



Georgia Standards of Excellence Frameworks

Mathematics

GSE Fourth Grade

Unit 2: Multiplication and Division of Whole
Numbers



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"Educating Georgia's Future"

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Unit 2: Multiplication and Division of Whole Numbers
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***Please note that all changes made to standards will appear in red bold type. Additional changes will appear in green.

OVERVIEW

In this unit students will:

- solve multi-step problems using the four operations
- use estimation to solve multiplication and division problems
- find factors and multiples
- identify prime and composite numbers
- generate patterns

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. The first unit should establish these routines, allowing students to gradually enhance their understanding of the concept of number and to develop computational proficiency.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “**Big Ideas**” be reviewed early in the planning process. A variety of resources should be utilized to supplement the tasks in this unit. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

For more detailed information about unpacking the content standards, unpacking a task, math routines and rituals, maintenance activities and more, please refer to the [Grade Level Overview for fourth grade](#).

STANDARDS FOR MATHEMATICAL PRACTICE

This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. This list is not exhaustive and will hopefully prompt further reflection and discussion.

1. **Make sense of problems and persevere in solving them.** Students make sense of problems involving multiplication and division.
2. **Reason abstractly and quantitatively.** Students demonstrate abstract reasoning about numbers, identifying which are prime and composite and explaining their identification.
3. **Construct viable arguments and critique the reasoning of others.** Students construct and critique arguments regarding number strategies including multiplication and division strategies.
4. **Model with mathematics.** Students use area models and rectangular arrays to model understanding of multiplication and division concepts.

5. **Use appropriate tools strategically.** Students select and use tools such as hundred charts and rectangular arrays. **Students will use hundreds charts, rectangular arrays,** and area models to identify types of numbers, factors and multiples and solve multiplication and division problems.
6. **Attend to precision.** Students attend to the language of real-world situations to determine if multiplication and division answers are reasonable.
7. **Look for and make use of structure.** Students relate the structure of an area model or rectangular array to determine the answers to multiplication and division problems.
8. **Look for and express regularity in repeated reasoning.** Students relate the structure of a hundred chart to identify prime and composite numbers, as well as, factors and multiples of numbers.

*****Mathematical Practices 1 and 6 should be evident in EVERY lesson.*****

STANDARDS FOR MATHEMATICAL CONTENT

Use the four operations with whole numbers to solve problems.

MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.

- a. **Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.**
- b. **Represent verbal statements of multiplicative comparisons as multiplication equations.**

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

MGSE4.OA.5 *Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.*

Use place value understanding and properties of operations to perform multi-digit arithmetic.

MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Geometric Measurement: understand concepts of angle and measure angles.

MGSE4.MD.8 *Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.*

BIG IDEAS

- Multiplication may be used to find the total number of objects when objects are arranged in equal groups.
- One of the factors in multiplication indicates the number of objects in a group and the other factor indicates the number of groups.
- Products may be calculated using invented strategies.
- Unfamiliar multiplication problems may be solved by using known multiplication facts and properties of multiplication and division. For example, $8 \times 7 = (8 \times 2) + (8 \times 5)$ and $18 \times 7 = (10 \times 7) + (8 \times 7)$.
- The properties of multiplication and division help us solve computation problems easily and provide reasoning for choices we make in problem solving.
- Multiplication may be represented by rectangular arrays/area models.
- There are two common situations where division may be used: fair sharing (given the total amount and the number of equal groups, determine how many/much in each group) and

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measurement (given the total amount and the amount in a group, determine how many groups of the same size can be created).

- Some division situations will produce a remainder, but the remainder will always be less than the divisor. If the remainder is greater than the divisor, that means at least one more can be given to each group (fair sharing) or at least one more group of the given size (the dividend) may be created.
- How the remainder is explained depends on the problem situation.
- The dividend, divisor, quotient, and remainder are related in the following manner:
 $\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$.
- The quotient remains unchanged when both the dividend and the divisor are multiplied or divided by the same number.
- Estimation is a helpful tool when finding the products of a 2- digit number multiplied by a 2-digit number.
- Multiplication and division can be represented using a rectangular area model.
- Multiplication may be used in problem contexts involving equal groups, rectangular arrays/area models, or rate.
- Multiply up to a 4-digit number by a 1-digit number using strategies.
- Divide whole-numbers quotients and remainders with up to four-digit dividends and remainders with up to four-digit dividends and one-digit divisors.

ESSENTIAL QUESTIONS Choose a few questions based on the needs of your students.

- What does it mean to factor?
- What is the difference between a prime and a composite number?
- What are multiples?
- How is skip counting related to identifying multiples?
- What is the difference between a factor and a product?
- How do we know if a number is prime or composite?
- How will diagrams help us determine and show the products of two-digit numbers?
- What patterns do I notice when I am multiplying whole numbers that can help me multiply more efficiently?
- What is a sensible answer to a real problem?
- **How is the area of a rectilinear figure calculated?**
- How can I ensure my answer is reasonable?
- What effect does a remainder have on a quotient?
- How can I mentally compute a division problem?
- What are compatible numbers and how do they aid in dividing whole numbers?
- How are multiplication and division related to each other?
- What are some simple methods for solving multiplication and division problems?
- What patterns of multiplication and division can assist us in problem solving?
- What happens in division when there are zeroes in both the divisor and the dividend?
- How are remainders and divisors related?
- What is the meaning of a remainder in a division problem?
- How can we use clues and reasoning to find an unknown number?
- How can we determine the relationships between numbers?
- How can we use patterns to solve problems?

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- How do multiplication, division, and estimation help us solve real world problems?
- How can we organize our work when solving a multi-step word problem?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Utilize the properties and patterns of multiplication (including the commutative, associative, and identity properties)
- Mentally solve basic multiplication problems using the distributive property. For example, 3×6 is 6 doubled and one more set of 6; $7 \times 4 = (2 \times 4) + (5 \times 4)$
- Fluently multiply within 100.
- Fluently divide within 100.

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: <http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf> and: <http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/>

STRATEGIES FOR TEACHING AND LEARNING

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.

SELECTED TERMS AND SYMBOLS

Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks. Common Core glossary: <http://www.corestandards.org/Math/Content/mathematics-glossary/glossary>. **The terms below are for teacher reference only and are not to be memorized by the students.**

- composite
- dividend
- divisor
- division (repeated subtraction)
- estimate
- factors
- multiplicand
- multiplier
- multiples
- partition division (fair-sharing)
- prime
- product
- properties
- quotient
- remainder

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they also may be used for teaching and learning.

Scaffolding Task	Tasks that build up to the learning task.
Constructing Task	Constructing understanding through deep/rich contextualized problem solving tasks.
Practice Task	Tasks that provide students opportunities to practice skills and concepts.
Performance Task	Tasks which may be a formative or summative assessment that checks for student understanding/misunderstanding and or progress toward the standard/learning goals at different points during a unit of instruction.
Culminating Task	Designed to require students to use several concepts learned during the unit to answer a new or unique situation. Allows students to give evidence of their own

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	understanding toward the mastery of the standard and requires them to extend their chain of mathematical reasoning.
Formative Assessment Lesson (FAL)	Lessons that support teachers in formative assessment which both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards.
CTE Classroom Tasks	Designed to demonstrate how the Career and Technical Education knowledge and skills can be integrated. The tasks provide teachers with realistic applications that combine mathematics and CTE content.
3-Act Task	A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the <i>Guide to Three-Act Tasks</i> on georgiastandards.org and the K-5 CCGPS Mathematics Wiki.

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Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)	Description
Factor Findings	Scaffolding Task <i>Partner Task</i>	Finding Factors	MGSE4.OA.4	Students will create a factor poster with array illustrations and factor rainbows.
My Son is Naughty	Constructing Task <i>Partner Task</i>	Finding Factors	MGSE4.OA.4	Students solve a problem involving finding factors of a number and addends of another number.
Investigating Prime and Composite	Scaffolding Task <i>Partner Task</i>	Prime and Composite Numbers	MGSE4.OA.4	Students learn about prime and composite by making arrays with color tiles.
Prime vs. Composite	Practice Task <i>Individual Task</i>	Prime and Composite Numbers	MGSE4.OA.4	Students list factors for numbers and determine whether the numbers are prime or composite given the list of factors.
Factor Trail Game	Practice Task <i>Individual/Partner Task</i>	Determining factor pairs	MGSE4.OA.4	Students earn points in the game Factor Trail as they practice finding all factors in a selected number.
* Rectilinear Robots	Constructing Task <i>Individual/Partner/Small Group Task</i>	Using factors to construct a rectilinear figure	MGSE4.OA.4 MGSE4.MD.8	Students will construct a rectilinear robot by making rectangles with given perimeters and calculating the total area of the robot.

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The Sieve of Eratosthenes	Practice Task <i>Individual/Partner Task</i>	Determining prime numbers less than 100	MGSE4.OA.4	Students will find all the prime numbers between 0-100 using colored pencils and a hundreds chart.
The Factor Game	Practice Task <i>Individual/Partner Task</i>	Recognizing factors as prime and composite numbers	MGSE4.OA.4	Students play a partner game by coloring in numbers and their factors on a hundreds chart.
Cicadas, Brood X	Constructing Task <i>Partner Task</i>	Prime and Composite Numbers	MGSE4.OA.4	Students learn that cicadas have a prime number of years in their life cycle which helps them survive.
Finding Multiples	Scaffolding Task <i>Individual Task</i>	Finding Multiples	MGSE4.OA.4	Students color in multiples of various one digit numbers on the hundreds chart and discuss numbers that are and are not multiples of the number colored in.
Finding Products	Constructing Task <i>Individual/Partner Task</i>	Understanding multiplicative comparisons with factors	MGSE4.OA.1 MGSE4.OA.4	Students will use intersection models to learn about multiplicative comparisons.
At the Circus	Constructing Task <i>Individual/ Partner Task</i>	Using Partial Products to Multiply	MGSE4.OA.2 MGSE4.NBT.5 MGSE4.OA.3	Students solve circus word problems to practice multiplying larger numbers.

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<u>School Store</u>	Constructing Task <i>Individual/ Partner Task</i>	Using Properties of Multiplication to Multiply	MGSE4.OA.1 MGSE4.OA.2 MGSE4.OA.3 MGSE4.NBT.5	Students apply what they know about multiplication to invent ways to multiply larger numbers.
<u>Sensible Rounding</u>	Constructing Task <i>Partner/Small Group Task</i>	Rounding	MGSE4.OA.3 MGSE4.NBT.6 MGSE4.NBT.3 MGSE4.MD.2	Students solve division word problems to discuss how to report the quotient when a remainder is involved.
<u>Compatible Numbers to Estimate</u>	Constructing Task <i>Individual/Partner Task</i>	Using compatible numbers to divide	MGSE4.OA.3 MGSE4.NBT.6	Students will learn about compatible numbers and how to use them to make reasonable estimates for division problems.
<u>Brain Only</u>	Scaffolding <i>Individual/Partner Task</i>	Patterns in Multiplication and Division	MGSE4.OA.2 MGSE4.NBT.5 MGSE4.NBT.6	Students use compare and contrast two problems in order to discover relationships between the dividend, divisor and quotient.
<u>What is $2500 \div 300$?</u>	Constructing <i>Individual/Partner Task</i>	Dividing with zeros	MGSE4.OA.2 MGSE4.OA.3 MGSE4.OA.5 MGSE4.NBT.6	Students learn about dividing when zeroes are involved in the calculation. Students also learn why a quotient is undefined when the divisor is zero.

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Boxes and Rolls	3 Act Task <i>Individual/Partner Task</i>	Multiplying Two Digit Whole Numbers, Dividing Whole Numbers	MGSE4.NBT.5 MGSE4.NBT.6	Students multiply and divide whole numbers to determine how many pennies are equivalent to the amount of money shown in an image.
Number Riddles	Constructing Task <i>Individual/Partner Task</i>	Factors and Multiples	MGSE4.OA.4	Students solve riddles to apply knowledge of factors, multiples and place value.
Earth Day Project	Constructing Task <i>Individual/Partner Task</i>	Generating Rules	MGSE4.OA.5	Students solve a real world problem involving patterns as students collect cans for a recycling project at school.
Culminating Task: School Newspaper	Performance Task <i>Individual Task</i>	Multiplication, Division and Rounding	MGSE4.OA.1 MGSE4.OA.2 MGSE4.OA.3 MGSE4.OA.5 MGSE4.NBT.5 MGSE4.NBT.6	Students plan how much paper to purchase in order to stay within a budget when producing a school newspaper.

If you need further information about this unit, please view the unit 2 webinar at www.georgiastandards.org.

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FORMATIVE ASSESSMENT LESSONS (FALS)

Formative Assessment Lessons are designed for teachers to use in order to target specific strengths and weaknesses in their students’ mathematical thinking in different areas. A Formative Assessment Lesson (FAL) includes a short task that is designed to target mathematical areas specific to a range of tasks from the unit. Teachers should give the task in advance of the delineated tasks and the teacher should use the information from the assessment task to differentiate the material to fit the needs of the students. The initial task should not be graded. It is to be used to guide instruction.

Teachers may use the following Formative Assessment Lessons (FALS) Chart to help them determine the areas of strengths and weaknesses of their students in particular areas within the unit.

Formative Assessments	FALS (Supporting Lesson Included)	Content Addressed	Pacing (Use before and after these tasks)
<u>Multi-digit Multiplication Strategies</u>	X	Multiplication Strategies	School Store Sensible Rounding Compatible Numbers to Estimate Brain Only What is $2500 \div 300$
<u>Array’s, Number Puzzles & Factor Trees</u>	X	Prime & Composite Factors & Multiples	My Son is Naughty The Factor Game Investigating Prime and Composite Numbers Prime vs. Composite Finding Multiples Number Riddles Earth Day
<u>Beads Under the Cloud</u>		Generate patterns of a given rule	Earth Day

Scaffolding Task: Factor Findings

Adapted from Illuminations provided by the National Council of Teachers of Mathematics

TASK CONTENT: In this lesson, students first create factor posters for a variety of different numbers that will be displayed in the classroom to be utilized as a resource throughout the school year.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconception should begin to clear up.

ESSENTIAL QUESTIONS

- What is a factor?
- What does it mean to factor?

MATERIALS

- color tiles
- graph paper
- crayons
- poster board
- scissors
- glue

GROUPING

Partner

NUMBER TALKS

Now that you have done several Number Talks throughout Unit One, they should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talk to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard) Students can utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 36$.

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

Doubling and halving is also a helpful strategy in building equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments:

In this lesson, students first create factor posters for a variety of different numbers that will be displayed in the classroom to be utilized as a resource throughout the school year. They make discoveries about factors using color tiles, represent their discoveries using graph paper, and display their information on poster board as find factors of an assigned number. The students’ factor poster will be used to assess the students’ understanding and ability to identify the factors of a given number using color tiles, graph paper and factor rainbows.

Throughout the lesson, circulate, observe and question the students as they create their factor poster. Put the number 24 on the board and have the students find the factors using one of the three methods and record them on a half sheet of paper at the end of the lesson. If you are going to use the number 24, try not to give a group 24 tiles during the lesson. Give students completed factor rainbows. Ask them to create the corresponding arrays and the original number.

Task Directions:

Begin the lesson by dividing the students into pairs. For this lesson it would work best to pair up students based on ability. Those students of similar abilities should be paired together. Give each pair of students 12 plastic color tiles. Instruct those students to arrange these 12 tiles into an array. Students will create one of the following:



Ask the students to identify how they would express these arrays in number sentences. (6×2 , 3×4 , 12×1) Some students will have 6×2 and others will have 2×6 . This is a great opportunity to discuss the commutative property. Bring attention to the fact that the arrays still look the same they are just positioned differently. Both problems still only provide the factors 6 and 2.

Then write just the numbers on the board: 1,2,3,4,6,12. Explain that these are the factors of 12 because they are the only numbers that can **divide 12 into equal groups** with no remainders. This is easily seen when you refer back to the arrays that **students** created.



A factor rainbow is a way of showing **all** factor pairs **for a number** in a list. Factor rainbows are used to check whether a list of factors is correct. To create a factor rainbow, the student must list the factors in order from least to greatest. They can then draw an arch that links the factor pairs. For square numbers, there will be no connecting arch in the middle; therefore the student can put a square around that number.

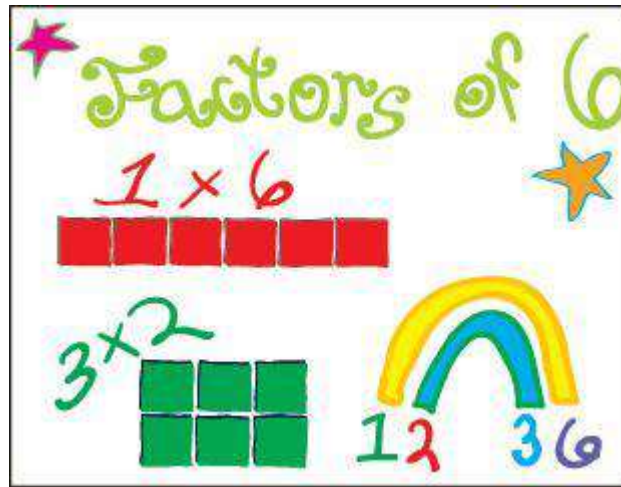
Provide the students with another example of finding factors of a number using the color tiles and then creating a factor rainbow. Possible numbers to use would be 16 or 9. Lead the students through the process for this example. Then have the pair of students decide on a number that they would like to find factors of. You may limit this number to **no more than 50**, depending on the amount of tiles you have available. When they have decided on their number, they can get that many tiles from a bucket of tiles located somewhere in the classroom and use them to find the factors of that number. Instruct the students to create the factor rainbow in their notebooks/journals. Check their factor rainbows to validate the students' understanding before you introduce the main activity.

For the main activity, instruct the students that they are each going to get a number and **that they must** find all of the factors for that number. **Explain to students that they will** create a poster **about the factors of their number** that the entire class can use throughout the year. Give each pair of students a bag of a different amount of color tiles. (ex. 18, 20, 24, 36, 40, 56, 60) Give the bags with the lower amounts to the pairs of students who may be struggling. Instruct the students using the following directions:

- Challenge the pairs of students to find as many different arrays as they can, using the color tiles they have been given. They must use all of the tiles each time.
- Each time they find an array they can then represent it on graph paper.

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- The students color one square on the graph paper for every plastic color tile in their array. This will form an array on their graph paper.
- They will cut it out and glue it to their poster and label it with the corresponding factors.
- They continue this process until they believe that they have found all of the factors.
- The students then check their factors by creating a factor rainbow at the bottom of their poster.
- The students must also include a title on their poster. Their poster might look similar to the following:



To conclude the lesson the students will display their posters on the wall and the class will have a gallery walk. During a gallery walk each student will walk around the room and look at everyone's work as if they were in a gallery. They will each be given some post-it notes they can use to anonymously comment on any piece of work and place on the poster. They will also write down two facts they discover after reviewing all of the posters. For example: Just because a number is larger doesn't mean that it has more factors than a smaller number. All even numbers have a factor of two. The number 16 is a perfect square, etc.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about the posters that have the number two listed as a factor?
- What would a factor poster of the number three look like?
- What is a number called when the only factors of that number are one and itself?
- When is it useful to know the factors of a number?

DIFFERENTIATION

Extension

- As an extension to this lesson the students can compare **and contrast factors on two different posters**. **Students can write what similarities and differences between the factors.**

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- Students can play the [Factor Game](#) on the computer.

Intervention

- Give bags to students that contain a number with a fewer factors.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L620> This links to the electronic version of this task. It could be used as an independent center activity or remediation.
- <https://learnzillion.com/lessons/782-find-all-factor-pairs-using-a-rainbow-factor-line>

Constructing Task: My Son is Naughty

Adapted from: My Son is Naughty, nzmaths

TASK CONTENT: This is an example of a problem where seemingly irrelevant information enables the solution to be found. It is a rare kind of problem at school level. However, we often get problems in life where apparently useless information turns out to be a key factor in its solution.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20. (5 and 4 are factors of 20 because there are 4 fives in 20.)

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5...24

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

3, 6, 9, 12, 15, 18, 21, 24

4, 8, 12, 16, 20, 24

8, 16, 24

12, 24

24

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4

- all numbers ending in 0 or 5 are multiples of 5

ESSENTIAL QUESTIONS

- What are factors?
- How do I determine the factors of a number?

MATERIALS

- My Son is Naughty recording sheet

GROUPING

- partner

NUMBER TALKS

Several number talk strategies can help student build a stronger understanding of multiplication and addition. For this task students could utilize several addition strategies from *Number Talks* such as compensation, adding up in chunks, near doubles etc. The table on pg. xxi is very useful for further information about strategies to support addition. In addition, doubling and halving are good strategies to support the multiplication necessary to complete this task. Several of those can be found on pgs. 276-282. Also a simple task like finding all the **factors** that equal 24 or three factors that equal 24 could further prepare them for this lesson. For example, during number talks simply write $\underline{\quad} \times \underline{\quad} = 24$ and then extend this to $\underline{\quad} \times \underline{\quad} \times \underline{\quad} = 24$

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Here we have a problem (problem follows in task description below) that looks impossible to solve at first glance. Ray was certainly right to say that the ‘36’ part is insufficient information to solve the problem. But surely knowing the additional information that Jack’s smallest son is very naughty can’t help at all? Where is the extra mathematical information there that enables Ray to solve the problem? It’s worth writing down *exactly* what key pieces of information you actually have. The trick is in first finding out as much as you can from the ‘36 and 13’ piece of information. Once you have done that what more do you need to know?

The extension of this activity is an even better example of a problem where seemingly irrelevant information enables the solution to be found. It is a rare kind of problem at school level. But we often get problems in life where apparently useless information turns out to be a key factor in its solution.

Solution

There are three key pieces of information here. These are:

- the product of the ages is 36;
- the sum of the ages is 13;
- the youngest of Jack’s sons is very naughty.

Let’s work with the three pieces of information separately.

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Suppose that the children are A and B. What can we tell about them from the fact that the product of their ages is 36? What two numbers multiplied together give you 36? How can you decompose 36 into two factors?

Perhaps the best way to do this is to work systematically as we have done in the table below. Start with 36, 1, and 1 and work downwards in the sense that the highest factor gets smaller.

But the second key idea is that the sum of the ages of the children is 13. How can we use this fact? In terms of the factors of 36, this just means that the sum of the factors is 13.

In the table we have listed the sums of all of the factors.

A's age	B's age	Sum of their ages
36	1	37
18	2	20
12	3	15
<u>9</u>	<u>4</u>	<u>13</u>
6	6	12

Task Directions

Students will follow the directions below from the “My Son is Naughty” recording sheet.

Jack and Ray were at the football game. Ray's team was winning so Jack decided to give Ray a hard time. Jack said, “Did you know that today is my two sons' birthday?” “How old are they?” Ray inquired. “I'll give you a hint. The product of their ages is 36,” Jack replied. “That's no help,” said Ray. So Jack weakened and gave him another clue. “Okay, my youngest son is very naughty, and the sum of their ages is 13.” “Nothing to it,” exclaimed Ray and he told Jack the correct ages of his sons.

How did Ray figure out the correct answer and what are Jack's sons' ages?

FORMATIVE ASSESSMENT QUESTIONS

- Suppose that the children are A and B. What can we tell about them from the fact that the product of their ages is 36?
- What two numbers multiplied together give you 36? How can you decompose 36 into two factors?
- What strategies might you be able to use **to solve the problem?**
- Do you think it is useful to make a table? If so, how?
- Do you think the last hint is very important? In what way?
- Have you considered all the possibilities?
- Have you checked your solutions? Does it look reasonable?

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- Are there any other solutions?

DIFFERENTIATION

Extension

- Jack and Ray were at the football game. Ray's team was winning so Jack decided to give Ray a hard time. So Jack said "Did you know that today is my **three** sons' birthday?" "How old are they?" Ray inquired. "I'll give you a hint. The product of their ages is 36 and the sum of their ages is 13," Jack replied. "That's no help," said Ray. So Jack weakened and gave him another clue. "O.K. My youngest son is very naughty." "Nothing to it," exclaimed Ray and he told Jack the correct ages of his sons. How did Ray figure out the correct answer and what are Jack's sons' ages?

Intervention

- Provide students with a table similar to the one above to fill in the information.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L620> This game involving finding factors could be used for additional practice of the concept.

Name _____ Date _____

My Son is Naughty

Jack and Ray were at the football game. Ray's team was winning so Jack decided to give Ray a hard time. Jack said, "Did you know that today is my two sons' birthday?" "How old are they?" Ray inquired. "I'll give you a hint. The product of their ages is 36," Jack replied. "That's no help," said Ray. So Jack weakened and gave him another clue. "Okay, my youngest son is very naughty, and the sum of their ages is 13." "Nothing to it," exclaimed Ray and he told Jack the correct ages of his sons.

How did Ray figure out the correct answer and what are Jack's sons' ages?

Scaffolding Task: Investigating Prime and Composite Numbers

TASK CONTENT: Students will discover the difference between prime and composite numbers through the making of arrays using color tiles or counters.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime when in fact it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1×7 and 7×1 , therefore it is a prime number) or by finding factors of the number.

ESSENTIAL QUESTIONS

- How do I identify prime numbers?
- How do I identify composite numbers?
- What is the difference between a prime and a composite number?

MATERIALS

- Counters or color tiles

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students will discover the difference between prime and composite numbers through the making of arrays using color tiles or counters.

Task Directions

Have students create a T-chart and label one side only two ways and the other more than two ways. Instruct students to answer the following questions and fill in the T-chart with the answers to the questions.

How many ways can you make 2? Use your counters.
How many ways can you make 8? Use your counters.
How many ways can you make 9? Use your counters.
How many ways can you make 11? Use your counters.
How many ways can you make 24? Use your counters.
How many ways can you make 41? Use your counters.
How many ways can you make 15? Use your counters.
How many ways can you make 13? Use your counters.

Have a discussion with students about their observations of the number of arrays made for each number. Introduce the vocabulary words prime and composite.

Have students complete the activity with the following comparisons.

Use your counters to determine if 21 is prime or composite. Explain your answer.
Use your counters to determine if 14 is prime or composite. Explain your answer.
Use your counters to determine if 7 is prime or composite. Explain your answer.
Use your counters to determine if 4 is prime or composite. Explain your answer.

FORMATIVE ASSESSMENT QUESTIONS

- What kind of number has only two ways it can be made?
- What kind of number has more than two ways it can be made?
- How do you know this number is prime? Composite?

DIFFERENTIATION

Extension

- Have students investigate using the counters to determine if 1 is prime or composite.
- Provide students with a list of numbers which are prime and a list of numbers which are composite. Have students prove the numbers are on the correct list by making arrays to determine the number of ways each number can be made.

TECHNOLOGY

- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm This resource has games dealing with prime and composite. It can be used to for additional practice.
- <http://www.prometheanplanet.com/en-us/Resources/Item/74922/prime-and-composite-with-the-area-model> This lesson introduces prime and composite using area model and relates it to tiling a floor. It can be used as an introduction to the concept or for remediation purposes.
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html This electronic sieve can be used to introduce the concept or for remediation purposes.

Practice Task: Prime vs. Composite

TASK CONTENT: This task refers to prime and composite numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime when in fact it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) or by finding factors of the number.

ESSENTIAL QUESTIONS

- How do I identify prime numbers?
- How do I identify composite numbers?
- What is the difference between a prime and a composite number?

MATERIALS

- Prime vs. Composite recording sheet

GROUPING

Individual

NUMBER TALKS

The concept of prime numbers can be explored with a number talk philosophy however the concept is unique and doesn't lend itself well to mental math, because the only factors are one and itself. Continue exploring the doubling and halving strategies found on pgs. 276-281 to build stronger understanding of factors.

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students will follow the directions below from the “Prime vs. Composite” recording sheet.

List the factors and draw lines to connect factor pairs. Write P for prime, C for composite, or N for neither.

What is the only even prime number? Use a diagram to explain how you know the number is prime.

How can you determine if a number is prime, composite, or neither by looking at the factors of the number? Explain your answer.

FORMATIVE ASSESSMENT QUESTIONS

- What are factors?
- How do you know you have found all the factors of that number?
- What kind of number has only two factors?
- What kind of number has more than two factors?
- How do you know this number is prime? Composite?
- What kind of diagram will you use to show the only even prime number?

DIFFERENTIATION

Extension

- Have students investigate larger prime numbers using <http://wonderopolis.org/wonder/what-is-a-prime-number/> .

Intervention

- Provide students with a list of numbers which are prime and a list of numbers which are composite. Have students prove the numbers are on the correct list by making arrays to determine the number of ways each number can be made.

TECHNOLOGY

- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm This resource has games dealing with prime and composite. It can be used to for additional practice.
- <http://www.prometheanplanet.com/en-us/Resources/Item/74922/prime-and-composite-with-the-area-model> This lesson introduces prime and composite using area model and relates it to tiling a floor. It can be used as an introduction to the concept or for remediation purposes.
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html This electronic sieve can be used to introduce the concept or for remediation purposes.

Name _____ Date _____

Prime vs. Composite

List the factors and draw lines to connect factor pairs. Write P for prime, C for composite, or N for neither.

Number	Factors	P, C, or N
1. 8		
2. 19		
3. 30		
4. 1		
5. 42		
6. 29		

7. What is the only even prime number? Use a diagram to explain how you know the number is prime.
8. How can you determine if a number is prime, composite, or neither by looking at the factors of the number? Explain your answer.

Practice Task: Factor Trail Game

Adapted from Illuminations provided by the National Council of Teachers of Mathematics

TASK CONTENT: When students play the Factor Trail game, they have to identify the factors of a number to earn points.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconceptions should begin to clear up.

ESSENTIAL QUESTIONS

- What is factor?
- What does it mean to factor?

MATERIALS

- Factor Trail Game (<http://illuminations.nctm.org/Lesson.aspx?id=2520>)
- Calculators (optional)
- Game board ([download the attachment at the Factor Trail Game link listed above](#))

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- Score Sheet (one per player - [download the attachment at the Factor Trail Game link listed above](#))
- Dice

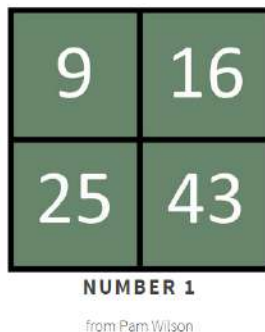
GROUPING

partner or small group

NUMBER TALKS

Now that you have done several Number Talks throughout Unit One, they should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students.

Number Talks can also be done using ideas from the “Which Doesn’t Belong” website. (<http://wodb.ca/shapes.html>) The website provides squares that are divided into four sections. Each section has a mathematical idea. Students look at the four ideas presented and decide which idea doesn’t belong. For example:



The “Which Doesn’t Belong” squares can be displayed for students on the board and treated as a Number Talk. After displaying the square above, give students a couple of minutes to look at it and develop some ideas about which number doesn’t belong. Students may give the thumbs up signal when they have a solution and can continue to look for other solutions that may be possible as time allows. The teacher can call on students to give solutions and defend their thinking about the solution they have selected. In the example shown above, students may say:

- 16 doesn’t belong because it is an even number. All the other numbers are odd.
- 43 doesn’t belong because it is prime and all the other numbers are composite.
- 43 doesn’t belong because it is not a square number and all the other numbers are square numbers.
- 9 doesn’t belong because it is only one digit and the other numbers have two digits.

The class conversation that results is a very rich in mathematical vocabulary and content that will help students grow as mathematicians.

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Task Directions

When students play the Factor Trail game, they have to identify the factors of a number to earn points. Built into this game is cooperative learning — students check one another's work before points are awarded. The score sheet used for this game provides a built-in assessment tool that teachers can use to check their students' understanding.

Explain to students that they will be playing a game involving factoring. Ask them what it means to factor a number, and then ask them to help you find all the factors of a number. You may want to choose a number with a lot of factors (24, 36, 60), or you can roll two dice or spin two spinners to generate the digits of a number randomly. However you choose a number, use the think-pair-share strategy to allow the class to identify the factors: first, give students one minute to "think" individually and come up with some factors of the number; then, give them another minute to discuss their lists with a partner; and finally, record the entire list of factors on the board or overhead projector via a class discussion.

You may then wish to give students a few more numbers to practice on their own before playing the game. Use these additional warm-up problems to determine how well students are able to factor numbers. Then, when students begin to play the game for practice, spend additional time with students who had difficulty.

After the warm-up, distribute the Factor Trail Game to all students. Note that two students can share the game board and rules that appear on the first and second pages, but all students will need their own score sheet.

Players move around the game board, landing on numbered squares. When landing on a square with a number, students should list all of the factors of that number on their score sheet. When a student believes that she has listed all of the factors, her opponent checks the list. If her opponent identifies any factors not on the player's list, or if the opponent identifies any number on the player's list that is not a factor of the number, the opponent receives 10 points for identifying the error. (If the opponent notices multiple errors, 10 points are earned for each error.) If the player made no errors, however, then she receives points for that turn equal to the sum of the factors of the number.

Example: A player lands on 18. On her score sheet, she lists 1, 2, 3, 4, 6, 9, and 18 as factors. Upon indicating that she has completed her list, her opponent points out that 4 is not a factor of 18. Consequently, the opponent receives 10 points for identifying the error. On the other hand, if she had not included 4 on her list, she would have correctly identified all of the factors and received $1 + 2 + 3 + 6 + 9 + 18 = 39$ points.

The game can be played with or without calculators. The use of calculators does not greatly influence the game, as students must still understand the concept and skill of factoring to be successful. However, if a secondary objective of the lesson is to have students practice mental arithmetic, then calculators should not be used.

As students play the game, circulate to offer assistance where necessary. This may involve settling a dispute between two students, or it may require intervening when you notice that students are making mistakes not caught by their opponents.

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With a few minutes left in class, pause all games and conduct a brief discussion using the questions that appear in the Formative Assessment Questions section below.

FORMATIVE ASSESSMENT QUESTIONS

- Which number on the game board has the most factors?
- For which number on the trail will a player earn the most points?
- In general, how many points are earned for a prime number?
- Collect the score sheets of all students. The score sheets can be used to determine if students were correctly finding the factors of numbers. One of the benefits of using the score sheet occurs when a 0 is entered in the "Points Earned" column. This indicates that the student made a mistake when finding the factors of that number, so it is easier to identify areas of difficulty.

DIFFERENTIATION

Extension

- Change the numbers on the game board. Note that all of the numbers are less than 100. For a more advanced game, include numbers in the hundreds or thousands.

Intervention

- Change the numbers on the game board. The numbers could change to being numbers below 50. The numbers will need to be frequently repeated, but the repetition will help students to develop the concept of factoring prior to working with larger numbers.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L620> This links to the electronic version of this task. It could be used as an independent center activity or remediation.
- <http://studyjams.scholastic.com/studyjams/jams/math/multiplication-division/prime-composite-numbers.htm> Study Jams: prime and Composite Numbers is an interactive presentation that gives information about the topic and presents students with “Try This” problems to check student understanding.

***CONSTRUCTING TASK: Rectilinear Robots**

Adapted from “*Rectangular Robots*” found on <http://www.k-5mathteachingresources.com/3rd-grade-measurement-and-data.html>

TASK CONTENT: Students will apply knowledge of factors to design a robot that is made of rectilinear figures on centimeter grid paper. Students will calculate the area of the robot.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

During this unit, students have become comfortable finding factors of numbers. In third grade, students learned how to find the area of squares and rectangles. Students will need both of these pieces of prior knowledge in order to design robots during this task.

ESSENTIAL QUESTIONS

- How can factors be used to solve problems?
- How is the area of a rectilinear figure calculated?

MATERIALS

- Centimeter grid paper
- Colored pencils, pencils or markers

GROUPING

Individual, partner or small group

NUMBER TALKS

Number Talks can also be done using ideas from the “Which Doesn’t Belong” website. (<http://wodb.ca/shapes.html>) The website provides squares that are divided into four sections. Each section has a mathematical idea. Students look at the four ideas presented and decide which idea doesn’t belong. For example:

121	16
9	73

NUMBER 5

from Isabelle & Noah Bourassa

The “Which Doesn’t Belong” square can be displayed for students on the board and treated as a Number Talk. After displaying the square above, give students a couple of minutes to look at it and develop some ideas about which number doesn’t belong. Students may give the thumbs up signal when they have a solution and can continue to look for other solutions that may be possible as time allows. The teacher can call on students to give solutions and defend their thinking about the solution they have selected. In the example shown above, students may say:

- 16 doesn’t belong because it is an even number. All the rest of the numbers are odd numbers.
- 9 doesn’t belong because it has no tens. All the rest of the numbers have tens.
- 73 doesn’t belong because it is prime. All the other numbers are composite.

The class conversation that results is a very rich in mathematical vocabulary and content that will help students grow as mathematicians. With the last bullet, a class discussion could be held about the factors of the three composite numbers in the square to prepare for the “Rectangular Robots” lesson.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Display the task directions for students. Have students silently read over the task directions and then discuss the directions as a whole group to assess student understanding of the expectations of the task.

Task Directions:

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Part A: You will construct a robot using a piece of centimeter grid paper. Your robot must meet the following criteria:

- The robot must be constructed using rectangles.
- The rectangles created must share a side of at least 1 centimeter in length.
- The robot must have a head, a body, two arms and two legs.
- The two arms and the two legs must be congruent.
- The perimeter of each body part must be one of the following measurements: 28 cm, 12 cm, 16 cm and 30 cm.

Part B: Calculate the area of each rectangle used to construct the robot.

Part C: Calculate the area of the robot.

Part D: Design another robot that meets the criteria in part A, but is different from the previous robot constructed.

Comments

After discussing the expectations for the task, ask students to think about how factors can be used to make the rectangles for the robot. Allow students to turn and talk about this idea for a couple of minutes. Then, discuss ideas as a whole group. An example of student thinking might be, “If I want to create a rectangle that has a perimeter of 12 cm, I would decompose 12 into 10 and 2. Since two of the sides need to be congruent but have a sum of ten, one factor is two (for the two sides) and the other factor is 5. The same is true for the other side. I have 2 as one factor (for the two sides) and that means the other factor is 1. That means I could make a rectangle using 5 cm by 1 cm for a body part on the robot.”

Have students work on the task in either partners or small groups. Students may also work individually for this activity, if desired. Facilitate the independent work time by checking to see if the robots being constructed meet the criteria listed in part A, as well as asking students questions about their robot and the criteria in part A. For example, you might as a small group, “What is the area of each leg on your robot? How did you determine the area of each leg? What does it mean that the two legs need to be congruent? How would you prove that your robot’s legs are congruent?” Teachers may also ask, “What factors have you found useful during the construction of your robot?”

FORMATIVE ASSESSMENT QUESTIONS

- What factors did you use to make rectangle body parts for the robot?
- How did you calculate the total area of the robot?

DIFFERENTIATION

Extension

- To extend this task, students can create another robot design as mentioned in Part D of the task. Students can compare and contrast the two robots built based on the data and the visual comparisons that can be made between the two robots.

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Intervention

- Students can work with smaller perimeters like 8 cm, 10 cm, 12 cm and 14 cm. Students could also make the robot with centimeter cubes or snap cubes if enough are available.

TECHNOLOGY

- <https://learnzillion.com/lessons/3905-find-area-by-counting-square-units> Learnzillion Video “Find Area by Counting Square Units” – This video from Learnzillion features rectilinear figures and models how to find the area using square units.
- <https://www.illustrativemathematics.org/content-standards/3/MD/C/7/tasks/1836> Illustrative Mathematics: Three Hidden Rectangles – Students can complete the task given the directions on the website to practice dividing a rectilinear figures into smaller rectangles to find the total area.

RECTILINEAR ROBOT

Task Instructions

Part A: You will construct a robot using a piece of centimeter grid paper. Your robot must meet the following criteria:

- The robot must be constructed using rectangles.
- The rectangles created must share a side of at least 1 centimeter in length.
- The robot must have a head, a body, two arms and two legs.
- The two arms and the two legs must be congruent.
- The perimeter of each body part must be one of the following measurements: 28 cm, 12 cm, 16 cm and 30 cm.

Part B: Calculate the area of each rectangle used to construct the robot.

Part C: Calculate the area of the robot.

Part D: Design another robot that meets the criteria in part A, but is different from the previous robot constructed.

Practice Task: The Sieve of Eratosthenes

TASK CONTENT: This task allows students to create a list of prime numbers between 1 and 100.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Many students may not understand why “one” is not a prime number. Be sure to allow for plenty of discussion about this. One is neither a prime nor a composite number. A prime number is one with exactly two positive, unique divisors - itself and one. One has only one positive, unique divisor. It cannot be written as a product of two factors, neither of which is the number itself, so one is also not composite. It falls in a class of numbers called units. These are the numbers whose reciprocals are also whole numbers. For more information go to:

<http://mathforum.org/dr.math/faq/faq.prime.num.html>

Students are asked to write an expression for given multiples in this task. Some may not be sure what the question is asking, providing a prime opportunity for discussion. Because all even numbers are multiples of two, the algebraic expression for multiples of two is $2n$. Similarly, the multiples of 5 can be expressed as $5n$.

ESSENTIAL QUESTIONS

- How do we know if a number is prime or composite?
- How can we determine whether a number is odd or even?
- How are factors and multiples defined?

MATERIALS

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- “Sieve of Eratosthenes” Recording Sheet
- “Exploring the Sieve of Eratosthenes” Recording Sheet
- Colored pencils, markers, highlighters or crayons

GROUPING

individual or partner

NUMBER TALKS

The concept of prime numbers can be explored with a number talks philosophy however the concept is unique and doesn't lend itself well to mental math, because the only factors are one and itself. Continue exploring the doubling and halving strategies found on pgs. 276-281 to build stronger understanding of factors.

Number Talks can also be done using ideas from the “Which Doesn't Belong” website. (<http://wodb.ca/shapes.html>) The website provides squares that are divided into four sections. Each section has a mathematical idea. Students look at the four ideas presented and decide which idea doesn't belong. For example:

17	26
44	65

NUMBER 3
from Mary Bourassa

The “Which Doesn't Belong” squares can be displayed for students on the board and treated as a Number Talk. After displaying the square above, give students a couple of minutes to look at it and develop some ideas about which number doesn't belong. Students may give the thumbs up signal when they have a solution and can continue to look for other solutions that may be possible as time allows. The teacher can call on students to give solutions and defend their thinking about the solution they have selected. In the example shown above, students may say:

- 17 doesn't belong because it is prime. All the other numbers are composite.
- 44 doesn't belong because it's tens digit is ten times more than the ones digit. All the other numbers have digits that are different.
- 65 doesn't belong because the sum of its tens and ones digit doesn't equal eight. All the rest of the numbers have a tens and a ones digit that have a sum of eight.

The class conversation that results is a very rich in mathematical vocabulary and content that will help students grow as mathematicians.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task allows students to create a list of prime numbers between 1 and 100.

Comments

Students should use a different color for each new prime number. Using different colors works best, but if you don't have enough crayons students might need to use a combination of crayons, highlighters, colored pencils, and markers. A different symbol could be used for each prime number (i.e., *, #, etc.), but using colors is more efficient and effective. Have students draw only one line through numbers so when they cross out a number more than once, they can do so in a different direction. This will help students find numbers that are multiples of several numbers. One way to introduce this task is to model the first one or two steps. Discuss the patterns students see. Write these on the board or chart paper as students share them.

Each student needs to complete his/her own Sieve of Eratosthenes. Let students share their observations. Record these on a chart or the board. The "Exploring the Sieve of Eratosthenes" student recording sheet will help students determine factors and multiples.

Task Directions

Use a 0-100 chart to complete the following:

1. Draw a line through the number 1, because it is not a prime number.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

FORMATIVE ASSESSMENT QUESTIONS

- What do all of the multiples of two have in common? Multiples of five?
- If you multiply by two will you always get an even answer (a multiple of two)? Why?
- If you multiply by five will you always get a multiple of five? Why?

DIFFERENTIATION

Extension

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- Students can write a letter to an “absent” classmate about how to use the Sieve of Eratosthenes to find factors of numbers.

Intervention

- Have students use a calculator or the computer to generate the multiples of numbers.

TECHNOLOGY

- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm This resource has games dealing with prime and composite. It can be used to for additional practice.
- <http://www.prometheanplanet.com/en-us/Resources/Item/74922/prime-and-composite-with-the-area-model> This lesson introduces prime and composite using area model and relates it to tiling a floor. It can be used as an introduction to the concept or for remediation purposes.
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html This electronic sieve can be used to introduce the concept or for remediation purposes.

Sieve of Eratosthenes

Eratosthenes (275-194 B.C., Greece) devised a 'sieve' to discover prime numbers. A sieve is like a strainer that you drain spaghetti through when it is done cooking. The water drains out, leaving your spaghetti behind. Eratosthenes's sieve drains out composite numbers and leaves prime numbers behind.

Use a hundred charts below to complete the following:

1. Put a square around the number 1, because it is neither prime number nor composite.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Name _____ Date _____

Exploring the Sieve of Eratosthenes Recording Sheet

Using your Sieve of Eratosthenes list the prime numbers up to 100.

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____,
 _____, _____, _____, _____, _____, _____, _____, _____, _____, _____,
 _____, _____, _____, _____, _____

Complete the table below. Choose a number between 40 and 60 for the last two rows, and then complete the chart using the numbers chosen.

Find this number on your hundreds board:	Prime or Composite?	List the factors of the number
3		
9		
12		
15		
17		
21		
24		
29		

Look at your chart and record 3 observations from your work on a separate sheet of paper.

Practice Task: The Factor Game

TASK CONTENT: This task requires students to demonstrate understanding of factors and multiples of whole numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (5 and 4 are factors of 20 because there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5...24

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

3, 6, 9, 12, 15, 18, 21, 24

4, 8, 12, 16, 20, 24

8, 16, 24

12, 24

24

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

ESSENTIAL QUESTIONS

- What are factors?
- How do I determine the factors of a number?

MATERIALS

- Crayons
- Game board

GROUPING

Partners

NUMBER TALKS

A powerful strategy for developing student understanding of factors is to utilize doubling and halving. See *Number Talks* pgs. 276-281. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

One way to introduce this task is to model the first one or two steps on the overhead. Discuss the patterns students see. Write these on the board or chart paper as students share them. Each group will complete a game board, however students need to answer questions separately. Let students share their observations. Record these on a chart or the board.

Task Directions

You and your partner will have a different colored crayon or colored pencil. The first move is granted to the person wearing the lightest colored shirt.

STEP 1: Partner A will use their color and color any number 1-99.

STEP 2: Partner B will then color in all the factors for the number that Partner A just colored in.

STEP 3: Once Partner B has colored in all factors, Partner B will color in any number 1-99, just like Partner A did in the beginning of the game.

STEP 4: Now it is Partner A's turn to color in all the factors for the number that Partner B filled in.

Continue the steps until the entire number chart is colored in.

STEP 5: Go back to your number chart and count how many prime numbers you have. You earn 2 points for each prime number you colored in.

STEP 6: Answer the questions on the back of this page.

****IMPORTANT: If a number is already colored in, you cannot color that box and you lose that number.**

FORMATIVE ASSESSMENT QUESTIONS

- How do you know you have found all of the factors?
- What representations did you use to find the factors of ____?

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- Did you notice any patterns in the factors you found?
- What strategies did you use to choose a number?

DIFFERENTIATION

Extension

- Students in need of an extension could extend the number chart to 150 or 200.
- Students in need of an extension could create restrictions for the game to make it more challenging, such as “no even numbers can be chosen for the first two turns.”

Intervention

- Students work in a small group setting with teacher and only use numbers 1-50 on the game board.

TECHNOLOGY

- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L620> This links to the electronic version of this task. It could be used as an independent center activity or remediation.
- <https://learnzillion.com/lessons/785-find-all-factor-pairs-of-a-number-using-a-t-chart> The Learnzillion video teaches how to find all the factor pairs for a given number using a t-chart.

THE FACTOR GAME

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

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Name _____

Date _____

1. List all of your composite numbers. _____

2. List all of your prime numbers. _____

3. If a number is divisible by 2, 3, 5 is it a composite number? How do you know?

4. Long ago, people observed the sun rising and setting over and over at about equal intervals. They decided to use the amount of time between two sunrises as the length of a day. They divided the day into 24 hours. Use what you know about factors to answer these questions:

a. Why is 24 a more convenient choice than 23 or 25? _____

b. If you were to select a number different from 24 to represent the hours in a day, what number would you choose? Why? _____

Constructing Task: Cicadas, Brood X

Adapted from Yummymath.com Cicadas

TASK CONTENT: This task refers to prime and composite numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8. A common misconception is that the number 1 is prime when in fact it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors. Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number) or by finding factors of the number.

ESSENTIAL QUESTIONS

- How do I identify prime numbers?
- How do I identify composite numbers?
- What is the difference between a prime and a composite number?

MATERIALS

- Cicadas, Brood X recording sheet

GROUPING

Partners

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students will complete the task below.

Task Directions

Cicadas are emerging again. Oh my!

In North America we have a kind of cicada that only emerges from the ground every 17 years. Isn't that weird? Can cicadas count?

1. Cicadas only emerge in North America every 17 years. Their most recent emergence happened in 2013.
 - a. What was the year of their last emergence prior to 2013?
 - b. What year will they emerge next after this year?
 - c. How old will you be when the Magicicada emerges again?

2. Before you read on, write down what you think is special about the number 17?

We did some research and found that emerging only every 17 years is another kind of survival technique. Some animals and insects can blend into their surroundings. Some animals or insects have sharp spines or hard shells so that they are dangerous or unpleasant to eat. Some insects look like something that tastes bad. Cicadas do something different. They use the number 17 to avoid predators.

These seem to be the important facts that contribute to the survival of cicadas:

- **Predators can't depend on cicadas for food in their life cycle.**
- **When the cicadas are finally present, there are too many to eat to impact cicada survival.**

First, you need to understand what a life cycle of an insect is: An insect's life-cycle length is the time that it takes for the insect egg to become an adult and then for the adult to lay eggs again. So, the life cycle of anything is the time it takes to go from being an egg to laying an egg.

Magicicada Septendecim (the North American cicada) has a life cycle of 17 years. Predators of the magicicada cicada are snakes, birds, rodents, small mammals, lizards, and fish. But since their life cycles don't coincide with the cicada's very long 17-year cycle, they need to be able to feed consistently on other insects and animals.

3. If an insect had a life cycle of 1 year, how many generations of that insect would have to go by before that insect species could depend on eating cicadas?
4. If a predator's life cycle was 2 years, how many generations would it take before it could eat cicadas?
5. If a predator's cycle was 3 years, how many generations would it take?
6. What if the predator had a life cycle of 5 years? 7 years? 11 years?

You should have noticed that 17 isn't divisible by any of these life cycles. That is none of these life cycles coincide with the 17-year cycle. Since these life cycles don't coincide with 17 evenly, no predator is going to start its life cycle or live its life cycle consistently with the cicadas. Thus it won't be able to rely on the cicadas as a food source. When a number is only divisible by itself and one it is a prime number. This prime number life cycle of the cicadas is its defense mechanism.

9. Why aren't we considering life cycles of 4, 6, 8, 9, 10 12, 14, 15 or 16 years?

10. What are some other life cycles that cicadas could have that would be a prime number life cycle?

11. Write a good definition for a prime number. Give examples and non-examples. Do you think one is a prime number?

FORMATIVE ASSESSMENT QUESTIONS

- What can you conclude about 2, 3, and 5 based on your findings?
- Why aren't we considering life cycles of 4, 6, 8, 9, 10 12, 14, 15 or 16 years?
- How do you know a number is prime? Composite?

DIFFERENTIATION

Extension

- Have students research other species with prime number life cycles.

Intervention

- Provide students with a list of numbers which are prime and a list of numbers which are composite. Have students prove the numbers are on the correct list by making arrays to determine the number of ways each number can be made.

TECHNOLOGY

- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm This resource has games dealing with prime and composite. It can be used to for additional practice.
- <http://www.prometheanplanet.com/en-us/Resources/Item/74922/prime-and-composite-with-the-area-model> This lesson introduces prime and composite using area model and relates it to tiling a floor. It can be used as an introduction to the concept or for remediation purposes.
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html This electronic sieve can be used to introduce the concept or for remediation purposes.

Name: _____

Date: _____

Cicadas, Brood X



Magicicada septendecim

Cicadas are emerging again. Oh my!

In North America we have a kind of cicada that only emerges from the ground every 17 years. Isn't that weird? Can cicadas count?

2. Cicadas only emerge in North America every 17 years. Their most recent emergence happened in 2013.
 - a. What was the year of their last emergence prior to 2013?
 - b. What year will they emerge next after this year?
 - c. How old will you be when the Magicicada emerges again?

2. Before you read on, write down what you think is special about the number 17.

We did some research and found that emerging only every 17 years is another kind of survival technique. Some animals and insects can blend into their surroundings. Some animals or insects have sharp spines or hard shells so that they are dangerous or unpleasant to eat. Some insects look like something that tastes bad. Cicadas do something different. They use the number 17 to avoid predators.

These seem to be the important facts that contribute to the survival of cicadas:

- **Predators can't depend on cicadas for food in their life cycle.**
- **When the cicadas are finally present, there are too many to eat to impact cicada survival.**

First, you need to understand what a life cycle of an insect is: An insect's life-cycle length is the time that it takes for the insect egg to become an adult and then for the adult to lay eggs again. So, the life cycle of anything is the time it takes to go from being an egg to laying an egg.

Magicicada Septendecim (the North American cicada) has a life cycle of 17 years. Predators of the magicicada cicada are snakes, birds, rodents, small mammals, lizards, and fish. But since their life cycles don't coincide with the cicada's very long 17-year cycle, they need to be able to feed consistently on other insects and animals.

3. If an insect had a life cycle of 1 year, how many generations of that insect would have to go by before that insect species could depend on eating cicadas?

4. If a predator's life cycle was 2 years, how many generations would it take before it could eat cicadas?

5. If a predator's cycle was 3 years, how many generations would it take?

6. What if the predator had a life cycle of 5 years? 7 years? 11 years?

You should have noticed that 17 isn't divisible by any of these life cycles. That is, none of these life cycles coincide with the 17-year cycle. Since these life cycles don't coincide with 17 evenly, no predator is going to start its life cycle or live its life cycle consistently with the cicadas. Thus it won't be able to rely on the cicadas as a food source. When a number is only divisible by itself and one it is a prime number. This prime number life cycle of the cicadas is its defense mechanism.

9. Why didn't we consider life cycles of 4, 6, 8, 9, 10, 12, 14, 15 or 16 years?

10. What are some other life cycles that cicadas could have that would be a prime number life cycle?

11. Write a good definition for a prime number. Give examples and non-examples. Do you think one is a prime number?

Scaffolding Task: Finding Multiples

TASK CONTENT: Students can utilize friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (5 and 4 are factors of 20 because there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5...24

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

3, 6, 9, 12, 15, 18, 21, 24

4, 8, 12, 16, 20, 24

8, 16, 24

12, 24

24

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

ESSENTIAL QUESTIONS

- What are multiples?
- How is skip counting related to identifying multiples?

MATERIALS

- Crayons
- Hundred chart

GROUPING

Individual

NUMBER TALKS

Students can utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 36$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Look at the patterns that a skip-counting sequence made on a hundreds chart, e.g., the sequence of threes makes a diagonal pattern like this:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Ask the students to predict other numbers in the threes sequence. Ask questions such as, “Will 41 be in the pattern? How do you know?” Point to numbers in the sequence (30 or less) and ask, “How

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many threes are in that number?” Ask the students to say or write the matching multiplication number sentence, e.g., “Seven times three is twenty-one” or $7 \times 3 = 21$.

Task Directions

Have students investigate the patterns for 2, 4, 5, and 6 using their hundred charts. Each sequence can be represented with a different color crayon.

FORMATIVE ASSESSMENT QUESTIONS

- What do you notice about this sequence?
- How many ____ are in that number?
- What number sentence represents this sequence?

DIFFERENTIATION

Extension

- Have students investigate the patterns for 7, 8, and 9 using their hundred charts. Each sequence can be represented with a different color crayon.

Intervention

- Students can practice the counting sequences in pairs with a calculator. If they enter + 3 and press the equals sign repeatedly, this will generate the sequence 3, 6, 9, 12, ... as the calculator adds three repeatedly.

TECHNOLOGY

- <http://illuminations.nctm.org/ActivityDetail.aspx?id=29> This game helps students exercise their skills with factors and multiples. It can be used for additional practice.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L273> This lesson provides instructions for students to create their own product game. It can be used for remediation or extending student understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L528> This lesson has students explore products when one of the factors is 6. They also create a personal multiplication chart. It can be used for remediation or as an extension.

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Practice Task: Finding Products

Adapted from Illuminations provided by the National Council of Teachers of Mathematics

TASK CONTENT: Students examine the role of the multiplicative comparisons, play a multiplication game, and explore products where one of the factors is 6.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.

- a. Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.**
- b. Represent verbal statements of multiplicative comparisons as multiplication equations.**

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconceptions should begin to clear up.

ESSENTIAL QUESTIONS

- What is a product?
- What is the difference between a factor and product?

MATERIALS

- Crayons
- Paper
- File Cards

GROUPING

Individual

NUMBER TALKS

Now that you have done several Number Talks throughout Unit One, they should be incorporated into the daily math routine. Continue utilizing the different strategies in number talks and revisiting them based on the needs of your students. Catherine Fosnot has developed problem “strings” that could be included in a number talk to further develop mental math skills. See Mini-lessons for Operations with Fractions, Decimals, and Percents by Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard. (Mini-lessons for Operation with Fraction, Decimal, and Percent, 2007, Kara Louise Imm, Catherine Twomey Fosnot and Willem Uittenbogaard)

Students can utilize the friendly number strategies to begin discussing multiples and specifically combining multiples to arrive at a larger product. For example, students could solve the following string of numbers:

$$1 \times 12 = 12$$

$$2 \times 12 = 24$$

by combining the products of 12 and 24 as well as combining the factors of 1 and 2 the students will produce another multiple of 12, namely $3 \times 12 = 36$

$$\begin{array}{r} 1 \times 12 = 12 \\ + 2 \times 12 = 24 \\ \hline 3 \times 12 = 36 \end{array}$$

Doubling and halving is also an effective strategy for building equivalent fractions. (Number Talks, 2010, Sherry Parrish).

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students examine the role of the multiplicative identity, play a multiplication game, and explore products where one of the factors is 6. They also create a "My Personal Multiplication Chart" to record products.

Task Directions:

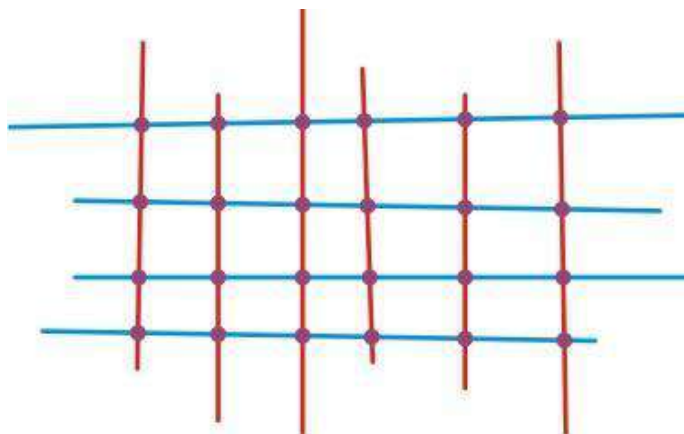
Provide students with paper and crayons and ask them to draw six blue vertical lines on the paper. Now ask them to draw four red horizontal lines intersecting the vertical lines. Ask them to circle in purple each place there is an intersection and count the number of intersections. Challenge them to

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identify what multiplication fact they have just demonstrated is if in this model, the number of rows is given first. [$4 \times 6 = 24$.] Ask them to turn their papers a quarter turn and name the multiplication fact now modeled. [$6 \times 4 = 24$.] **Discuss multiplicative comparisons with students. Explain that in third grade, students learned that multiplication is equal groups. This year they will learn that multiplication is also stating a comparison. For example, there are 24 intersections on the model below. 24 is six times as much/much as four. 24 is also four times as much/much as six.**



Encourage them to generate other facts where one factor is 6, including 6×0 and 6×1 .

Repeat with 7 as a factor.

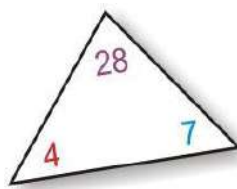
Distribute index cards to each pair and ask each student to make a set of 10 cards numbered 0 to 9, one to a card.

them to take turns turning over the top card, multiplying the number drawn by 6 and then saying the **multiplicative comparison** and the product. **For example, if a student turned over a four, she would say, “Six times as much/much as four is 24.”** As each card is used, it should be returned to the bottom of the deck. Give students time to play, and then ask the class to skip count in unison by 6. Encourage them to do so without looking at the game board. Repeat for 7 as a factor.

Ask students to save the numbered cards for later use.

Next, ask students to make a deck of triangle fact cards for the 6 and 7 tables by putting 2 factors, one of them a 6 or a 7, on 2 of the corners and the product in the third corner. They may wish to use red and blue for the factors and purple for the product.

When they have made triangle fact cards for the facts 0×6 to 9×6 and 0×7 to 9×7 , ask each student to cover the product on one card with his or her thumb, show the card to the other student, and ask him or her to tell the product. Encourage the students to separate the cards with the facts they know from those they are less certain. In the card below, if the 28 is covered, students can ask, “What number is seven times as much as four?” If the seven is covered, students could ask, “What is four times as much as seven?”



FORMATIVE ASSESSMENT QUESTIONS

- If you spin a 5, what would the product of 6 and that number be? **How would you describe that as a comparison statement?**
- If you spin a 1, what would the product of that number and 6 be? Why are you sure of that?
- How could you model with intersections the multiplication facts $3 \times 6 = 18$ and $6 \times 3 = 18$? What is alike between these multiplication sentences? What is different?
- What numbers do you say when you skip count by 7's to 70? Which of these are even numbers?

DIFFERENTIATION

Extension

- What products can you get when you multiply by 7? How many are even? Will you get an even product when you multiply by 3? By 6? By 8? By 9? How can you tell if the product will be even?
- How many ways can you have a product of 6? Of 12? Of 25? Of 42? Of 1?

Intervention

- Students who need additional practice may use the [Times Table](#) tool.

TECHNOLOGY

- <http://illuminations.nctm.org/ActivityDetail.aspx?id=29> This game helps students exercise their skills with factors and multiples. It can be used for additional practice.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L273> This lesson provides instructions for students to create their own product game. It can be used for remediation or extending student understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L528> This lesson has students explore products when one of the factors is 6. They also create a personal multiplication chart. It can be used for remediation or as an extension.

Constructing Task: At the Circus

TASK CONTENT: In this task, students will make diagrams to discover and demonstrate the answers to 2-digit by 2-digit or 4-digit by 1-digit multiplication story problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

MGSE4.NBT.5 Multiply a whole number of up to four digits by one –digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular, arrays, and /or area models.

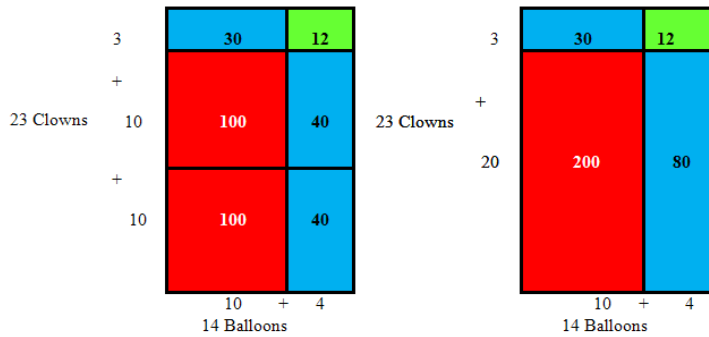
STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

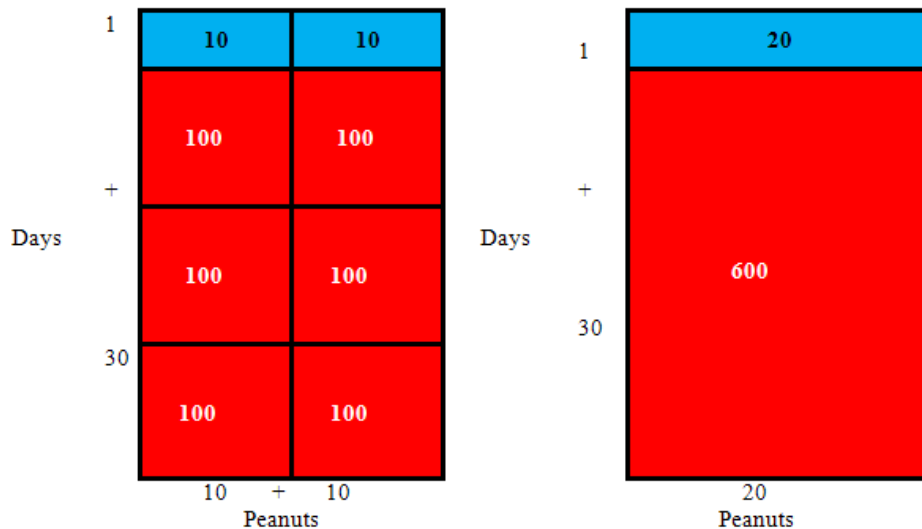
Students should understand how to use grid paper and partial product area models to determine multiplication products with numbers larger than 10. Use this task or another one similar to it to help students make the transition from depending on manipulatives for determining products of larger numbers to being able to determine these products through self-made diagrams. Based on their understanding of base ten blocks, students should draw a model similar to either of these for the clown and balloons problem:

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To get the final answer the students can add the areas:
 $100 + 100 + 30 + 40 + 40 + 12 = 322$ or $200 + 80 + 30 + 12 = 322$. There will be 322 balloons.

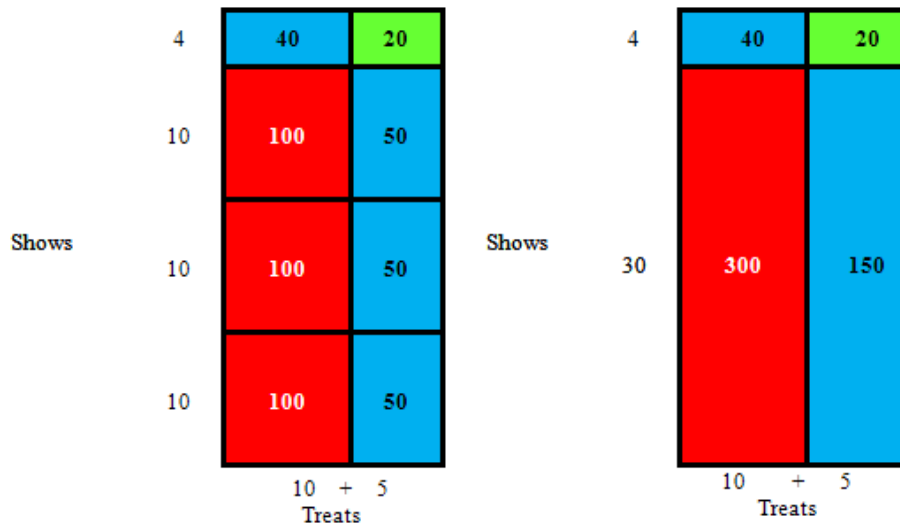
Below are two possible solutions for the Jumbo the elephant problem:



To get the final answer students can add the areas:
 $100 + 100 + 100 + 100 + 100 + 100 + 10 + 10 = 620$ or $600 + 20 = 620$.
 Jumbo's trainer needs to carry 620 peanuts with him.

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Below are two possible solutions for dancing bear family problem:



To get the final answer the students can add the areas:
 $100 + 100 + 100 + 50 + 50 + 50 + 40 + 20 = 510$ or $300 + 150 + 40 + 20 = 510$.
 The dancing bear family will receive 510 treats from the trainer during 15 shows.

ESSENTIAL QUESTIONS

- How will diagrams help us determine and show the products of **larger** numbers?

MATERIALS

- Colored pencils, markers, or crayons
- Centimeter Grid Paper
- “At the Circus” recording sheet

GROUPING

Partner or Individual Task

NUMBER TALKS

Chapter 7 of Sherry Parrish’s *Number Talks* provides additional background knowledge about array models and partial products. Also the talks on pgs. 272-275 provide students with more practice utilizing a partial product strategy. However, avoid having students multiply numbers with two digit factors until after the School Store Task. Students are expected to discover their own strategy for multiplying a two digit number by another two digit number with that task.

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will make diagrams to discover and demonstrate the answers to 2-digit by 2-digit or 4-digit by 1-digit multiplication story problems.

Comments

This task provides opportunities for students to work with arrays in real world situations as they work with larger numbers. The recording sheet also asks students to develop a story problem of their own.

The idea of moving beyond building arrays with base-ten blocks to drawing rectangles on paper or grid paper is critical. At this point students must begin to visualize the multiplication process without the blocks. As students begin to work, they may realize that modeling problems such as these can require a large number of base-ten blocks. Ask them to think of ways to do the same problem without having to utilize base-ten blocks.

Task Directions

Students will follow the directions below from the “At the Circus” recording sheet.

Solve the following problems. Show your thinking using words, pictures, and/or numbers.

1. There are 110 clowns at the circus. Each clown is carrying 4 balloons. How many balloons are there altogether?
2. The dancing bear family loves when their trainer gives them little treats to reward them for a good performance. If the trainer gives the dancing bear family 34 treats each show, how many treats will the trainer need for 22 shows?
3. Jumbo the elephant loves peanuts. His trainer gives him 6 peanuts **after every performance**. If **Jumbo has performed in 1342 shows**, how many peanuts **has the trainer given Jumbo**?
4. The husband and wife team, The Flying Pendulum Duo, has to perform a daring trapeze act. Mrs. Pendulum has to complete a total of 15 flips while in the air for the performance. They will travel to 32 cities to conduct one performance a night. Once the circus stint is over, how many flips will Mrs. Pendulum have completed?
5. Create your own circus problem and solution to share with the class.

FORMATIVE ASSESSMENT QUESTIONS

- What are the dimensions of the array? How do you know?
- Describe what each dimension in your array represents.
- What groups are being counted in your word problem? How do you know?
- How many times is each group in your word problem being counted? How do you know?
- How are you using colors to keep your data organized?
- How can colors help you identify the number of groups and the number of times each group is counted?

DIFFERENTIATION

Extension

- Ask students to try to solve a multiplication problem using a different method. Encourage students to apply another student's strategy to solve a given problem.

Intervention

- All students will not be ready at the same time to discontinue use of the base-ten manipulatives. The teacher will need to be conscientious about monitoring each student's level of understanding to know when the student will be ready to transition to diagrams without manipulatives support. If students experience frustration or uncertainty during this process, the manipulatives should be kept available for use as reinforcement or as a way to check the diagram.

TECHNOLOGY

- <http://www.ixl.com/math/grade-4/multiply-a-2-digit-number-by-a-2-digit-number-word-problems> This resource provides word problems involving the multiplication of two 2 - digit numbers. It can be used as additional practice of the concept.
- <http://www.teachersdomain.org/resource/vt107.math.algebra.pat.arrange1/> This resource is a video demonstrating the concept of multiplication.
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Name _____

Date _____

At the Circus

Solve the following problems. Show your thinking using words, pictures and/or numbers.

1. There are 110 clowns at the circus. Each clown is carrying 4 balloons. How many balloons are there altogether?

2. The dancing bear family loves when their trainer gives them little treats to reward them for a good performance. If the trainer gives the dancing bear family 34 treats each show, how many treats will the trainer need for 22 shows?

3. Jumbo the elephant loves peanuts. His trainer gives him 6 peanuts **after every performance**. If **Jumbo has performed in 1342 shows**, how many peanuts **has the trainer given Jumbo**?

4. The husband and wife team, The Flying Pendulum Duo, has to perform a daring trapeze act. Mrs. Pendulum has to complete a total of 15 flips while in the air for the performance. They will travel to 32 cities to conduct one performance a night. Once the circus stint is over, how many flips will Mrs. Pendulum have completed?

5. Create your own circus problem and solution to share with the class.

Constructing Task: School Store

TASK CONTENT: Students explore their understanding of multiplication and how it applies to multiplying 2 digit numbers by 2-digit numbers and 1-digit up to 4-digit whole numbers.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.

- a. Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.
- b. Represent verbal statements of multiplicative comparisons as multiplication equations.

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

MGSE4.NBT.5 Multiply a whole number of up to four digits by one–digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular, arrays, and /or area models.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
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3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
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BACKGROUND KNOWLEDGE

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays Area	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

ESSENTIAL QUESTIONS

- How can I effectively explain my mathematical thinking and reasoning to others?
- What patterns do I notice when I am multiplying whole numbers that can help me multiply more efficiently?
- What real life situations require the use of multiplication?

MATERIALS

- “School Store” recording sheet

GROUPING

Partner or Individual Task

NUMBER TALKS

Chapter 7 of Sherry Parrish’s *Number Talks* provides additional background knowledge about array models and partial products. Also the talks on pgs. 272-275 provide students with more practice utilizing a partial product strategy. However, avoid having students multiply numbers with two digit factors until after this task. Students are expected to discover their own strategy for multiplying a two digit number by another two digit number with this task.

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students explore their understanding of multiplication and how it applies to multiplying 2 digit numbers by 2-digit numbers and 1-digit up to 4-digit whole numbers.

Comments

This is an opportunity for students to use what they know about multiplication to find the product of 1-digit and 2-digits by up to 4-digit whole numbers. This task should be completed **before** students have any experiences with the standard algorithm for multiplying two digit numbers.

There should be no instruction on how to multiply 1 to 2 digit by up to 4-digit before giving students this problem. An example of how students may solve this type of problem is provided below.

As students work on this problem, talk with the students about their thinking and how they know their answers are correct. Also, it is important for students to share their thinking with their classmates

Task Directions

Students will follow the directions below from the “School Store” recording sheet.

Use what you know about multiplication to solve the following problems.

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1. Jeni sells 25 pencils each day at the school store. How many pencils will she sell after 14 days?
2. Marquis runs the school store. He noticed they sold around 521 pens every day. If he wants to order as few pens as possible to get through 3 weeks of school, how many pens should he order?
3. There are 1,647 students at Eagle Creek Elementary School. Sixty-eight students shopped at the Eagle's Nest school store each day. How many students shopped at the school store after 5 days?

Below is one possible way students can solve the problems for this task. This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.

Examples:

Unknown Product: A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? ($3 \times 6 = p$).

Group Size Unknown: A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? ($18 \div p = 3$ or $3 \times p = 18$).

Number of Groups Unknown: A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? ($18 \div 6 = p$ or $6 \times p = 18$).

When distinguishing multiplicative comparison from additive comparison, students should note that Additive comparisons focus on the difference between two quantities (e.g., Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?). A simple way to remember this is, "How many more?"

Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other (e.g., Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?). A simple way to remember this is "How many times as much?" or "How many times as many?"

FORMATIVE ASSESSMENT QUESTIONS

- How do you know your answer is correct?
- What strategies are you using to solve this problem?
- What properties of multiplication have you used to solve this problem?

DIFFERENTIATION

Extension

- Have students show as many different representations as they can and develop additional strategies as needed.
- Have students practice each of the strategies with numbers in the hundreds and thousands. Numbers larger than this can usually be handled with the use of the

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calculator, which should also be introduced once students have mastered the basic algorithm.

Intervention

- Have students solve the problem by inserting smaller numbers into the problem, then having them to choose two of the strategies to solve the problems

TECHNOLOGY

- <http://www.ixl.com/math/grade-4/multiply-a-2-digit-number-by-a-2-digit-number-word-problems> This resource provides word problems involving the multiplication of two 2-digit numbers. It can be used as additional practice of the concept.
- <http://www.teachersdomain.org/resource/vt107.math.algebra.pat.arrange1/> This resource is a video demonstrating the concept of multiplication.
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Name _____ Date _____

School Store



Use what you know about multiplication to solve the following problems.

1. Jeni sells 25 pencils each day at the school store. How many pencils will she sell after 14 days?

2. Marquis runs the school store. He noticed they sold around 521 pens every day. If he wants to order as few pens as possible to get through 3 weeks of school, how many pens should he order?

3. There are 1,647 students at Eagle Creek Elementary School. Sixty-eight students shopped at the Eagle's Nest school store each day. How many students shopped at the school store after 5 days?

Constructing Task: Sensible Rounding

TASK CONTENT: This task provides several contexts in which students will have to determine the best estimation for the situation.

STANDARDS OF MATHEMATICAL CONTENT

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

MGSE4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

STANDARDS OF MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

A problem such as $44 \div 7$ produces a calculator answer of 6.285714286 and the traditional division algorithm will produce 6r2. For any practical purposes, this number must be rounded. Yet there is no one rule for doing this. This is because the context always suggests the reason for rounding and the degree of precision required of the answer. For example, if the context is money, the degree of precision might be to the nearest hundredth (penny). The students need to display very good number sense and understanding of real problems to round calculator answers sensibly.

ESSENTIAL QUESTIONS

- What is a sensible answer to a real problem?
- What information is needed in order to round a whole number to any place?
- How can I ensure my answer is reasonable?
- What effect does a remainder have on my rounded answer?

MATERIALS

- Sensible Rounding recording sheet
- calculator

GROUPING

Partner or small group

NUMBER TALKS

Several number talks strategies can help student build a stronger understanding of division. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information refer to pgs. 286-299 in *Number Talks*. (Number Talks, 2010, Sherry Parrish)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments:

For students to be able to round accurately, “rounding should be flexible and well understood conceptually” (Van de Walle, 246). In order for students to conceptually understand rounding, they must be engaged in context to allow them to make sense of this concept. This task provides several contexts in which students will have to determine the best estimation for the situation. With these estimations, students will use the most familiar form of estimation, rounding (Van de Walle, 241).

Task directions:

Students will follow the directions below from the “Sensible Rounding” recording sheet.

Use an empty number line to work through the following problems. Discuss the answers carefully with your partner or group.

“Jane has 44 liters of milk to share among seven families. How much does she measure out for each family?” (*In traditional rounding “rules” students would round this amount to 7 liters. However, this context proves this rounded amount would be inaccurate because there is not enough milk for each family to receive 7 liters.*)

“The market gardener sends 44 tons of potatoes to eight supermarkets. How much does he send to each?” The market gardener would need x amount of tons of potatoes in order for each supermarket to receive 6 tons of potatoes. What amount does x represent? How do you know?

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(Students will determine each supermarket will receive about 5 tons of potatoes with 4 tons left over for the gardeners. Students can use the remainder to help determine the value for x , or they can use the relationship of 6×8 to help them identify 48 as the value of x .)

“The service station sells seven large pizzas for \$44. About how much does a pizza cost?” Assume the service station charges to the nearest \$1.

“John shares 44 Tootsie Pops among seven children. How many does each child get?”
(You cannot cut up Tootsie Pops, so $6r6$ Tootsie pops ≈ 6 Tootsie pops. Have students discuss how many would be left over and who would get the extra Tootsie pops.)

“Joel has 44 cookies to share among seven people. He needs to get rid of all the cookies. How many cookies does each person receive?”*(Here $6r6$ is very inappropriate as an answer because he must get rid of all the cookies. Everyone receives six cookies. There are two whole cookies left over so probably these should be cut. Students need to decide if this amount can be considered 6 cookies or 7 cookies.)*

FORMATIVE ASSESSMENT QUESTIONS

- What is the problem asking you?
- Does your answer make sense? How do you know?
- How does rounding help you in this context?
- Did you get the same answer for $44 \div 7$ each time you encounter it? Why or why not?

DIFFERENTIATION

Extension

- For each of these division problems, create word problems that are solved by the division yet the rounding rules change with the context. $2,225 \div 17$; $4,567 \div 29$; $7,888 \div 11$...
- Have students use a calculator to divide and discuss how the results given by the calculator are not sensible answers. Students must explain a more sensible answer based on the context.

Intervention

- Allow students to use manipulatives to simulate the division contexts and discuss a sensible rounded answer.

TECHNOLOGY

- <http://www.bbc.co.uk/schools/ks1bitesize/numeracy/division/index.shtml> This resource provides differentiated activities for 3 levels. It can be used for remediation or additional practice.
- http://www.mathplayground.com/TB_MD/tb_md5_iFrame.html Watch a short video that explains how to use thinking blocks to interpret remainders when solving division word problems. Then practice modeling and solving problems. The video can be used to show other strategies for dividing or as an introduction to the task.

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- <http://illuminations.nctm.org/ActivityDetail.aspx?ID=224> This is a game which can be used to practice the concept of division.

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Sensible Rounding

Directions: Use an empty number line to work through the following problems. Discuss the answers carefully with your partner or group.

1. Jane has 44 liters of milk to share among seven families. How much does she give to each family?

2. The market gardener sends 44 tons of potatoes to eight supermarkets. How much does he send to each?

The market gardener would need x amount of tons of potatoes in order for each supermarket to receive 6 tons of potatoes. What amount does x represent? How do you know?

3. The service station sells seven large pizzas for \$44. About how much does a pizza cost? Assume the service station charges to the nearest \$1.

4. John shares 44 Tootsie Pops among seven children. How many does each child get?

5. Joel has 44 cookies to share among seven people. All cookies must be shared. How many cookies does each person receive?

Constructing Task: Compatible Numbers to Estimate

TASK CONTENT: This task deals with using friendly numbers to divide.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.OA.3 **Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.**

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Compatible numbers are easy to use and are chosen to simplify the calculation of an estimate. Students should understand that sensibly chosen compatible numbers are not right or wrong, though some may yield better estimates than others. For example, the quotient $407 \div 5$ may be estimated using the compatible numbers $400 \div 5$, $420 \div 6$, $350 \div 7$, $400 \div 4$, or any of several other pairs. The first two, however, yield estimates closer to the actual quotient than the others do.

ESSENTIAL QUESTIONS

- How can I mentally compute a division problem?
- What are compatible numbers and how do they aid in dividing whole numbers?

MATERIALS

- “Compatible Numbers” recording sheet

GROUPING

Individual or partner

NUMBER TALKS

Several number talks strategies can help student build a stronger understanding of division. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information refer to pgs. 286-299 in *Number Talks*. (Number Talks, 2010, Sherry Parrish)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Comments

Ask students how they could estimate the number of small prizes each of Mr. Wong’s 9 students would receive if he had exactly 893 prizes to give away. If no one mentions compatible numbers, remind the class that they can estimate the answer to a problem by replacing the numbers in the problem with numbers that are easier to calculate with. Such easier numbers are called compatible numbers. You might show these two examples of compatible numbers:

- To find an estimate for $263 \div 5$, students may say that 263 is close to 300. 300 is a compatible number for the divisor 5 because 300 is a multiple of 5. $300 \div 5$ is a problem that can be solved mentally. The estimated quotient will be about 60. Some students may want to round 263 to 250. 250 is also compatible with 5 because it is a multiple of 5. $250 \div 5$ can also be solved mentally. $25 \div 5$ is 5. 250 is ten times more than 25. Since the divisor is still 5, the quotient will be ten times more than $25 \div 5$, which is 50. Discuss the fact that now, you have a range that the quotient will fall in between. The actual quotient will be more than 50, but less than 60.
- To estimate $3,456 \div 7$, students might recognize 3,456 is close to 3,500 and choose compatible numbers 3,500 and 7. So, $3,456 \div 7$ is about $3,500 \div 7$, or 500.

Task Directions

Students will follow the directions below from the “Compatible Numbers” recording sheet.

1. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year. He will give away as many prizes as possible. Estimate the number of small prizes each of Mr. Wong’s 9 students would receive if he had exactly 893 prizes to give away.
2. At Hatfield Elementary School, there are 504 students in 7 classes. Each class has the same number of students. What is a good estimate of the number of students in each class? Explain your reasoning.
3. Marcel worked 9 hours and earned \$232. What is a good estimate of the amount that he earned each hour? Explain your reasoning.

FORMATIVE ASSESSMENT QUESTIONS

- What compatible numbers are you using? **Why did you choose those numbers?**
- How did these compatible numbers make solve the problem easier?
- Do you think that is a reasonable estimate? Why?

DIFFERENTIATION

Extension

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- Have students solve the following problem with an estimate which fits the context. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year.. He will give away as many prizes as possible. What is the greatest number of prizes that could be left over? Is it possible for each student to get 200 prizes?

Intervention

- Have students link basic division facts to identifying compatible numbers. You can begin with $35 \div 7$, then $350 \div 7$. Make explicit the connection of the compatibility between 35 and 7 and how it can be applied to 350 and 7.

TECHNOLOGY

- <http://www.bbc.co.uk/schools/ks1bitesize/numeracy/division/index.shtml> This resource provides differentiated activities for 3 levels. It can be used for remediation or additional practice.
- http://www.thinkingblocks.com/ThinkingBlocks_MD/TB_MD_Main.html Watch a short video that explains how to use thinking blocks to interpret remainders when solving division word problems. Then practice modeling and solving problems. The video can be used to show other strategies for dividing or as an introduction to the task.

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Compatible Numbers

Use compatible numbers to help you estimate the answers for the following problems.

1. Mr. Wong has between 300 and 1,000 small prizes to divide evenly among his 9 students over the course of the school year. He will give away as many prizes as possible. Estimate the number of small prizes each of Mr. Wong's 9 students would receive if he had exactly 893 prizes to give away.

2. At Hatfield Elementary School, there are 504 students in 7 classes. Each class has the same number of students. What is a good estimate of the number of students in each class? Explain your reasoning.

3. Marcel worked 9 hours and earned \$232. What is a good estimate of the amount that he earned each hour? Explain your reasoning.

Scaffolding Task: Brain Only!

TASK CONTENT: In this task, students analyze multiplication and division expressions to find patterns and make connections among division and multiplication problems.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NBT.5 Multiply a whole number of up to four digits by one –digit whole number, and multiply two tow digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular, arrays, and /or area models.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.OA.2 **Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.**

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example: A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will be in each box?

Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

Using Place Value: $260 \div 4 = (200 \div 4) + (60 \div 4)$

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Using Multiplication: $4 \times 50 = 200$, $4 \times 10 = 40$, $4 \times 5 = 20$; $50 + 10 + 5 = 65$; so $260 \div 4 = 65$

Some students may need to use manipulatives and other strategies to model their thinking until they begin to “see” the patterns and understand what is happening. The teacher should not point these relationships out for the students. Instead, they should guide the thinking of the students through questions and allow students to discuss their thinking with peers; the teacher should act as a facilitator.

Example:

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many students are on each team?

<u>Student 1</u>	<u>Student 2</u>	<u>Student 3</u>
592 divided by 8 There are 70 8s in 560 $592 - 560 = 32$ There are four 8s in 32 $70 + 4 = 74$	592 divided by 8 I know that ten 8s is 80 If I take out fifty 8s that is 400 $592 - 400 = 192$ I can take out 20 more 8s which is 160 $192 - 160 = 32$ 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74.	Student 3 I want to get to 592 $8 \times 25 = 200$ $8 \times 25 = 200$ $8 \times 25 = 200$ $200 + 200 + 200 = 600$ $600 - 8 = 592$ I had 75 groups of 8 and took one away, so there are 74 teams.

ESSENTIAL QUESTIONS

- How are multiplication and division related to each other?
- What are some simple methods for solving multiplication and division problems?
- What patterns of multiplication and division can assist us in problem solving?

MATERIALS

- “Brain Only!” recording sheet

GROUPING

Small Group or Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students analyze multiplication and division expressions to find patterns and make connections among division and multiplication problems.

Comments

It is critical for students to understand the relationship that exists between multiplication and division as well as the strong relationship between the dividend, divisor, and quotient. This task is designed to allow students to further explore these relationships.

Task Directions

Students will follow the directions below from the “Brain Only!” recording sheet.

It’s true! You can really use one problem to solve related problems just by using our brain!

Use this problem to answer the following ones with your brain.

$$1240 \div 4 = 310$$

1. Be able to explain the relationship of each problem to the one above.

$$1240 \div 8 = q$$

$$620 \div d = 155$$

$$155 \times 4 = p$$

$$620 \div 2 = q$$

$$310 \times 8 = p$$

$$620 \div d = 310$$

2. Make up at least 3 more problems that are related to these.
-

3. Swap with your partner and see if you can use only your brain to solve their related problems.

4. Be able to explain the relationship of each of your partner’s problems, too.

FORMATIVE ASSESSMENT QUESTIONS

- What patterns do you notice in the sets of numbers?
- How is each multiplication or division expression related to the others?
- What shortcuts do you think you can learn from analyzing these expressions?
- Can you think of other related multiplication or division problems?

DIFFERENTIATION

Extension

- Ask students if they think these ideas extend to larger numbers. Encourage them to use four and five digit numbers in the problems they create for the final step. Have them explain their thinking when choosing the numbers they did and provide evidence that the pattern continues even when the numbers become much larger.

Intervention

- The multiplication chart is an excellent visual cueing device that can be used to help students discover patterns in multiplication and division. Do not neglect to use this excellent teaching tool frequently for students who have not developed the ability to see connections and relationships between numbers easily.

TECHNOLOGY

- <http://illuminations.nctm.org/Lesson.aspx?id=2621> In this lesson, students generate products using a number line model. It can be used as remediation, additional practice or in an independent center.
- <http://www.mathcats.com/grownupcats/ideabankmultiplication.html> This resource provides additional ideas for approaching multiplication instruction. This could be used in small group review or additional practice.

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Brain Only!

It's true! You can really use one problem to solve related problems just by using our brain!

Use this problem to answer the following ones with your brain.

$$1240 \div 4 = 310$$

1. Be able to explain the relationship of each problem to the one above.

$$1240 \div 8 = q$$

$$620 \div d = 155$$

$$155 \times 4 = p$$

$$620 \div 2 = q$$

$$310 \times 8 = p$$

$$620 \div d = 310$$

2. Make up at least 3 more problems that are related to these.

3. Swap with your partner and see if you can use only your brain to solve their related problems.

4. Be able to explain the relationship of each of your partner's problems, too.

Constructing Task: What is $2,500 \div 300$?

TASK CONTENT: Students explore why dividing by zero is undefined.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.OA.2 **Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.**

MGSE4.OA.3 **Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.**

MGSE4.OA.5 **Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.**

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

When students simply memorize the rule to simplify division involving numbers ending with zeroes by eliminating the same number of 0s from both numbers, they often produce an incorrect answer as shown in this problem. They do not realize that $2,500 \div 300$ and $25 \div 3$ are equivalent expressions. When we solve $2,500 \div 300$, we can think about using hundreds as a unit. Thus, when we use $25 \div 3$ to solve $2,500 \div 300$, we are indeed asking, “How many groups of 3 hundreds can we make with 25 hundreds?” Thus, the remainder must also be interpreted with the unit of the dividend and the divisor; that is, there is 1 hundred leftover. The remainder for $2500 \div 300$ must be 100.

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	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays Area	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

ESSENTIAL QUESTIONS

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- How can we find evidence to support our conclusions?
- What happens in division when there are zeroes in both the divisor and the dividend?
- How are remainders and divisors related?
- What is the meaning of a remainder in a division problem?

MATERIALS

- “What is $2,500 \div 300$?” recording sheet

GROUPING

Partner or Individual Task

NUMBER TALKS

Several number talks strategies can help student build a stronger understanding of division. Repeated subtraction, partial quotients, multiplying up and proportional reasoning are all valuable strategies that students can explore through number talks. For more information refer to pgs. 286-299 in *Number Talks*. (Number Talks, 2010, Sherry Parrish)

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Part 1

Students explore why dividing by zero is undefined.

Task Directions

Start this task with a whole group discussion regarding division and zero. Remind students about the relationship between multiplication and division and the product when one of the factors is zero. Then pose the following problem.

What happens when you divide by zero?
Use multiplication to justify your answer.
Were you surprised by your findings? Why or why not?

Regardless what approach the students elect to take with this task, when justifying using multiplication, they should notice that something strange happens. For instance, if they choose to divide using a dividend of 12, they may discover something similar to the following:

$12 \div 12 = 1$	$12 = 12 \times 1$
$12 \div 6 = 2$	$12 = 6 \times 2$
$12 \div 4 = 3$	$12 = 4 \times 3$
$12 \div 3 = 4$	$12 = 3 \times 4$
$12 \div 2 = 6$	$12 = 2 \times 6$
$12 \div 1 = 12$	$12 = 1 \times 12$

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$$12 \div 0 = ? \qquad 12 = 0 \times ?$$

If students choose to use a fact family they will have similar results.

If $2 \times 4 = 8$, then $8 \div 4 = 2$.

If $4 \times 2 = 8$, then $8 \div 2 = 4$.

Following the same pattern:

If $0 \times 8 = 0$, then $0 \div 8 = 0$

If $8 \times 0 = 0$, but $0 \div 0 \neq 8$

After students have had time to explore division with zero and share their findings, explain that because any number multiplied by zero is zero, the last situation in both examples is impossible. **Therefore, division by zero is not possible and we call it “undefined”.**

Some students may have noticed that occasionally when using a computer or calculator, they are given an error message. Frequently that is caused by situations such as dividing by zero. Now that they understand why this may happen, they may enjoy playing with a computer or calculator to verify this message.

History also has interesting situations that have occurred due to division by zero. The USS Yorktown had a divide by zero error on September 21, 1997. This caused the ship’s propulsion system to fail.

Part II

Students determine whether a child’s work is mathematically sound and give evidence for their conclusions.

Task Directions

Students will follow the directions below from the “What is $2,500 \div 300$?” recording sheet. Steven says the answer to $2500 \div 300$ is 8, with a remainder of 1. He said, “My reason is because you can just cross out two 0s in both numbers to make it $25 \div 3$. The answer to $25 \div 3$ is 8, with a remainder of 1.” Is he correct? Why or why not?

FORMATIVE ASSESSMENT QUESTIONS

- What is the value of the one left over? How do you know?
- What is the number expression you are solving?
- Is it helpful to cancel out zeroes if you have them in both the dividend and divisor? Why or why not?
- What effect does a set of zeroes in the divisor and dividend have on the quotient? On the remainder?

DIFFERENTIATION

Extension

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- Using a problem such as 208 divided by 30, have students explain and give evidence for ignoring the zeroes or using them as a shortcut to solving this division problem. Require an explanation that uses multiple representations to explain their conclusions.

Intervention

- Have students to use smaller numbers 50 divided by 10, have student to model or illustrate the problem and explain their thought process in solving the problem.

TECHNOLOGY

- <http://www.homeschoolmath.net/operation-game.php> This resources uses a variety of operations to solve problems. This can be used as practice.
- <http://www.toonuniversity.com/flash.asp?err=517&engine=13> This is an activity students can complete to practice solving multiplication problems with multiples of ten. It can be used as an introduction to this task or for additional practice.

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What is $2,500 \div 300$?

Steven says the answer to $2500 \div 300$ is 8, with a remainder of 1. He said, “My reason is because you can just cross out two 0s in both numbers to make it $25 \div 3$. The answer to $25 \div 3$ is 8, with a remainder of 1.” Is he correct? Why or why not?

3 ACT TASK: Boxes and Rolls

TASK CONTENT: Determine the amount of money contained within boxes and rolls of coins. Approximate time, 1 class period.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

BACKGROUND KNOWLEDGE

This task follows the 3-Act Math Task format originally developed by Dan Meyer. More information on this type of task may be found at <http://blog.mrmeyer.com/category/3acts/>. A Three-Act Task is a whole-group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three. More information along with guidelines for 3-Act Tasks may be found in the *Guide to Three-Act Tasks* on georgiastandards.org and the K-5 CCGPS Mathematics Wiki.

ESSENTIAL QUESTIONS

- How can we use clues and reasoning to find an unknown number?
- What patterns do I notice when multiplying whole numbers which help me multiply more efficiently?

MATERIALS

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- 3Act Student Recording Sheet
- Act 1 image
- Act 2 images

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments

In this task, students will be shown a picture and then tell what they noticed. Next, they will be asked to discuss what they wonder about or are curious about. Their curiosities will be recorded as questions on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Task Directions

Act I – Whole Group - Pose the conflict and introduce students to the scenario by showing Act I picture.

1. Students are shown the Act 1 image.
2. Pass out the 3 Act recording sheet.
3. Ask students what they wonder about and what questions they have about what they saw. Students should share with each other first before sharing aloud and then record these questions on the recording sheet (think-pair-share). The teacher may need to guide students so that the questions generated are math-related.

Anticipated questions:

- How many pennies are there?
 - How many rolls are there?
 - How much money is in the boxes?
 - How much money is in the rolls?
 - *How many pennies are in the boxes and rolls?
***Main question to be answered.**
4. As the facilitator, you can select which question you would like every student to answer, have students vote on which question the class will answer or allow the students to pick which question they would like to answer. Once the question is selected ask students to estimate answers to their questions (think-pair-share). Students will write their best estimate, then write two more estimates – one that is too low and one that is too high so that they

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establish a range in which the solution should occur. Instruct students to record their estimates on a number line.

Act II – Student Exploration - Provide additional information as students work toward solutions to their questions.

1. Ask students to determine what additional information they will need to solve their questions. The teacher provides that information only when students ask for it:
 - Each box has 50 rolls of pennies.
 - Show students the images of penny rolls.

2. Ask students to work in small groups to answer the question(s) they created in Act I. The teacher provides guidance as needed during this phase by asking questions such as:
 - Can you explain what you've done so far?
 - What strategies are you using?
 - What assumptions are you making?
 - What tools or models may help you?
 - Why is that true?
 - Does that make sense?

Act III – Whole Group - Share student solutions and strategies as well as Act III solution.

1. Ask students to present their solutions and strategies.
2. Share solution in Act III solution. (*6500 pennies*)
3. Lead discussion to compare these, asking questions such as:
 - How reasonable was your estimate?
 - Which strategy was most efficient?
 - Can you think of another method that might have worked?
 - What might you do differently next time?

Comments

Act IV is an extension question or situation of the above problem. An Act IV can be implemented with students who demonstrate understanding of the concepts covered in acts II and III. The following questions and/or situations can be used as an Act IV:

- How much money in dollars and cents is in the boxes and rolls?
- **What are different combinations of coins that could be used instead of 6,500 pennies?**

FORMATIVE ASSESSMENT QUESTIONS

- What models did you create?
- What organizational strategies did you use?

DIFFERENTIATION

Extension

- Students can complete **a suggested Act IV situation or create their own Act IV situation.**

Intervention

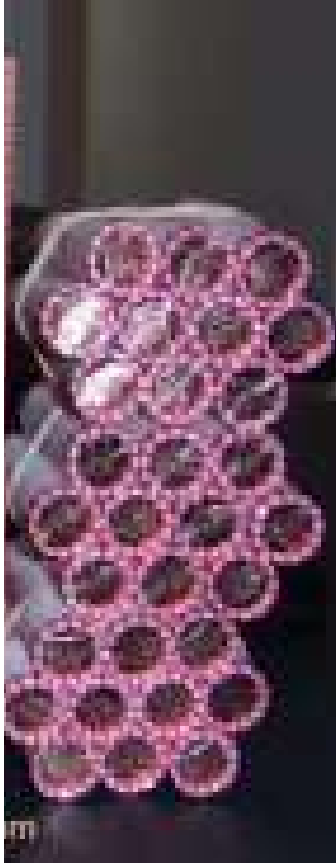
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- Allow students to model a roll of pennies using manipulatives.

Act One Image



Act 2 Images



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Name: _____

Adapted from Andrew Stadel

Task Title:

ACT 1

What questions come to your mind?

Main question:

On an empty number line, record an estimate that is too low, just right and an estimate that is too high. Explain your estimates.

ACT 2

What information would you like to know or need to solve the MAIN question?

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Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Record the actual answer on the number line above containing the three previous estimates.

ACT 4 (use this space when necessary)

Constructing Task: Number Riddles

TASK CONTENT: Students use logic, specifically deductive reasoning, to solve number riddles. This task gives students an opportunity synthesize their skills with multiples, factors, place value, and numeric expressions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. Students should understand the process of finding factor pairs so they can do this for any number 1-100,

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Multiples: 1, 2, 3, 4, 5...24

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

3, 6, 9, 12, 15, 18, 21, 24

4, 8, 12, 16, 20, 24

8, 16, 24

12, 24

24

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of 4
- all numbers ending in 0 or 5 are multiples of 5

ESSENTIAL QUESTIONS

- How can we use clues and reasoning to find an unknown number?

MATERIALS

- “Number Riddles” Recording Sheet

GROUPING

Individual/Partner Task

NUMBER TALKS

This task allows kids to review several number talks strategies. Revisit chapter 8 and explore whatever strategy is most appropriate for your class, as well as any addition and subtraction strategies that seem applicable and appropriate.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Students use logic, specifically deductive reasoning to solve number riddles. This task gives students an opportunity to synthesize their skills with multiples, factors, place value, and numeric expressions.

Comments

This task provides students with an opportunity to consolidate their understanding of factors and multiples, prime and composite numbers, place value, and problem solving. In order for students to be successful with this task, they should have an understanding of the elements required to solve the problem on the task sheet.

Solutions given below:

Riddle #1 463	Riddle #2 43	Riddle #3 7632
Riddle #4 485	Riddle #5 193	Riddle #6 347
Riddle #7 813	Riddle #8 1205	

Task Directions

Students work individually or in pairs to solve the number riddles provided on the “Number Riddles” task sheet.

FORMATIVE ASSESSMENT QUESTIONS

- How can you find the factors of a number? Multiples of a number?
- How do you know if a number is even? Odd?
- What operation do you use if you are finding a sum? Difference?

DIFFERENTIATION

Extension

- Have students write their own riddles on separate index cards. These can be collected and used as warm-ups, centers, problem solving activities, etc.
- These types of riddles can be used as warm-up problems to reinforce critical thinking and mathematical vocabulary. They can be adapted to include a variety of mathematical concepts.

Intervention

- Have students use the “Number Riddles” Recording Sheet as a tic-tac-toe task where students need to solve three riddles in a row, column, or diagonal.

TECHNOLOGY

- <http://calculationnation.nctm.org/Games/Game.aspx?GameId=A0537FC6-3B08-4AFC-9AD6-0CC5E3BC9B86> Factor Dazzle is a strategic game to help students learn about finding factors and multiples. It can be used to extend understanding or for remediation purposes.
- <http://www.teachersdomain.org/resource/vtI07.math.algebra.pat.lparrange/> This resource requires students to use manipulatives to figure out patterns in problems. It can be used to extend understanding of the concept.

Number Riddles Recording Sheet

Use the clues to solve these number riddles.

<p>Riddle #1 – I am a 3-digit whole number between 400 and 650. My hundreds digit is divisible by 2 but not 3. My tens digit is a multiple of 3. My ones digit is $\frac{1}{2}$ of my tens digit. All of my digits are different. The sum of my digits is 13. What number am I?</p>	<p>Riddle #2 – I'm a 2-digit whole number between 30 and 80. My tens digit is one more than my ones digit. I am a prime number. What number am I?</p>	<p>Riddle #3 – I am a four-digit whole number greater than 6000. My thousands digit is prime. My ones digit is the only even prime number. I am divisible by 4. My tens digit is $\frac{1}{2}$ my hundreds digit. All of my digits are different. What number am I?</p>
<p>Riddle #4 – I'm an odd number between 250 and 700. I am divisible by 5. My tens digit is 3 more than my ones digit. The sum of my digits is 17. What number am I?</p>	<p>Riddle #5 – I am between 150 and 375. All of my digits are odd. My tens digit is 3 times my ones digit. My hundreds digit is my smallest digit. None of my digits is the same. What number am I?</p>	<p>Riddle #6 – I am a three-digit whole number between 300 and 500. My ones digit is the largest single digit prime number. My tens digit is even. The sum of my digits is 14. What number am I?</p>
<p>Riddle #7 – I am a 3-digit odd number greater than 800. My tens digit is 2 less than my ones digit. I am divisible by 3 but not 5. The sum of my digits is 12. What number am I?</p>	<p>Riddle #8 – I am a four-digit whole number. I am divisible by 5 but not 10. My thousands digit is neither prime nor composite. My hundreds digit is 2 more than my tens digit. The sum of my digits is 8. What number am I?</p>	<p style="text-align: center;"><u>Challenge</u></p> <p>Create two riddles of your own. Include the answer to your riddle on the back of the paper.</p>

Constructing Task: Earth Day Project

TASK CONTENT: In this activity, students consider a real-world situation involving a set of data. Using the data, students determine the pattern formed by the numbers in the data set. Then they extend the pattern and use the pattern to make predictions.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

BACKGROUND KNOWLEDGE

Students should have prior experiences working with and extending patterns. Also, students should be able to graph points easily. After points are graphed, ask students if it is appropriate to connect the points. (In this situation it is not appropriate because students collect cans just once a day and they do not (typically) collect a fraction of a can. However, students may want to line up the points along the edge of a ruler or sheet of paper to make predictions using the graph.

ESSENTIAL QUESTIONS

- How can we determine the relationships between numbers?
- How can we use patterns to solve problems?
- How can we describe a pattern?

MATERIALS

- “Earth Day Project” student recording sheet, 3 pages
- Blocks to use to build the pattern (for those students who wish to use them)

GROUPING

Individual/Partner Task

NUMBER TALKS

This task allows kids to review several number talks strategies. Revisit chapter 8 and explore whatever strategy is most appropriate for you class, as well as any addition and subtraction strategies that seem applicable and appropriate. (Number Talks, 2010, Sherry Parrish)

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this activity, students consider a real-world situation involving a set of data. Using the data, students determine the pattern formed by the numbers in the data set. Then they extend the pattern and use the pattern to make predictions.

Comments

This activity may be used as an assessment, as a learning task, or as an independent follow-up activity for reinforcement or review, depending on the instruction that occurred prior to this task.

For Teacher information only:

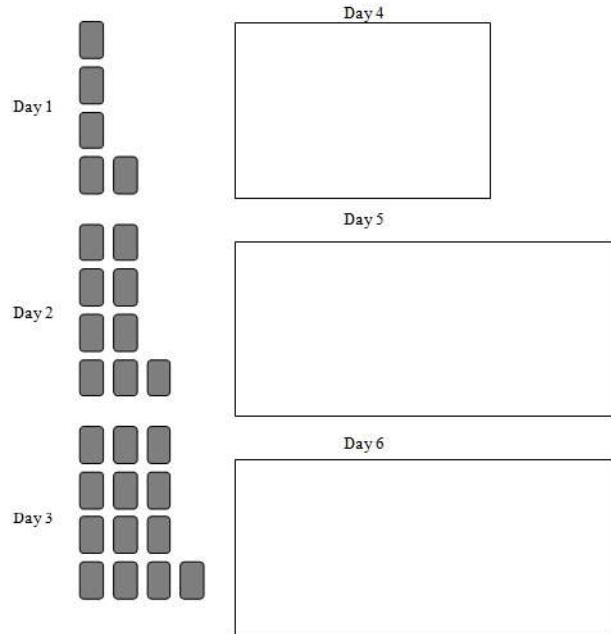
- Teachers should give some thought to this pattern before presenting this problem to their students. Start by looking at the relationship of the numbers in the two columns. Teachers should try to express this relationship in words. See the examples below.
- Some students may think about the pattern in this way:
 - Day 1 shows a column of 4 and 1 more.
 - Day 2 shows 2 columns of 4 and 1 more.
 - Day 3 shows 3 columns of 4 and 1 more
 - Day 4 will show 4 columns of 4 and 1 more.Therefore the pattern is generated by $4 \times n + 1$, where n represents the number of the day. While it is not expected that students will be able to generalize this pattern to an expression (except possibly as an extension for some students), asking students to talk about what they see changing/growing in the pattern is important to help them develop an awareness of the structure of a pattern.
- Keep in mind some students may see the pattern differently. For example, it is possible for students to describe it as follows:
 - Day 1 shows a 2×4 rectangle with 3 missing.
 - Day 2 shows a 3×4 rectangle with 3 missing.
 - Day 3 shows a 4×4 rectangle with 3 missing.
 - Day 4 will show a 5×4 rectangle with 3 missing.Of course, this can be written as $(n + 1) \times 4 - 3$, with the n representing the number of the day. Using the distributive property gives you $4 \times n + 4 - 3$, which is the same as $4 \times n + 1$. Asking students about their thinking is a good way to understand how students see the relationship of the numbers in the two columns.

Task Directions

Students will follow the directions below from the “Earth Day Project” student recording sheet.

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Fourth graders in Ms. Smith’s class have decided to start a recycling project for Earth Day. They put a bin in the cafeteria to collect used aluminum cans. At the end of each school day, they take the bin back to their classroom and count the cans collected for the day. Ms. Smith’s class is keeping notes about how many cans are being collected. It seems that the number of cans collected each day follows a pattern. If the pattern continues, sketch the number of cans collected on days 4-6 in the boxes below.



The students recorded the number of cans they collected each day in the t-table below. When they collect one hundred cans, the students can turn them in to the recycling center and earn money to be used for an upcoming field trip. If the pattern continues how many days will it take to collect at least 100 cans?

Day	Cans
1	5
2	9
3	13
4	17
5	21
6	_____
7	_____
8	_____
9	_____
10	_____
⋮	_____
⋮	_____
⋮	_____
20	_____
⋮	_____
⋮	_____
100	_____

- Use what you know about the cans collected in the first five days to make a prediction about how many days it will take to collect at least 100 cans. Show your work and explain in words why you predicted the number of days that you chose.
- Continue the pattern in the t-table. Fill in the missing values.
- Explain how you found the missing values in the t-table.
- How many days will it take the class to collect enough cans for the field trip. Show all work and explain your thinking.
- On the graph paper below, label the horizontal axis “Number of Days,” label the vertical axis “Number of Cans.” Label the horizontal axis 1-25 by 1s; label the vertical axis 1-100 by 5s. Make sure you start at zero. Plot the number of cans collected each day for days 1 - 5.

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If the pattern continues, use the graph to predict the number of cans the students will collect on the 25th day.

FORMATIVE ASSESSMENT QUESTIONS

- What is changing each day in the pattern?
- How many cans will be collected on day 4? How do you know? How will the pattern look?
- How did you complete the chart? How do you know you are correct?
- What do you notice about the numbers in each column? What do you notice about how the numbers in each row are related?
- How did you find the number of cans collected on day 20? On day 100? How do you know your answers are correct?
- How do you plot points on a coordinate plane?

DIFFERENTIATION

Extension

- Asks students to write in words what is happening in the pattern (i.e. each day the number of cans increases by 4; the number of cans each day can be found by multiplying the day number by 4 and adding 1 or the expression $4 \times \text{day number} + 1$ where is the day number.). Also, ask students to make other predictions based on the graph and check their predictions using the expression $4 \times \text{day number} + 1$.

Intervention

- Some students will benefit by using manipulatives to help them demonstrate the problem with concrete objects prior to drawing a model or attempting to extend the pattern.

TECHNOLOGY

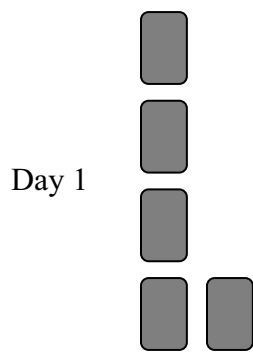
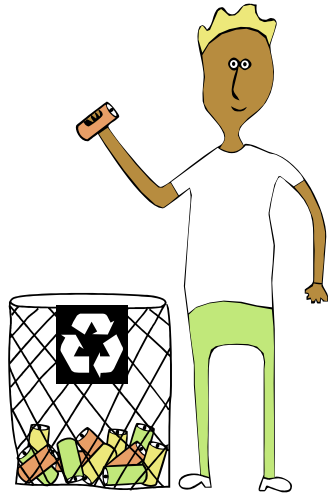
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L305> This lesson requires students to analyze numeric patterns. It can be used for additional practice, remediation or extending understanding of the concept.
- <http://illuminations.nctm.org/LessonDetail.aspx?ID=L327> This interactive lesson has students find patterns in the multiplication tables. IT can be used for additional practice, remediation or extending understanding.
- <http://www.prometheanplanet.com/en-us/Resources/Item/74887/function-machines> This activity can be used to assess basic knowledge of function machines following a lesson.

Name _____ Date _____

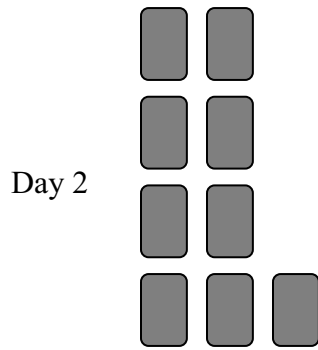
Earth Day Project

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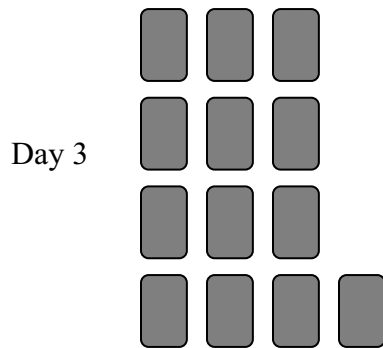
It seems that the number of cans collected each day follows a pattern. If the pattern continues, sketch the number of cans collected on days 4-6 in the boxes below.



Day 4



Day 5



Day 6

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The students recorded the number of cans they collected each day in the t-table below. When they collect one hundred cans, the students can turn them in to the recycling center and earn money to be used for an upcoming field trip. If the pattern continues how many days will it take to collect at least 100 cans?

- Use what you know about the cans collected in the first five days to make a prediction about how many days it will take to collect at least 100 cans. Show your work and explain in words why you predicted the number of days that you chose.

Day	Cans
1	5
2	9
3	13
4	17
5	21
6	_____
7	_____
8	_____
9	_____
10	_____
.	_____
20	_____
.	_____
100	_____

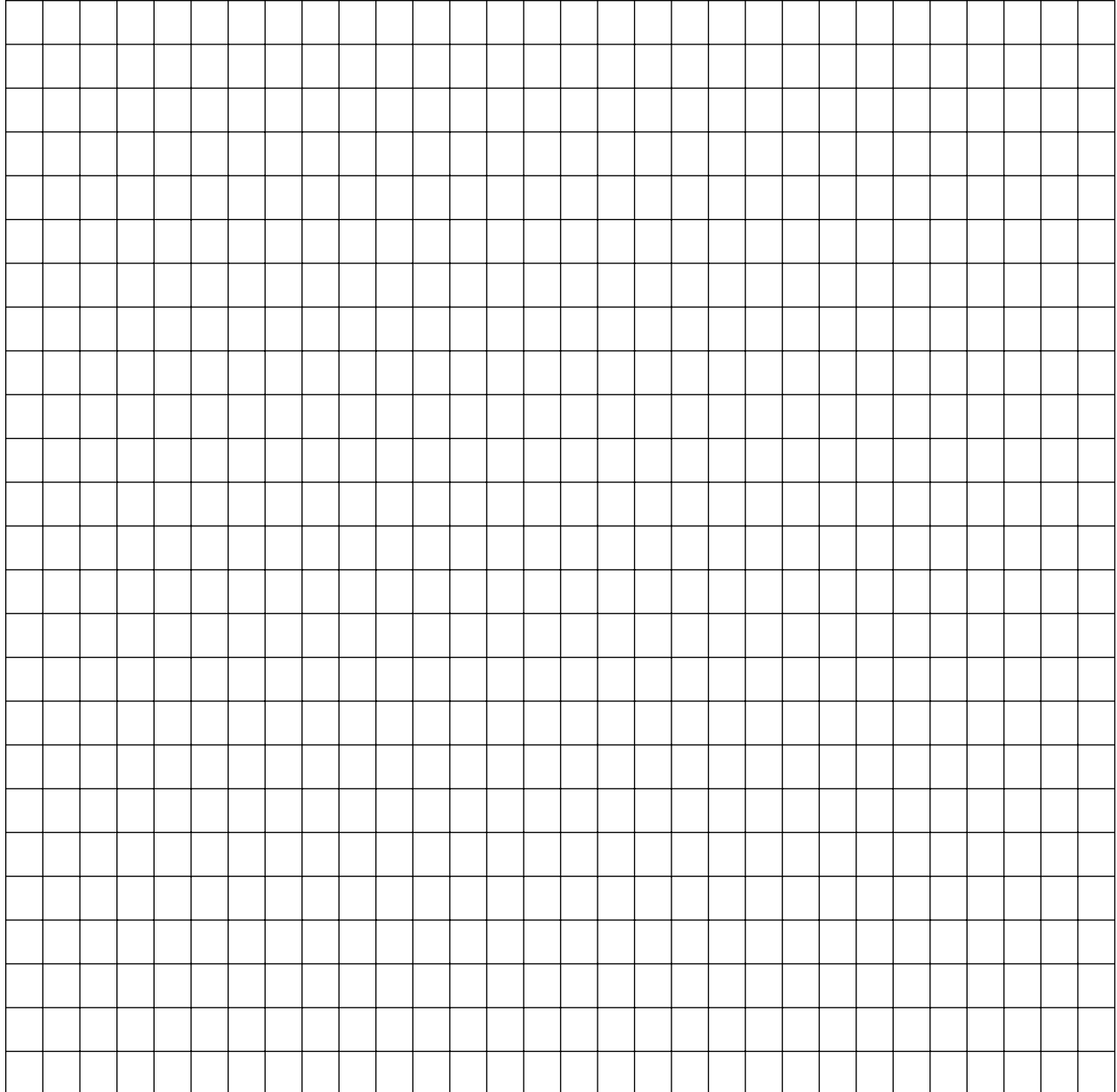
- Continue the pattern in the t-table. Fill in the missing values.
- Explain how you found the missing values in the t-table.

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5. On the graph paper below, label the horizontal axis “Number of Days,” and label the vertical axis “Number of Cans.” Label the horizontal axis 1-25 by 1s; label the vertical axis 1-100 by 5s. Make sure you start at zero. Create a bar graph of the number of cans collected each day for days 1 - 5. If the pattern continues, use the graph to predict the number of cans the students will collect on the 25th day.



Culminating Task: School Newspaper

TASK CONTENT: Multiplication, division, and rounding are essential elements of this task.

STANDARDS FOR MATHEMATICAL CONTENT

MGSE4.OA.1 Understand that a multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity.

- a. Interpret a multiplication equation as a comparison e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.
- b. Represent verbal statements of multiplicative comparisons as multiplication equations.

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison. Use drawings and equations with a symbol or letter for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Explain informally why the pattern will continue to develop in this way. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers.

MGSE4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

STANDARDS FOR MATHEMATICAL PRACTICE TO BE EMPHASIZED:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

ESSENTIAL QUESTIONS

- How do multiplication, division, and estimation help solve real world problems?
- How can we organize our work when solving a multi-step word problem?
- How can a remainder affect the answer in a division problem?

MATERIALS

“School Newspaper” recording sheet

GROUPING

Partner or Individual Task

NUMBER TALKS

This task allows students to continue to review several number talks strategies. You may wish to revisit chapter 8 of Number Talks by Sherry Parish, and explore strategies appropriate for your class, as well as any addition and subtraction strategies that seem applicable and appropriate.

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

Comments

Multiplication, division, and rounding are essential elements of this task. You may wish to use the actual number of students within your school to make this task more meaningful for your class. Also, you may need to adjust the other numbers within the task to ensure the essential elements of the task are addressed.

While this task is intended to serve as a summative assessment, it also may be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them. This task is appropriate to use in a variety of ways, including:

- Peer Review
- Display
- Portfolio

Task Directions

Students will follow the directions below from the “School Newspaper” recording sheet.

Your class has agreed to publish a 6 page school-wide newspaper for each of the 1,740 students. Both sides of the paper will be printed to help save money. A package of 100 sheets of special newsprint paper costs \$1.

- Find the cost of publishing the newspaper if you can make the copies at school for no additional cost other than purchasing the paper. Explain how you know.

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- The students in your class do not have the money to pay for this much paper and your teacher does not have it in her budget. To solve this problem, it was agreed to ask local businesses to purchase advertisements to place in the newspaper. The prices that your class decided to charge are:
 - Full-page \$50
 - Half-page \$25
- What is the fewest number of advertisement pages that you will need in order to pay for the paper and not make more than \$25 over your expenses? How do you know?
- What is the largest number of pages that you will need to pay for the paper without going more than \$25 over what you need to spend? Explain your thinking.
- Show a combination of both sizes that would pay for the paper without using more pages than the largest number of pages needed to pay for the paper.

FORMATIVE ASSESSMENT QUESTIONS

- Can you describe your strategy for solving this problem?
- Explain how you know your answer is correct
- Is there any mental math you are using? Explain.
- How are you organizing the information in the problem to help you solve it?
- How do the remainders in this problem affect the answers?

DIFFERENTIATION

Extension

- How many extra copies of the newspaper could you make using the paper that you purchased? How do you know?
- If the school decided to charge \$20 for the use of the copy machine, how would this change your results?
- If each of the teachers in your school also wanted a copy of the newspaper, show how that would change your results.
- What would you need to charge for the advertisements if you wanted them to use no more than 2 pages? Tell why.
- If the school will not allow you to use the copy machine and you have to use a local printer, how would this affect the costs?

Intervention

- Have students work in small groups or with a partner.
- Adjust the number of copies needed for the newspapers.

Name _____ Date _____

School Newspaper

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