 whole rectangles.



ESSENTIAL QUESTIONS

- What strategies can be used for finding products when multiplying a whole number by a fraction?
- How can I model the multiplication of a whole number by a fraction?

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MATERIALS

- Colored pencils or crayons
- Area Model recording sheet

GROUPING

Individual/Partner

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will use area models to demonstrate their conceptual understanding of multiplying a whole number by a fraction. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

Comments

The area model of representing fraction is an important way for students to understand both multiplication and division of fractions. This task would allow students some freedom to create their own models and create a miniature book to represent the difference between multiplying a whole number by a fraction and dividing a whole number by another whole number.

Task Directions

Have students follow the directions on the area model recording sheet. Use the square below to draw an area model to represent the following multiplication problems. Use your area model to help you compute the answer to each problem.

FORMATIVE ASSESSMENT QUESTIONS

- What did you notice when you multiplied a whole number by a fraction? Did this surprise you? Why or why not?
- How is multiplying a whole number by a fraction different than multiplying a whole number by another whole number?

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DIFFERENTIATION

Extension

• Students can extend this activity by creating other models, such as a set model for multiplying a whole number by a fraction.

Intervention

• Students could begin by multiplying whole numbers with unit fractions (fractions that have a numerator of 1, such as 1/4, 1/2, 1/3) before moving on to fractions that are not unit fractions.

Intervention Table

Name _____

_____Date _____

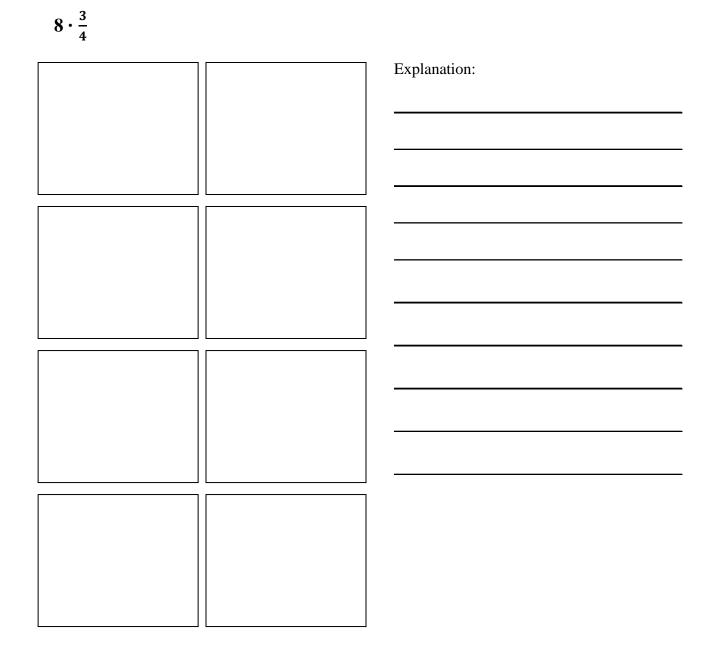
Area Models: Multiplication

Use the squares below to draw an area model to represent the following multiplication problems. Use your area model to help you compute the answer to each problem.



	Explanation:

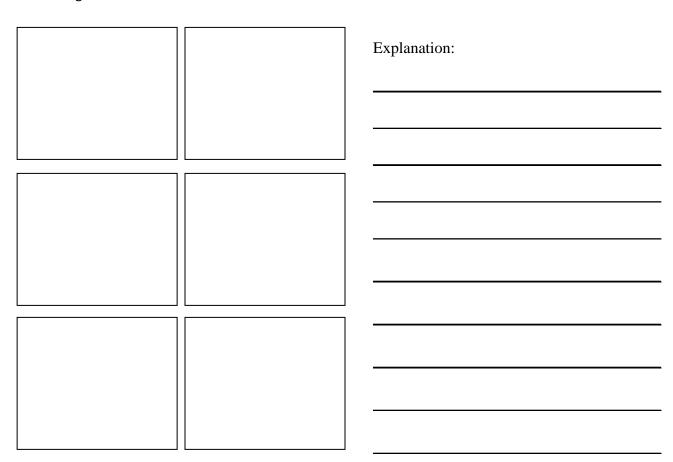
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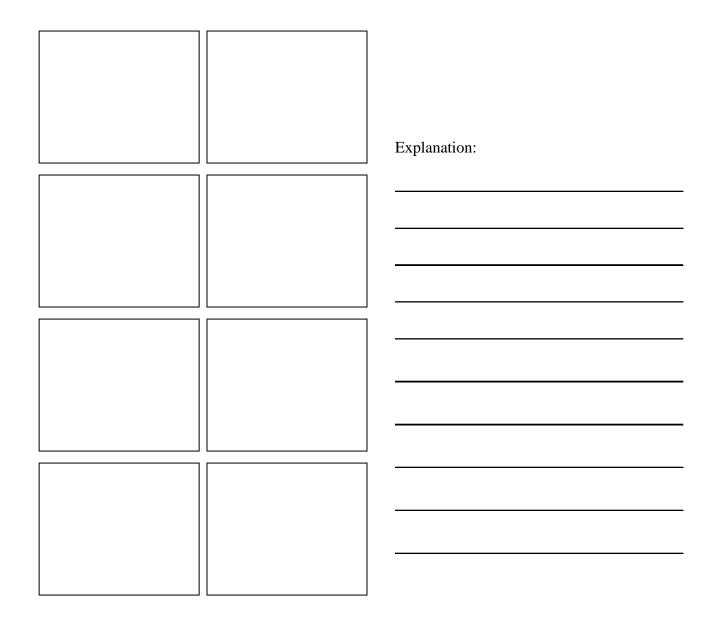
Georgia Standards of Excellence Framework GSE Fourth Grade · Unit 4: Operations with Fractions

 $6 x \frac{2}{5}$



Use the square below to draw an area model to represent the following operations. Use your area model to help you compute the answer to each problem. What happens if you switch the equation around to read $\frac{1}{2}$ times 8?

 $8 \cdot \frac{1}{2}$



PERFORMANCE TASK: How Many CCs?

Back to Task Table

In this task, students must determine various fractions of different whole numbers to complete the provided table.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

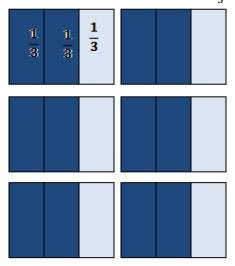
Students are often taught at younger ages that the operation of multiplication produces a larger product and the operation of division produces a smaller quotient. Multiplying whole numbers does produce larger numbers and dividing whole numbers does produce smaller numbers. However, when students begin using these operations with fractions the exact opposite occurs. This task will illustrate that idea. Below is one example of how the task could be accomplished. Mathematics • GSE Grade 4 • Unit 4: Operations with Fractions

Richard Woods, State School Superintendent July 2017 • Page 134 of 191 All Rights Reserved Draw an area model to represent each of the following operations. Use your area model to help you find the product for each problem.

$$6 \cdot \frac{2}{3}$$

Possible Solution

A possible solution for $6 \cdot \frac{2}{3}$ is below. This model shows six rectangles with each having $\frac{2}{3}$ of their area shaded. The results show $\frac{12}{3}$ shaded which is equivalent to 4 whole rectangles.



When numbers become too large to use, models are no longer sensible. Students can decompose the whole number using the fraction as a guide. An example is below.

$$6 \cdot \frac{2}{3}$$

2/3 of 6 is 4 because $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{12}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = 4$

ESSENTIAL QUESTIONS

• How do I determine the product using a fraction of a whole number?

MATERIALS

• "How Many CCs?" recording sheet

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GROUPING

Individual or partner task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

To complete this task correctly, students must understand the amount of drainage fluid and Albumin are equivalent for each reading. Clues are provided within the table to help the students determine the cc of fluid for each. On one occasion, students use the amount of Albumin to determine the drainage fluid.

You can explain to the students "cc" are the cubic centimeters commonly used to measure fluids in the medical field. It is not necessary to attempt to convert cc to another unit of measure.

Students may run into difficulty when trying to determine 4/8 of 100cc. Allow students to struggle, which will provide an opportunity to apply understanding of equivalent fractions. An easy equivalent would be $\frac{1}{2}$. Again, allow students to discover this through struggle.

TASK

Registered Nurse Molly has a patient with a drainage tube in her abdomen. The tube was inserted after a recent surgery. The volume measured in the tube has to be replaced by a blood product called albumin through an IV line. Nurse Molly checks the drainage every 6 hours. The chart below shows the amount of fluid measured in the drainage tube at the checked times. Help Nurse Molly determine how much Albumin to give the patient after each checkpoint.

Time	Drainage Fluid	Albumin
6:00am	100cc	100cc
12 noon	$\frac{3}{4}$ of the volume at 6am	75cc
6:00pm	75cc	75cc
12 midnight	$\frac{2}{3}$ of the volume at 6pm	50cc
6:00am	$\frac{4}{8}$ of 100cc	50cc
12 noon	25cc	$\frac{1}{2}$ of 50cc =

FORMATIVE ASSESSMENT QUESTIONS

- How did you determine ³/₄ of 100?
- How can you model ³/₄ of 100?
- What equivalent fraction can help find the product of $\frac{4}{8}$ of 100?
- What is the least amount of fluid replaced?
- If the drainage at 12 noon was ¹/₄ of 6am's drainage would the amount of Albumin be the same as it is at ³/₄ of 6am? Why?

DIFFERENTIATION

Extension

• The Albumin is administered from a syringe containing 100cc of the fluid using an automatic pump. If the nurse sets the pump for 30 minutes, it will dispense 100cc over the course of the 30 minutes. If the nurse sets it for 15 minutes, how many cc of Albumin will be dispensed? How much time is needed to dispense 25cc? 75cc?

Intervention

• Have students determine the fraction of drainage fluid using unit fractions first. For example, ¹/₄ of 6am before figuring ³/₄ of 6am.

Intervention Table

TECHNOLOGY

• <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L341</u> Class Attributes: This lesson reinforces fractional parts. It can be used to extend understanding of the concept.

Name: _____

Date:_____

How Many CCs?

Registered Nurse Molly has a patient with a drainage tube in her abdomen. The tube was inserted after a recent surgery. The volume measured in the tube has to be replaced by a blood product called Albumin through an IV line. Nurse Molly checks the drainage every 6 hours. The chart below shows the amount of fluid measured in the drainage tube at the checked times. Help Nurse Molly determine how much Albumin to give the patient after each checkpoint.

Time	Drainage Fluid	Albumin
6:00am	100cc	
12 noon	$\frac{3}{4}$ of the volume at 6am	
6:00pm		75cc
12 midnight	$\frac{2}{3}$ of the volume at 6pm	
6:00am	$\frac{4}{8}$ of 100cc	
12 noon		$\frac{1}{2}$ of 50cc =

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<u>CONSTRUCTING TASK:</u> Who Put the Tang in Tangram?

Back to Task Table

Adapted from a lesson on the Utah Education Network <u>www.uen.org</u> Tangram clip art from <u>http://www.who.int/world-health-day/previous/2005/infomaterials/en/</u>

TASK CONTENT: Students will multiply a fraction by a whole number utilizing repeated addition.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.3 Understand a fraction $\frac{a}{b}$ with a numerator >1 as a sum of unit fractions $\frac{1}{b}$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

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BACKGROUND KNOWLEDGE

Students should now have a strong understanding of fraction multiplication. Through investigations they have explored how repeated addition of fraction is similar to multiplication of fractions. However, often the tasks and situations encountered ask students to multiply a whole number by a fraction. Modeling the multiplication of a fraction by a whole number can pose a challenge and some students may see it as a different context. For example, if 4 people share 2 pounds of ground beef equally to cook a recipe then each person will have a $\frac{1}{2}$ lb portion of ground beef. Some students may see this differently. Students may think that if 4 people each contributed $\frac{1}{2}$ pound of ground beef for a recipe that totals 2 pounds of ground beef. The context will ask for students to either arrive at a fraction or arrive at a whole number. This task further develops their ability to repeatedly add a fraction until you arrive at a larger mixed number or whole number, as opposed to decomposing a whole number or mixed number into smaller fractions.

Some misconceptions that students may still be struggling with is that the denominator stays the same when repeatedly adding a unit fraction such as $\frac{1}{2}$, for example, $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$ as opposed to $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{6}$ which is incorrect. Some students may still be discovering the relationship between the numerator and denominator in relation to 1. When the numerator is greater than the denominator we have passed the benchmark of 1 whole.

Students will need to approach this task with the following prerequisite knowledge:

- Experience with common plane figures and the identification of their sides and angles.
- Familiarity with tangram puzzles
- Knowledge of area and congruence

ESSENTIAL QUESTIONS

- What happens when we use repeated addition to add fractions?
- How can we use multiplication rather than repeated addition when adding fractions?
- What happens to a fraction when it is larger than 1?

MATERIALS

- *The Warlord's Puzzle* by Virginia Walton Pilegard, or a similar book about tangrams
- "Who Put the Tang in Tangrams? Finding Areas" student recording sheet (2 pages)
- Tangram sets

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)*

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task will introduce students to the value of a triangle compared to a square, revisit and reinforce student understanding of area, and the goal of this lesson is to provide students with further opportunities to repeatedly add a fraction, decompose fractions, and ultimately develop a strong foundation for multiplying fractions. The class begins the activity by discussing how we measure area, namely in square units. The teacher then provides the students with tangram puzzle pieces and through manipulating them students will discover that the small triangle is half the size of the square unit and is therefore assigned a value of $\frac{1}{2}$. Students determine the area of tangram pieces without using formulae by determining how many small triangles will cover up each shape.

Part 1 – Area of Tangram Pieces

Comments

As an introduction to this task, the book *The Warlord's Puzzle* by Virginia Walton Pilegard or a similar book about tangrams can be read to the students. After the story, guide students to create their own tangram pieces through paper folding. Directions with illustrations can be found below and at the following web site: <u>http://mathforum.org/trscavo/tangrams/construct.html</u>.

- Fold a 9 x 12 piece of art paper to form a square. Cut off the extra piece at the bottom and discard.
- Cut the square in half on the diagonal fold to form two triangles.
- Take one of the triangles and fold it in half to form two smaller congruent triangles. Cut along the fold.
- Take the other large triangle and make a small pinch crease in the middle of the baseline (longest side) to identify the center. Take the apex of the triangle (the vertex opposite the longest side) and fold it to touch the center of the baseline. This forms a trapezoid.
- Cut along the fold line. This gives you a trapezoid and a small triangle.
- Fold the trapezoid in half (two congruent shapes) and cut along the fold line.
- Take one half of the trapezoid and fold the pointed end to form a small square. Cut along the fold. This will give you a small square and a small triangle.
- Take the remaining half of the trapezoid. Fold one of the corners of the square end to form a small triangle and a parallelogram. Cut along the fold.

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As you deconstruct the square, discuss the relationships between the pieces. Once a full set is completed (One small square, two small congruent triangles, two large congruent triangles, a medium size triangle and a parallelogram), ask students to experiment with the shapes to create new figures. If you have no plastic tangrams, have students cut these from card stock so that they may be traced as follows.

To start this task, give each student a set of plastic tangrams to use for this task. (They are easier to trace than the paper ones.) Ask students to find the two small congruent triangles and review the definition of congruent: same size, same shape. Put them together to make a square. Ask students what the area of this shape would be and ask them to explain how they know. (Because the square formed with the two small triangles is congruent to the tangram square, its area must be the same: $1 u^2$) Next ask students to take just one of the small triangles. Ask students what its area would be and ask them to explain how they know. Remember to relate it to the square. (The area of each small triangle is half of the area of the square, so its area is $\frac{1}{2} u^2$). Give students time to try to determine how to make the shapes before students share their work. This process time for thinking and experimenting will help students develop their spatial reasoning. Next, give each student the "Who Put the Tang in Tangram? Finding Areas" student recording sheet. Ask students to work with a partner to find the area of the given shapes. Once most students have completed finding the area of the figures, encourage partners to model how they found the areas and ask the class if they agree or disagree, requiring students to explain their

thinking.

Students will be figuring the area of each of the tangram pieces by comparing them to the small square. Sample solutions for the "<u>Who Put the Tang in Tangram</u>? Finding Areas" student recording sheet are shown below.

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Area of Figure Figure Show your work (in square units) The triangle is half of the square so it Two small triangles can be Small Triangle arranged to make a square must have an area of $\frac{1}{2}$ u². congruent to the square with an area of 1 u². Medium Triangle Two small triangles can be Since the area of each small triangle is arranged as shown to u², the area of the medium triangle must make a triangle congruent be 1 u². to the medium triangle. Large Triangle The square and two small Since the area of each small triangle is -(Use the square) triangles can be arranged u² and the area of the square is 1 u², the as shown to make a area of the large triangle must be triangle congruent to the $2 u^{2} \left(\frac{1}{2} u^{2} + \frac{1}{2} u^{2} + 1 u^{2}\right).$ large triangle. Two small triangles can be Parallelogram Since the area of each small triangle is $\frac{1}{2}$ arranged as shown to u², the area of the parallelogram must be 1 make a parallelogram u^2 congruent to the given parallelogram. Trapezoid A small triangle and the Since the area of a small triangle is $\frac{1}{2}u^2$ parallelogram can be and the area of the parallelogram 1 u2, the combined as shown to area of the large triangle must create the trapezoid. be $1\frac{1}{2}u^2\left(\frac{1}{2}u^2 + 1u^2\right)$. Two small and one medium Two small and one Since the area of each small triangle is $\frac{1}{2}$ triangle medium triangle can be u² and the area of the medium triangle is 1 combined as shown to u2, the area of the triangle must be 2 make this square. $u^{2}\left(\frac{1}{2}u^{2}+\frac{1}{2}u^{2}+1u^{2}\right)$ Rectangle Two small triangles and Since the area of each small triangle is 1 the parallelogram can be u² and the area of the parallelogram is 1 combined as shown to u², the area of the rectangle must be create this rectangle. $2 u^2 \left(\frac{1}{2} u^2 + \frac{1}{2} u^2 + 1 u^2 \right).$

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Figure Sketch it below	Show your work	Area of Figure (in square units)
Triangle congruent to a large triangle (Do not use the square)	Two small triangles and the medium triangle can be arranged to make a triangle congruent to the large triangle.	Since the area of each small triangle is $\frac{1}{2} \underbrace{u^2}_{u^2}$ and the area of the medium triangle is 1 u ² , the area of the large triangle must be 2 u ² $\left(\frac{1}{2} u^2 + \frac{1}{2} u^2 + 1 u^2\right)$.
Trapezoid (Different from the one page 1)	One possible trapezoid can be made using two small triangles and the parallelogram.	Since the area of each small triangle is $\frac{1}{2}u^2$ and the area of the parallelogram is 1 u ² , the area of the trapezoid must be 2 u ² $\left(\frac{1}{2}u^2 + \frac{1}{2}u^2 + 1u^2\right)$.
Parallelogram (Different from the one on page 1)	One possible parallelogram can be made as shown using two small triangles and the parallelogram.	Since the area of each small triangle is $\frac{1}{2}u^2$ and the area of the parallelogram is $1u^2$, the area of the created parallelogram must be $2u^2(\frac{1}{2}u^2 + \frac{1}{2}u^2 + 1u^2)$.
Pentagon	One possible pentagon can be made as shown using two small triangles, the square and the medium triangle.	Since the area of each small triangle is $\frac{1}{2}u^2$, the area of the square is 1 u ² , and the area of the medium triangle is 1 u ² , the area of the pentagon must be 3 u ² $\left(\frac{1}{2}u^2 + \frac{1}{2}u^2 + 1 u^2 + 1 u^2\right)$.
Square using all 7 pieces	One possible square can be made as shown all seven pieces.	Since the area of each small triangle is $\frac{1}{2}$ u ² , the area of the square is 1 u ² , the area of the medium triangle is 1 u ² , the area of the parallelogram is 1 u ² , and the area each of the large triangles is 2 u ² , the area of the square must be 8 u ² ($\frac{1}{2}$ u ² + $\frac{1}{2}$ u ² + 1 u ² + 1 u ² + 1 u ² + 2 u ² + 2 u ²).

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Task Directions

Students will follow the directions in the "<u>Who Put the Tang in Tangram?</u> Finding Areas" student recording sheet.

FORMATIVE ASSESSMENT QUESTIONS

- What is the area of this shape? How do you know?
- What shapes have the same area as the area of this shape?
- What shapes did you use to create a figure congruent to this figure?
- What could you add to find the area of this shape?
- What is the value of a triangle, square, medium triangle, trapezoid, etc.?
- What is another way you could make that same shape?
- What addition number sentence could you write to help determine the area of your shape?
- Some shapes have a size of $\frac{5}{2}$. What is another way we can write that?
- Some shapes have a size of $2\frac{1}{2}$. What is another way we can write that?
- Where would you find $2\frac{1}{2}$ on a number line?
- Where would you find $\frac{5}{2}$ on a number line?
- What multiplication number sentence could you write to help you determine the area of your shape?

DIFFERENTIATION

Extension

- There are different ways to create many of the shapes on the "<u>Who Put the Tang in</u> <u>Tangram?</u> Finding Areas" student recording sheet. Allow students to explore these shapes to see if they can find different ways to create them using the tangrams.
- Challenge students by changing the value of the square unit and continue to explore the relative area sizes of each shape. For example, if they entire square puzzle had a value of 1 what would be the size of a square? (one eighth) What would be the size of small triangle? (one sixteenth)

Intervention

- For students with fine motor control difficulties do not have them trace the shapes. Just let them manipulate the tangrams. Also, students may be given a copy of the tangram puzzle so they just have to cut out the shapes, not fold to make the shapes.
- It might be helpful to give some students two sets of tangrams in different colors so they can more easily see the relationships between the shapes.
- Some students may benefit from thinking of the large triangles as $\frac{1}{2}$ a square unit, or the medium triangles as $\frac{1}{2}$ a square unit.

Intervention Table

TECHNOLOGY

• <u>http://pbskids.org/cyberchase/math-games/tanagram-game/</u> Tangram Game: Students can view a picture made out of tangram pieces and try to replicate it using the interactive tangram pieces provided.

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Name _____

Date _____

Who Put the Tang in Tangram?

Find the area of the following figures.



Figure	Show your work	Area of Figure (in square units)
Small Triangle		
Medium Triangle		
Large Triangle		
Parallelogram		
Trapezoid		
Two small and one medium		
triangles		
Rectangle		

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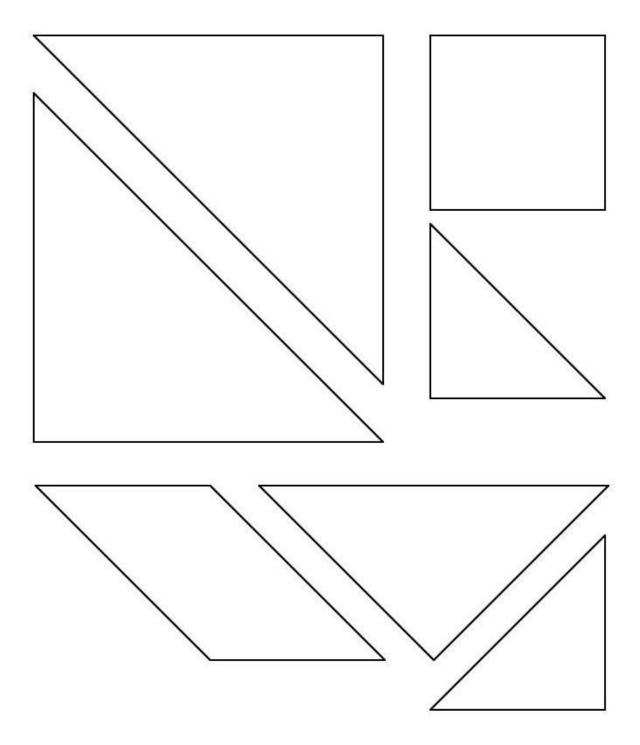
Figure Sketch it below	Show your work	Area of Figure (in square units)
Triangle congruent to a large triangle (Do not use the square)		
Trapezoid (Different from the one page 1)		
Parellelogram (Different from the one on page 1)		
Pentagon		
Square using all 7 pieces		

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Pick 4 of the shapes that you successfully determined the area of and complete the chart below.

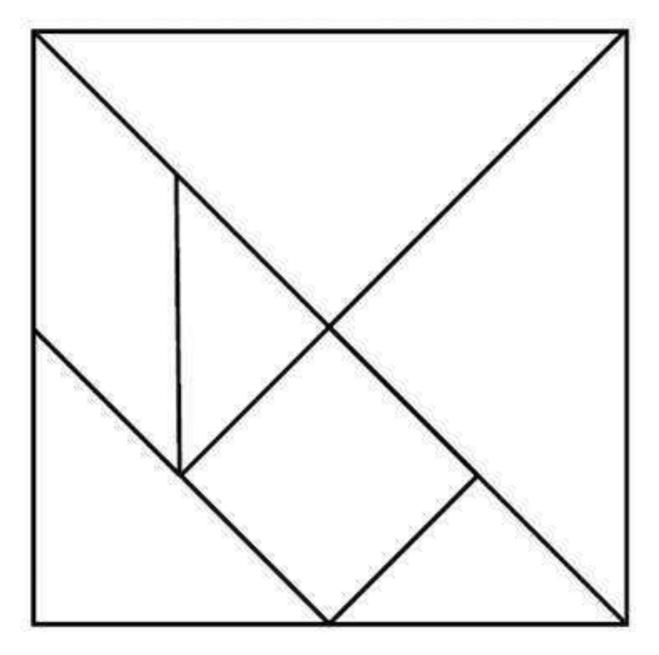
Figure	Addition Number Sentence	Multiplication Number Sentence
Draw the shape and name it.	Write an addition sentence that describes the area of your shape.	Write a multiplication sentence that describes the area of your shape.

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CONSTRUCTING TASK: Birthday Cookout

Back to Task Table

Students will solve story problems that involve the multiplication of fractions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.

b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

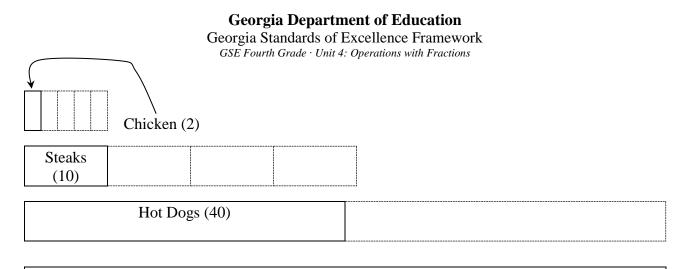
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

You may want to review problem solving strategies with your students as they begin work on this task. Strategies such as making a table and working backward are two approaches to this task. Another suggestion for solving fraction word problems such as this is to utilize the Singapore Math strategy of drawing bars that are proportionate to the values in the problem. For example, we know that 80 people ordered hamburgers so we can draw a large bar to represent the hamburgers. We can then draw a bar $\frac{1}{2}$ the size of our "hamburger" bar to represent the number of people that want hot dogs. Next, we can draw a bar that is $\frac{1}{4}$ the size of our "hot dog" bar to represent the number of people that want steak. Finally, we can draw a bar $\frac{1}{5}$ the size of our "steak" bar to represent the number of people that want chicken.



Hamburgers (80)

ESSENTIAL QUESTIONS

- How can we use fractions to help us solve problems?
- How can we model answers to fraction problems?
- How can we write equations to represent our answers when solving word problems?

MATERIALS

• "Birthday Cookout" student recording sheet

GROUPING

Partner/Small Group Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task asks students to use problem solving strategies and their knowledge of fractions to solve a real-world problem involving food for a birthday party.

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Comments

The setting of this task is likely a familiar one for students. You may want to begin with a discussion of how math is used when planning a birthday party. The discussion may include a wide range of mathematical ideas such as number of invitations, amount of food, and the amount of money needed to purchase food.

Solutions are given below:

- How many people asked for chicken? $(\frac{1}{5} \text{ of } 10 \text{ is } 2)$
- How many people asked for steak? $(\frac{1}{4} \text{ of } 40 \text{ is } 10)$
- How many people asked for hot dogs? $(\frac{1}{2} \text{ of } 80 \text{ is } 40)$

Task Directions

Have students follow the directions on the "Birthday Cookout" student recording sheet.

FORMATIVE ASSESSMENT QUESTIONS

- What problem solving strategies will you use to solve this problem?
- What models will you use to determine what the chef needs to know?
- How are you using fractions to help solve this problem?

DIFFERENTIATION

Extension

- Have students research and determine the cost of the items the chef needs.
- Have students create their own menu and create a new problem involving fractions.
- Have students determine the percentage of guests who chose each menu item.
- Students could explore this problem with larger numbers such as 320 hamburgers and then look for patterns.
- Students could be given different information to begin with other than the number of hamburgers. How would the problem change if we only knew that 40 people asked for steak?

Intervention

- Use smaller numbers, for example instead of 80 hamburgers, use 40 hamburgers.
 - How many people asked for chicken? $(\frac{1}{5} \text{ of } 5 \text{ is } 1)$
 - How many people asked for steak? $(\frac{1}{4} \text{ of } 20 \text{ is } 5)$
 - How many asked for hot-dogs? $(\frac{1}{2} \text{ of } 40 \text{ is } 20)$
- These tasks can always be physically performed with fraction tiles for the more kinesthetic learners. Many fraction bars are labeled and students can turn them over to assign them the value of the hamburgers, hot dogs, etc.

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look at Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> Identify Fractional Parts of a Group: This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> Another Look at the Set Model Using Attribute Pieces: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

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Name _____

Date _____

Birthday Cookout

Bob is turning 60 this year! His family is celebrating by having a cookout. Marcy took orders and found one fifth as many people wanted chicken as wanted steaks, one fourth as many people wanted steaks as wanted hot dogs, and one half as many people wanted hot dogs as wanted hamburgers. She gave her son-in-law, the chef, an order for 80 hamburgers.

The chef needs more information. He has to know:

- How many people asked for chicken?
- How many people asked for steak?
- How many asked for hot-dogs?

Use words, pictures and numbers to tell the chef what he needs to know. Be prepared to share!



PERFORMANCE TASK: A Chance Surgery

Back to Task Table

In this task, students will use models to multiply a whole number by a fraction. This information will be used to complete the multi-step task, which involves multiplication of whole numbers as well.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students need to be able to apply their understanding of mathematical concepts with multi-step problems. In accordance with SMP 1, students need to make sense of the problems in an attempt to solve them. For this task it is essential that students make sense of the information provided, as students will need to apply their fractional understanding in order to multiply accurately. The operation of multiplication is applied to various kinds of numbers.

ESSENTIAL QUESTIONS

• How can I model the multiplication of a whole number and a fraction?

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- What fractional understanding do I need to multiply a fraction by a whole number?
- How do I solve a multi-step problem?

MATERIALS

• "A Chance Surgery" recording sheet

GROUPING

Individual task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students will need to determine $\frac{2}{3}$ and $\frac{1}{3}$ of 15 in order to correctly complete the multi-step task.

TASK

Dr. Clifton is a surgeon at Children's Healthcare of Atlanta Egleston. In 2012, he performed 15 surgeries to treat biliary atresia, a condition in which bile cannot be expelled from the liver.

Studies have shown that $\frac{2}{3}$ of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton's patients last year, how many will eventually need a liver transplant? (10) According to the statistic, how many patients will not need a transplant? (5) If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many

If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year? (60 hours)

If Dr. Clifton's caseload doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant? (20) How many hours should he anticipate conducting the surgery in 2013? (120 hours)

FORMATIVE ASSESSMENT QUESTIONS

- Explain how you determined the number of patients who will need a transplant. How did you determine the number of patients who will not need a transplant?
- How did you determine the hours spent in surgery in 2012?
- How did you model multiplying a fraction by a whole number?
- How does doubling the caseload affect all other information?

DIFFERENTIATION

Extension

- Determine the amount of patients who will not need a liver transplant and who will need a liver transplant if Dr. Clifton's caseload quadrupled. How many hours would he be in surgery?
- Create a function table that shows the effect of the caseload on the amount of hours in surgery. Include at least 5 data points within the table.

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Intervention

• Allow students to use manipulatives such as counters, beans, bears, etc. to represent the 15 patients.

Intervention Table

TECHNOLOGY

- <u>https://www.illustrativemathematics.org/content-standards/4/NF/B/4/tasks/857</u> Sugar in Six Cans of Soda: This task from Illustrative Mathematics is also a problem based situation which students can apply multiplying a fraction by a whole number to find a solution.
- <u>http://3-5cctask.ncdpi.wikispaces.net/4.NF.3-4.NF.4</u> Public Schools of North Carolina has tasks that are also problem solving opportunities that students would benefit from completing.

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Name ___

_____ Date_____

A Chance Surgery

Dr. Clifton is a surgeon at Children's Healthcare of Atlanta Egleston. In 2012, he performed 15 surgeries to treat biliary atresia, a condition in which bile cannot be expelled from the liver. Studies have shown that $\frac{2}{3}$ of the patients treated for biliary atresia eventually need a liver transplant. Of Dr. Clifton's patients last year how many will eventually need a liver transplant? According to the statistic, how many patients will not need a transplant?

If it typically takes Dr. Clifton 4 hours to complete a biliary atresia surgery, about how many hours did he perform this operation last year?

If Dr. Clifton's case load doubles in 2013, how many patients can he expect to have successful surgery and possibly not need a liver transplant? How many hours should he anticipate conducting the surgery in 2013?

Georgia Department of Education Georgia Standards of Excellence Framework

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PRACTICE TASK: Fraction Pie Game

Back to Task Table

Students will practice writing number sentences to show multiplication of fractions.

STANDARDS OF MATHEMATICAL CONTENT

MGSE4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

When asked what is greater, $\frac{1}{2}$ or $\frac{1}{4}$, many students will say that $\frac{1}{4}$ is larger. When asked why, they will say, "Because four is a greater than two." Students need a lot of exposure to fractions to discover various essential fractional concepts. One important concept involves students discovering the relationship between the size of a fractional piece and the size of the denominator. The greater the denominator, the smaller the unit fraction. *Be careful not to teach this as a blanket rule, as this idea can become a misconception when students compare two fractions and neglect to consider numerators as well as denominators. For example, when comparing 1/10 and 50/100, a student may say 50/100 is smaller because the greater the denominator, the smaller the fractions. This game is intentionally broken up into two parts so students can see that the greater the denominator, the greater the number of divisions, which means the size of the piece is smaller. Young students also typically enjoy games of chance. Probability is a skill that also builds fractional understanding. Having students play with both 6-sided and 10-sided dice will also build this skill and they will begin to gain some understanding of both fractions and their odds when playing games of chance.*

Before asking students to work on this task, be sure students are able to:

- Use repeated addition to add fractions with the same denominator.
- decompose fractions, for example $\frac{4}{4} = \frac{1}{2} + \frac{1}{2}$ or $\frac{1}{4} + \frac{3}{4}$

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• understand that the whole can be any number and the fractions always depend on taking a portion of this whole

ESSENTIAL QUESTIONS

- What does it mean to take a fractional portion of a whole number?
- How is multiplication of fractions similar to repeated addition of fraction?
- What is the relationship between the size of the denominator and the size of each fractional piece (i.e. the numerator)?

MATERIALS

- Fraction circles
- Colored pencils or crayons
- 6- or 10-sided dice
- Fraction Pie Game student recording sheet.

GROUPING

Group/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)*

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students will play a simple game of chance to see who can fill up 15 wholes on their score card and game board first. Children at this age enjoy games of chance, however this game also gives students a chance to practice using their fractional understandings and also requires them to use logical thinking and problem solving strategies.

Comments

Colored fraction circles should be made available to the students as much as possible. However, if these manipulatives are not available, then student's score cards may also be colored in. Fraction bars can be used as an alternative manipulative for this task. Students may even utilize the fraction bars as a strategy for keeping track of their score.

This task could be introduced by playing similar commercially bought dice games, such as Yahtzee, which are great, but often reinforce whole number operations and not fractions. Some

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students might be familiar with these games. To teach the students how to play the game, the teacher could model with a student and play one round. To play the game, a player rolls the dice. The value of the roll indicates the number of fraction pieces you can shade in.

The game is designed to build some fractional thinking alongside some strategy. For example, if a player rolled a 6, they could shade in a fraction with a denominator such as tenths or twelfths. Students will quickly realize that the greater the denominator, the more challenging it is to cover up that fraction.

Task Directions

Students will follow directions below from the Fraction Pie Game recording sheet. The object of the game is to be the first person to 15 wholes. You have to complete a whole for it to count toward the 15. You need to be able to prove your answer.

2-4 person game

1. Each person takes turns rolling the dice.

2. After you have rolled, you must pick what type of pie you choose to color in and then you may color in fractional pie for the value of the role (for example, if you rolled a 6 and chose halves then you may shade in 6 halves)

- 3. You are also responsible for filling in your score card as you play.
- 4. Play several rounds, but be sure to share your strategy when you are done.
- 5. Be sure you can prove your answer.

FORMATIVE ASSESSMENT QUESTIONS

- What fraction circles are you trying to fill up first? Why?
- What strategies do you have for this game?
- If you roll a number that is greater, what fraction circle might you try to fill in?
- If you roll a number that is less, what fraction circle might you try to fill in?

DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson could be extended to use a larger value then 15, although more score cards will need to be reproduced. For example, if two score cards were made available, students could play to 20 or 30.
- Students could also play with additional dice.
- Students could be introduced to fractions such as $\frac{1}{25}$, $\frac{1}{50}$, and $\frac{1}{100}$.

Intervention

- Students may need to play to a smaller whole than 15.
- Students could benefit from number lines such as the ones included in the fraction strips below.

Intervention Table

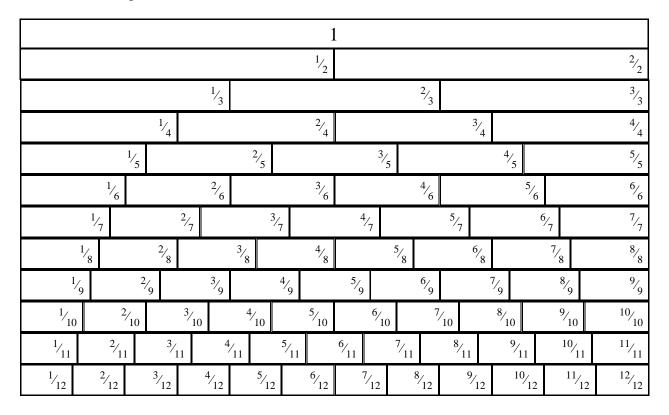
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TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look at Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> Fractional Parts of a Group: This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> Another Look at the Set Model Using Attribute Pieces: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.



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Name:	Date:	
	Fraction Pie Game Part 1	
The object of the game is t	o be the first person to make 15 wholes. Yo	ou have to complete a

whole for it to count toward the 15. You need to be able to prove your answer.

2-4 person game

1. Each person takes turns rolling the dice.

2. After you have rolled, you must pick what type of pie you choose to color in and then you may color in fractional pies for the value of the roll (for example, if you rolled a 6 and chose halves then you may shade in 6 halves)

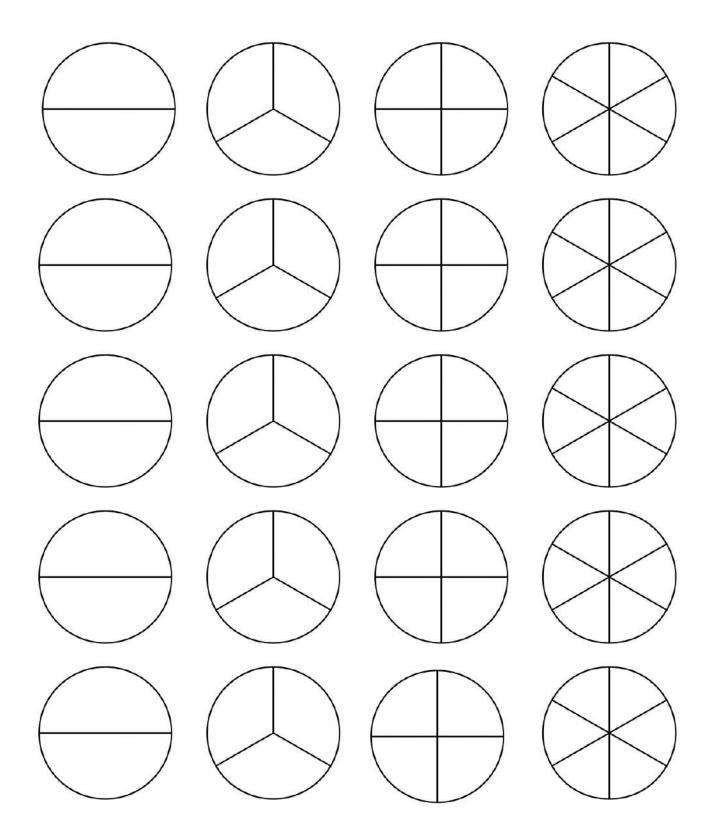
3. You are also responsible for filling in your score card as you play.

4. Play several rounds, but be sure to share your strategy when you are done.

5. Be sure you can prove your answer.

Fraction	Tally	Fraction x Tally	Product
1			
2			
1			
3			
1			
4			
1			
6			

TOTAL: _____



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Name:	Date:	
	Fraction Pie Game Part 2	

The object of the game is to be the first person to 15 wholes. You have to complete a whole for it count toward the 15. You need to be able to prove your answer.

2-4 person game

1. Each person takes turns rolling the dice.

2. After you have rolled, you must pick what type of pie you choose to color in, and then you may color in fractional pie for the value of the roll (for example, if you rolled a 6 and chose halves, then you may shade in 6 halves)

3. You are also responsible for filling in your score card as you play.

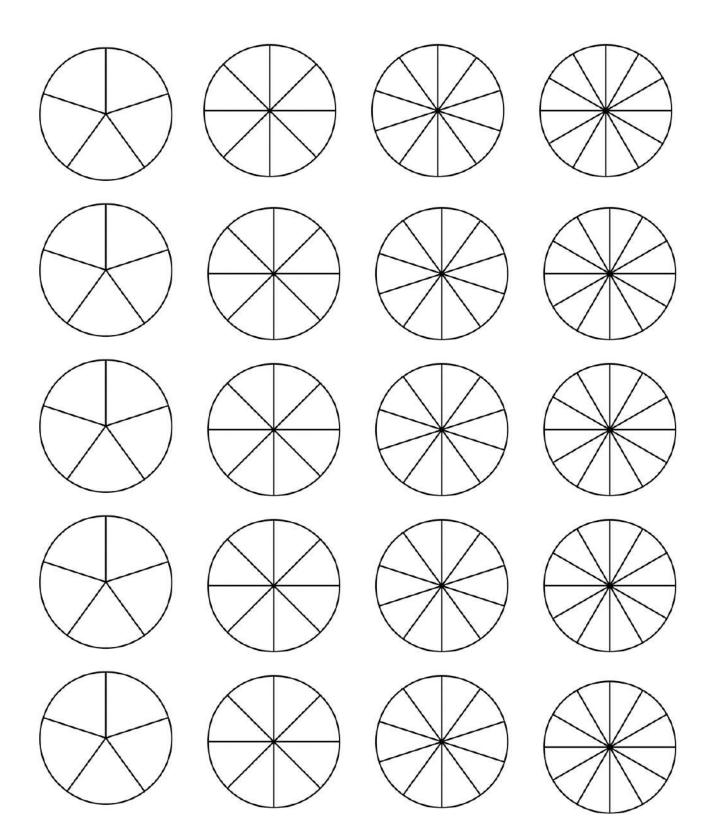
4. Play several rounds, but be sure to share your strategy when you are done.

5. Be sure you can prove your answer.

Fraction	Tally	Fraction x Tally	Product
1			
5			
1			
8			
1			
10			
1			
12			

TOTAL: _____

Was it easier to get to 15 in Part 1 or Part 2? Why do you suppose this is?



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<u>3-ACT TASK:</u> How Much Sugar?

Back to Task Table

Adapted from www.gfletchy.wordpress.com

TASK CONTENT: Students will determine the fractional part of sugar in a can of soda.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at <u>http://blog.mrmeyer.com/category/3acts/</u>. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the *Guide to Three-Act Tasks* on georgiastandards.org and the K-5 Georgia Mathematics Wiki.

Before assigning the task, students should have had several experiences with finding a fraction of a set. Also, students should have some degree of fluency with multiplication facts with a product of 24 or less.

ESSENTIAL QUESTIONS

- How can I be sure fractional parts are equal in size?
- What do the numbers (terms) in a fraction represent?
- How does the number of equal pieces affect the fraction name?
- How can I represent a fraction of a discrete model (a set)?
- How are multiplication, division, and fractions related?

MATERIALS

• How Much Sugar Student Recording sheet

GROUPING

Individual or partner task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will watch the video and then tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. Their curiosities will be recorded as questions on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Task Directions

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I video.

- 1. Show Act I video to students. <u>http://vimeo.com/86619134</u>
- 2. Pass out the 3 Act recording sheet.
- 3. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first before sharing them aloud and then record Mathematics • GSE Grade 4 • Unit 4: Operations with Fractions

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these questions on the recording sheet (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.

- 4. Anticipated questions students may ask and wish to answer: How much sugar is in a case of Mountain Dew? How much sugar is in a can of Mountain Dew?
- 5. As the facilitator, you can select which question you would like every student to answer, have students vote on which question the class will answer or allow the students to pick which question they would like to answer. Once the question is selected ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates one that is too low and one that is too high so that they establish a range in which the solution should occur. Instruct students to record their estimates on a number line.

Act II – Student Exploration - Provide additional information as students work toward solutions to their questions.

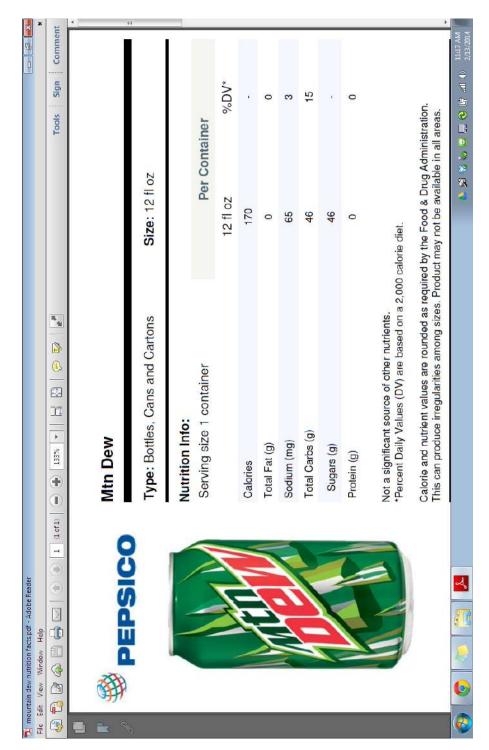
1. Ask students to determine what additional information they will need to solve their questions. The teacher provides this information only when students ask for it:

46 grams of sugar = approximately 1/5 of a cup of sugar

1 can of Mountain Dew = 1/5 of a cup of sugar

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- 2. Ask students to work in small groups to answer the questions they created in Act I. The teacher provides guidance as needed during this phase by asking questions such as:
 - a. Can you explain what you've done so far?
 - b. What strategies are you using?
 - c. What assumptions are you making?
 - d. What tools or models may help you?
 - e. Why is that true?
 - f. Does that make sense?

Act III – Whole Group - Share student solutions and strategies as well as Act III solution.

- 1. Ask students to present their solutions and strategies.
- 2. Share solution.
- 3. Lead discussion to compare these, asking questions such as:
 - a. How reasonable was your estimate?
 - b. Which strategy was most efficient?
 - c. Can you think of another method that might have worked?
 - d. What might you do differently next time?

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FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

- Challenge students to determine the amount of sugar in a 2-liter bottle of Mountain Dew.
- Encourage students to research other can sodas such as Pepsi, Sprite or Coca-Cola and determine the amount of sugar in a case of the soda.

Intervention

• Encourage students to begin looking at the amount of sugar in a six pack of Mountain Dew.

Intervention Table

TECHNOLOGY

• <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L341</u> Class Attributes: This lesson reinforces fractional parts. It can be used to extend understanding of the concept.

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Name: _____

Adapted from Andrew Stadel

Task Title: ACT 1

What questions come to your mind?

On an empty number line, record an estimate that is too low, just right and an estimate that is too high. Explain your estimates.

<u>ACT 2</u>

What information would you like to know or need to solve the MAIN question?

Use this area for your work, tables, calculations, sketches, and final solution.

<u>ACT 3</u>

What was the result?

Record the actual answer on the number line above containing the three previous estimates.

ACT 4 (use this space when necessary)

PERFORMANCE TASK: Fraction Farm

Back to Task Table

In this task students will solve a story problem that involves the multiplication of a whole number by a fraction using pattern blocks.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

A great strategy for this type of problems solving task is to construct some sort of table. Students have been exposed to tables throughout this unit, however, the tasks have always included a table and students are then asked to fill it in. For this task, a table has not been included intentionally, with the hope the students will have learned the value of organizing their information. You may ask, "How could you organize your information?" to guide students.

For this task, students should be allowed to use pattern blocks because rearranging the pattern blocks will allow them to make four hexagons and therefore start to determine the fractional value of each piece. Once the fraction is determined, students then begin multiplying the total value of \$1,200 by each fraction. For example, Field C is a hexagon and represents ¹/₄ of the property and is therefore valued at ¹/₄ x \$1,200 = \$300. If students are not allowed to manipulate pattern blocks they may struggle with finding each fractional value. Alternatively, students could be provided with isometric pattern block paper, draw the field, and determine the fractional value that way. Isometric pattern block paper has been included with this task.

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Answer Key:

Field	Shape	Fraction of Whole	Dollar Value
Т	Trapezoid	¹ / ₈ Or ³ / ₂₄	\$150
В	Parallelogram	¹ / ₁₂ OF ² / ₂₄	\$100
С	Hexagon	1/4 or %24	\$300
D	Parallelogram	¹ / ₁₂ OF ² / ₂₄	\$100
E	Triangle	1/24	\$50
F	Parallelogram	¹ / ₁₂ OF ² / ₂₄	\$100
G	Trapezoid	¹ / ₈ or ³ / ₂₄	\$150
Н	Trapezoid	¹ / ₈ Or ³ / ₂₄	\$150
Ι	Triangle	1/24	\$50
J	Triangle	1/24	\$50
			TOTAL: \$1,200

ESSENTIAL QUESTIONS

- How can I multiply a whole number by a fraction?
- What is the relationship between multiplication by a fraction and division?
- How can I represent multiplication of a whole number?

MATERIALS

- Pattern Blocks
- Fraction Farm student handout
- Crayons or colored pencils

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem

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Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students are asked to determine the value of the fractional pieces of a farm. The farm has been divided into several plots, each representing a fraction of the whole.

Task Directions

- The farm fields above are worth a total of \$1,200. The fields are divided up like pattern blocks; hexagons, trapezoids, parallelograms and triangles. Each field's value is based on its size.
- What part of the whole farm does each field represent?
- What is the dollar value of each field?
- Show and explain all of your mathematical thinking.

FORMATIVE ASSESSMENT QUESTIONS

- How can pattern block or isometric pattern block graph paper help you determine each fraction?
- Is there another way to represent that fraction?
- How are you going to organize your work?
- What is the value of the hexagon and what shapes can cover a hexagon?

DIFFERENTIATION

Extension

- Once students have completed the task above, this lesson can be extended by changing the value of the land to \$2,400 or \$3,600.
- Students could also be encourage to create their own field with subplots of land using pattern blocks. This would be an open-ended assignment and require a higher degree of critical thinking, but very worthwhile. It would also self- differentiate. Students ready for a more complicated fractional field could build larger designs with lots of pattern blocks.
- Students could work in pairs to design their own farm, then trade their design with another pair of students and solve another's design.

Intervention

- If necessary, students could make a smaller design that is less complicated.
- Students struggling with multiplying a whole number by a fraction could begin with a smaller whole number such as \$240.
- Some students may still need to be provided with a table. See below:

Intervention Table

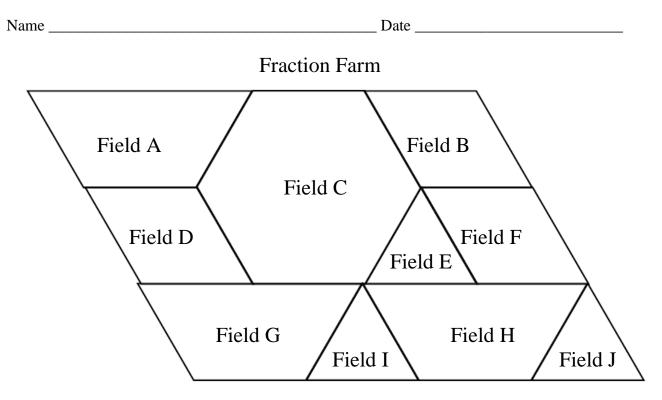
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Field	Shape	Fraction of Whole	Dollar Value
Т	Trapezoid	¹ / ₈ Or ³ / ₂₄	\$150
В	Parallelogram	$\frac{1}{12}$ or $\frac{2}{24}$	\$100
С	Hexagon	¹ / ₄ or ⁶ / ₂₄	\$300
D	Parallelogram	¹ / ₁₂ Or ² / ₂₄	\$100
E	Triangle	1/24	\$50
F	Parallelogram	¹ / ₁₂ Or ² / ₂₄	\$100
G	Trapezoid	¹ / ₈ Or ³ / ₂₄	\$150
Н	Trapezoid	¹ / ₈ Or ³ / ₂₄	\$150
Ι	Triangle	1/24	\$50
J	Triangle	1/24	\$50
			TOTAL: \$1,200

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look at Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
- <u>http://www.visualfractions.com/Identify_sets.html</u> Fractional Parts of a Group: This site would be best utilized with a lot of guidance from the teacher. It can be used to reinforce the concept within a small group or extend understanding of the concept.
- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L339</u> Another Look at the Set Model Using Attribute Pieces: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.

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The farm fields above are worth a total of \$1,200. The fields are divided up like pattern blocks; hexagons, trapezoids, parallelograms and triangles. Each field's value is based on its size (its fractional relationship to the whole plot of farm land). What part of the whole farm does each field represent? What would a fair price for each field be, based on its fractional value? Show and explain all of your mathematical thinking.

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CCSS Mathematics Content Standards Rubric - Grades 3 & 4

Students apply mathematical concepts, reasoning, and procedural skills in problems-solving situations and support their solutions using computations, mathematical language, and appropriate representations/ modeling.

CCSS	using computations, mathematical language, and appropriate representations/ modering.			
Math Criteria by Strand	Novice	Apprentice	Practitioner	Expert
Number & Operations in Base Ten , Number & Operations – Fractions, & Operations & Algebraic Thinking	Applies flawed strategies (e.g., attempts to form groups when multiplying, but does not use equal sized groups) Selects the incorrect operation to perform or major inaccuracies in computation lead to an incorrect solution Still demonstrates limited knowledge of place value or number sense (e.g., difficulty estimating, representing part-whole relationships; cannot determine reasonableness of an answer; does not see relationship between multiplication-division) A correct answer may be stated, but is not supported by student work or explanations	Some parts of problem correct and those parts are supported by student work Uses visual models (number line, area models, set models) to represent and compare parts of a whole, but stops short of applying concepts in problem solving contexts Demonstrates relationships between multiplication and division with whole numbers using number facts, objects, visuals, and symbols/equations May include limited/ partial explanations for solutions Uses 4 operations with small whole numbers, but displays some inaccuracies in computations of large numbers (multi-digit) and small numbers (fractions, decimals)	Expresses whole numbers as fractions (gr 3) and fractions as decimals with 10 or 100 as denominator (gr 4) 3.NF-3 4.NF-5, 6 Uses place value to round whole numbers 3.NBT-1 4.NBT-3 Expresses fractions and equivalent fractions (gr 3-4) and decimal-fraction equivalents (gr 4); explains/ illustrates why they are/are not equivalent (e.g., number lines, area models, sets; compare to benchmarks) 3.NF-1, 2, 3 4.NF-1, 2, 5, 6, 7 Uses addition, subtraction, and multiplication to solve problems with fractions (gr 4) 3.NF-1, 2, 3 4.NF-3, 4 Uses 4 operations in solving multi-step problems and word problems with whole numbers (e.g., using equations, arrays, explaining patterns using whole numbers, following a rule) 3.OA-3, 4, 5, 6, 7, 8, 9 3.NBT-2, 3 4.OA-2, 3, 5 May be some minor flaws when performing multi-step computations, <u>but</u> <u>procedural and conceptual</u> <u>understanding is clearly</u> evident	All parts of problem correct, precise, and supported by student work or explanations Extends understanding of equivalence of fractions by identifying proper and improper fractions Interprets meaning of the products when multiplying (gr 3-4) and remainders when dividing (gr 4) Uses a variety of representations (e.g., concrete models, diagrams, equations), strategies (e.g., place value, properties of operations), and algorithms to solve problems or represent solutions in multiple ways Uses additive or multiplicative reasoning to solve or interpret most problems

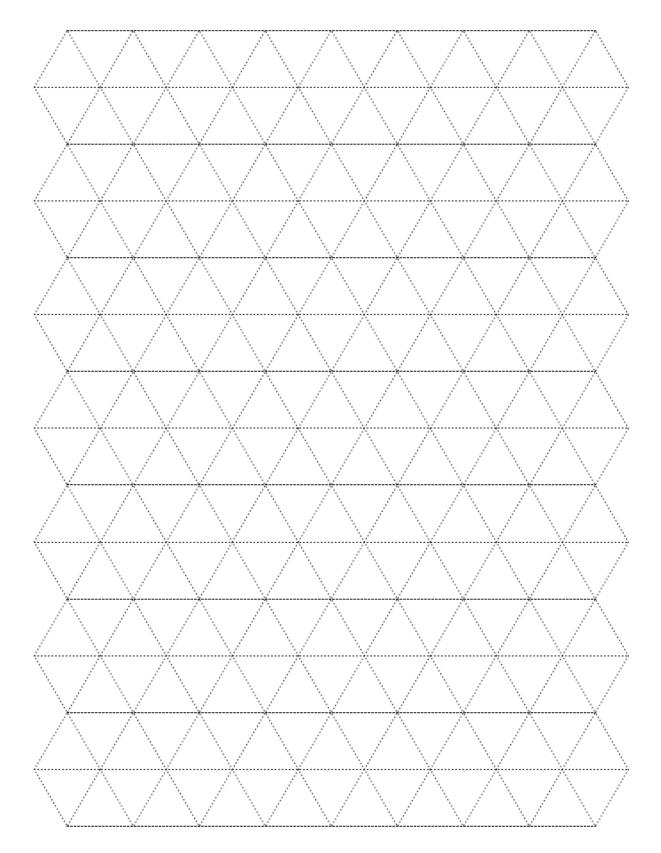
NOTE: Anchor papers will illustrate how descriptors for each performance level are evidenced at each grade.

Working Drafts of math content rubrics for assessing CCSS mathematics standards ---- Developed by Karin Hess, National Center for Assessment using several sources: CCSS for mathematics; NAAC mathematics LPFs (2010); First Steps in mathematics series; Math Exemplars rubrics --- (10.2011)

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Georgia Department of Education Georgia Standards of Excellence Framework

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CULMINATING TASK: Land Grant

Back to Task Table

In this culminating task students will multiply whole numbers by fractions and represent products. Students will then use their mathematical thinking to make a decision about a proposed land use situation.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$. b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

For this activity students will gain a lot of practice multiplying a whole number. In this particular task, the whole number is 12. Students will also be asked to determine the remainder of the land after they complete the first two tasks in each problem. For example, the first problem states, "the remaining land would be left for gardening and a walking path". For this part of the problem students must also realize that they can subtract to determine that there are seven acres left. In addition to practicing these skills, students will organize their information in tables to better understand it, and make comparisons. They will be asked to write out number sentences or brief explanations to show their products (i.e. $\frac{1}{4}$ of 12 or $\frac{1}{4} \times 12$). Last, but not least, students will use the information they collected to make a sound argument for what plan is best for the community. Students need to have many opportunities to use math to not only justify their answers but also to

justify their decisions in life. Adults do this constantly when we earn and spend. Students also need to know that math is often the deciding factor in many issues that we face today.

Before asking students to work on this task, be sure students are able to:

- use repeated addition to add fractions with the same denominator.
- decompose fractions, for example $\frac{4}{4} = \frac{1}{2} + \frac{1}{2}$ or $\frac{1}{4} + \frac{3}{4}$
- understand that the whole can be any number/size and the fractions always depend on taking a portion of this whole
- find equivalent fractions such as $\frac{1}{6} = \frac{2}{12}$.

ESSENTIAL QUESTIONS

- What does it mean to take a fraction portion of a whole number?
- How is multiplication of fractions similar to division of whole numbers?
- How do we determine the whole amount when given a fractional value of the whole?
- How do we determine a fractional value when given the whole number?

MATERIALS

- Colored Tiles
- Colored pencils or crayons
- "Land Grant" student recording sheet.

GROUPING

Individual/Partner Task

NUMBER TALKS

Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem "strings" which may be included in number talks to further develop mental math skills. See *Mini-lessons for Operations with Fractions, Decimals, and Percents* by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (*Mini-lessons for Operations with Fractions, Decimals, and Percents*, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

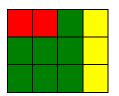
TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will decide how best to use a 12-acre plot of land that has been donated to the local community. Students will have to use what they have learned about addition, subtraction, and multiplication of fractions in order to successfully complete this task. In addition to this, students will also practice multiplying whole numbers and use amounts, such as acreage and building costs, to determine what plan is best for the community.

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Comments

Color tiles should be available for students to use. Students may want to just pick out three colors, one color for each purpose outlined in the activity. For example, in the first scenario the local library wants to build another branch. The 12 acres would be used as follows: the library itself would be $\frac{1}{6}$ of the land or 2 acres; the playground would be $\frac{1}{4}$ of the land or 3 acres and the remaining $\frac{7}{12}$ or 7 acres would be used for the garden. If a student chose to have red tiles for the library, yellow tiles for the playground and green tiles for the garden than their model could look like the one below.



This task could be introduced by bringing in local newspaper articles or by looking at how land is being used in the local community. Blue prints and maps could also help students understand this task better. Land use is always a contentious issue and newspapers such as The Atlanta Journal Constitution constantly publish articles about how local communities are using land. Utilize www.ajc.com for more information.

Task Directions

Students will follow directions below from the Land Grant task sheet.

- Obtain a set of colored tiles.
- Draw each plan using colored pencils or crayons.
- Work with a partner or small group to determine the products, equivalent fractions and/or sums for each step.
- Complete the tables included in the student recording sheets.
- Calculate construction given a specific cost per acre.
- Make an argument that utilizes math to help the city decide how to best use the land.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know how many acres the building, garden, pool, etc. will be?
- What strategies did you use to determine your product? (i.e. equivalent fractions, division of whole numbers, repeated addition)
- Do you think this is a good use of the land? Why or why not?
- What is another way that you can make this design work?

DIFFERENTIATION

Extension

• Once students have completed the task above, this lesson could be extended to use a greater acreage, such as 24, 30, or 36.

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- Students could use more practice solving problems where the numerator is a number other than 1, for example ⁵/₆ of 30.
- Students could solve more challenging problems that involve not just greater areas of land, but also more than three features on the land. For example, a 24 acre piece of land that will house a sports field, recreation center, pool, tennis courts, basketball courts, parking lot, etc.

Intervention

- Students may use repeated addition to solve these problems.
- Students could begin with a smaller number of acres, such as 6 or 8 and work up to the task as it is written.
- Students could solve for just two parts of the problems, such as the buildings and parking lot.

Intervention Table

TECHNOLOGY

- <u>http://illuminations.nctm.org/LessonDetail.aspx?ID=L342</u> Another Look at Fractions of a Set: In this lesson, students identify fractions in real-world contexts from a set of items that are not identical. It can be used as an extension of the concept.
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Name:		Date:
	Land Grant Part 1	

Someone recently donated 12 acres of land to the City Council. The donor wants the land to be used for the public good. Three different organizations have proposed plans for the land below. The local government needs your help deciding how this land could be best used. Make a drawing of each plan and determine how many acres they will need for both their buildings and their landscapes.

1. The Local Library wants to build a new library and use the remaining land for a community garden and a playground. The Library would take up one-sixth of the land, the playground would take up one-fourth of the land and the remaining land would be left for the community garden and a walking path. Determine how many acres of land each for the building, the playground and the garden.

	Fraction of the Whole	Acres
Building		
Playground		
Garden		

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2. The Parks and Recreation Department would also like to use the land, but for different purposes. They hope to use one third of the land for a Recreation Center that would include dance and fitness rooms, an art room, a gymnasium and locker rooms. In addition to this building, they also want to use one sixth of the land to build a swimming pool and tennis courts. Finally, they would use one half of the land for playing fields and a parking lot. Determine how many acres of land each for the Recreation Center, the pool, the tennis courts, and the playing fields.

	Fraction of the Whole	Acres
Recreation		
Center		
Swimming Pool		
& Tennis Courts		
Playing Fields		
and Parking Lot		

3. Last but not least, The National Forest Service wants to use the land to preserve the trees there. They would leave two-thirds of the land untouched but available for hiking trails. One fourth of the land would be used for a visitor center which would include a small museum and office. The remaining land would become a parking lot. Determine how many acres of land each for the hiking trails, the visitor center and the parking lot.

	Fraction of the Whole	Acres
Hiking Trails		
Visitor Center		
Parking Lot		

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Name:		Date:
	Land Grant Part 2	

The City Council has to provide running water and electricity to any building that is built. They use taxes to pay for this but it is expensive and will cost \$1,200 per acre. Determine how much tax money the City Council will need for each of the plans.

Organization	Tax Money Required
Library	
Parks and Recreation Department	
National Forest Service	

Now that we know the details of each organizations proposal, including how much tax money each will cost the city, please decide which land proposal you think is best.

- You must include fractions in your answer.
- You must include cost in your answer.
- You must consider what is best for the community.
- You may include any other information you feel is important.